

Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

2nd Lecture / 2. Vorlesung

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Notation and Field Quantities / Notation und Feldgrößen

Vector / Vektor:
Electric Field Strength / Elektrische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = \underline{\mathbf{E}}_x(\underline{\mathbf{R}}, t) + \underline{\mathbf{E}}_y(\underline{\mathbf{R}}, t) + \underline{\mathbf{E}}_z(\underline{\mathbf{R}}, t)$$

3 Vector Components /
3 Vektorkomponenten

$$= E_x(x, y, z, t) \underline{\mathbf{e}}_x + E_y(x, y, z, t) \underline{\mathbf{e}}_y + E_z(x, y, z, t) \underline{\mathbf{e}}_z$$

mit $\{x, y, z\} = \{x_1, x_2, x_3\}$

$$= \sum_{i=1}^3 E_{x_i}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i}$$

$$= E_{x_i}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i}$$

Dyad / Dyade:
Permittivity Dyad / Permittivitätsdyade

$$\underline{\underline{\epsilon}}(\underline{\mathbf{R}}, t) = \underline{\underline{\epsilon}}_{xx}(\underline{\mathbf{R}}, t) + \underline{\underline{\epsilon}}_{xy}(\underline{\mathbf{R}}, t) + \underline{\underline{\epsilon}}_{xz}(\underline{\mathbf{R}}, t)$$

$$+ \underline{\underline{\epsilon}}_{yx}(\underline{\mathbf{R}}, t) + \underline{\underline{\epsilon}}_{yy}(\underline{\mathbf{R}}, t) + \underline{\underline{\epsilon}}_{yz}(\underline{\mathbf{R}}, t)$$

$$+ \underline{\underline{\epsilon}}_{zx}(\underline{\mathbf{R}}, t) + \underline{\underline{\epsilon}}_{zy}(\underline{\mathbf{R}}, t) + \underline{\underline{\epsilon}}_{zz}(\underline{\mathbf{R}}, t)$$

9 Dyadic Components /
9 dyadische Komponenten

$$= \epsilon_{xx}(x, y, z, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_x + \epsilon_{xy}(x, y, z, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_y + \epsilon_{xz}(x, y, z, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_z$$

$$+ \epsilon_{yx}(x, y, z, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_x + \epsilon_{yy}(x, y, z, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_y + \epsilon_{yz}(x, y, z, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_z$$

$$+ \epsilon_{zx}(x, y, z, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_x + \epsilon_{zy}(x, y, z, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_y + \epsilon_{zz}(x, y, z, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_z$$

mit $\{x, y, z\} = \{x_1, x_2, x_3\}$

$$= \sum_{i=1}^3 \sum_{j=1}^3 \epsilon_{x_i x_j}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i} \underline{\mathbf{e}}_{x_j}$$

$$= \epsilon_{x_i x_j}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i} \underline{\mathbf{e}}_{x_j}$$

with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

Einstein's Summation Convention: If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. /
Einsteinsche Summenkonvention: Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.

Governing Equations of Electromagnetic Fields and Waves / Grundgleichungen elektromagnetischer Felder und Wellen

Governing Equations in Differential Form / Grundgleichungen in Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = \frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) + \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \rho_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = \rho_m(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \rho_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \rho_m(\underline{\mathbf{R}}, t)$$

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}} - \iint_S \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}}$$

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dR}} = \iint_S \frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}} + \iint_S \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}}$$

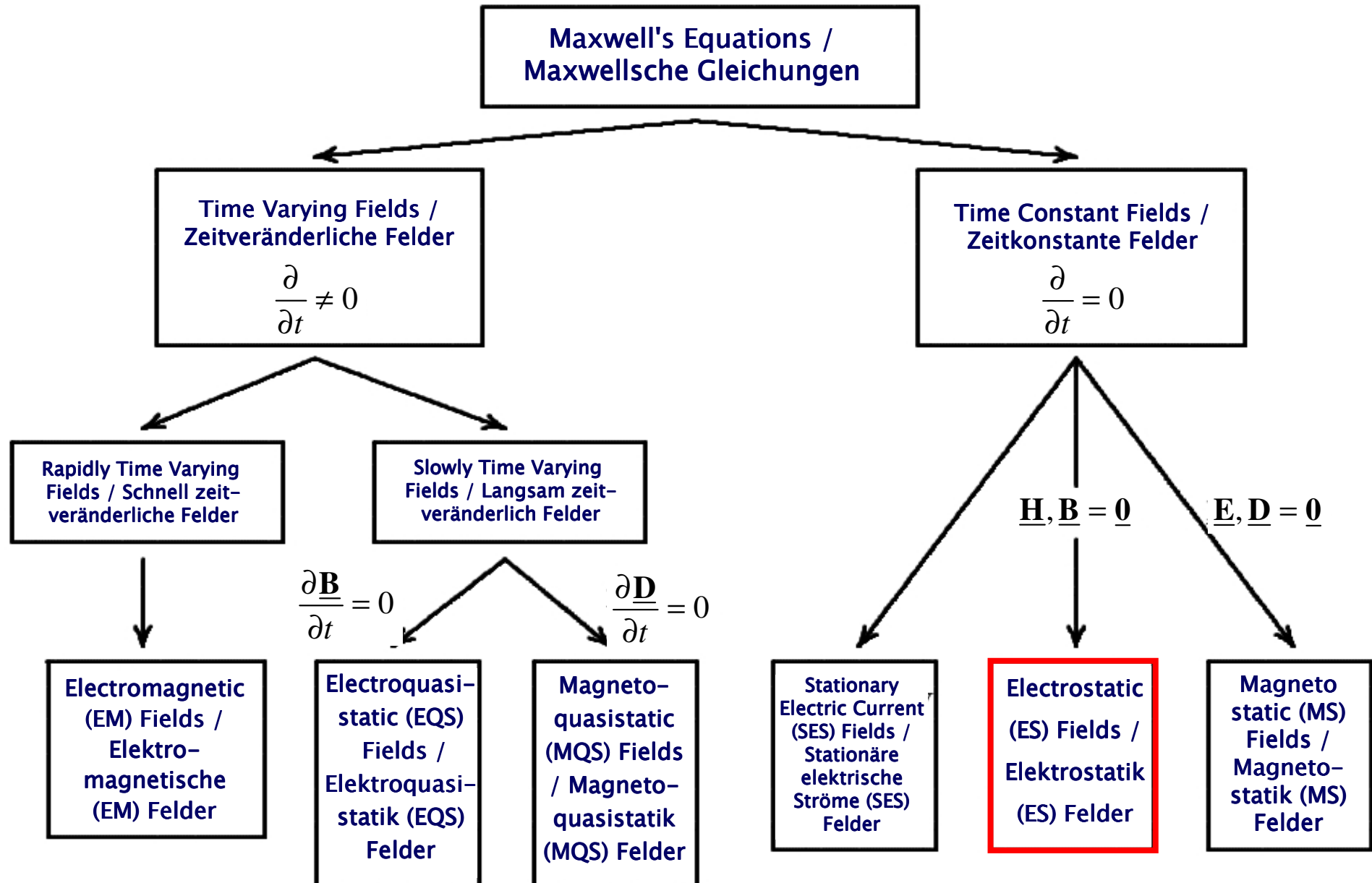
$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}}, t) dV$$

$$\oiint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_m(\underline{\mathbf{R}}, t) dV$$

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t} \rho_e(\underline{\mathbf{R}}, t) dV$$

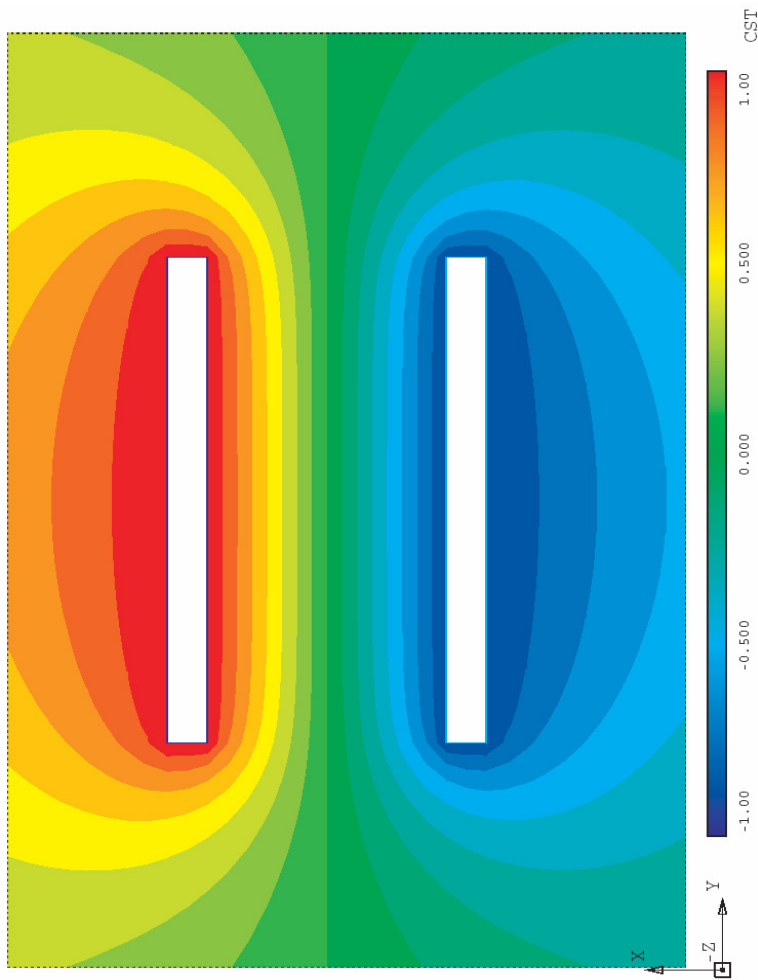
$$\oiint_{S=\partial V} \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t) \cdot \underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t} \rho_m(\underline{\mathbf{R}}, t) dV$$

Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen

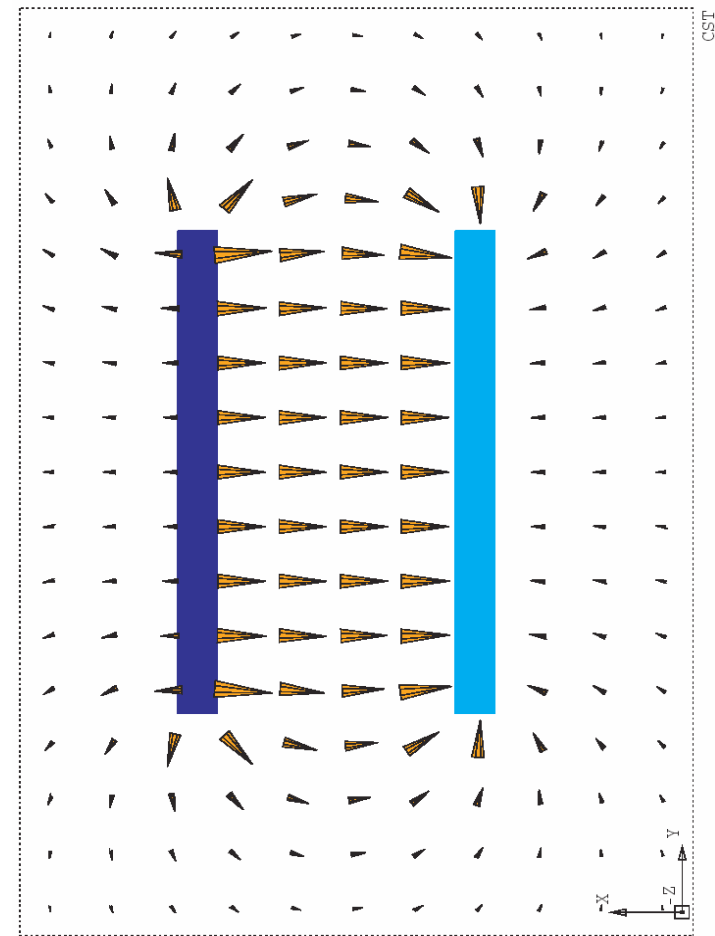


Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

Scalar Field: Electrostatic Potential /
Skalarfeld: Elektrostatisches Potenzial

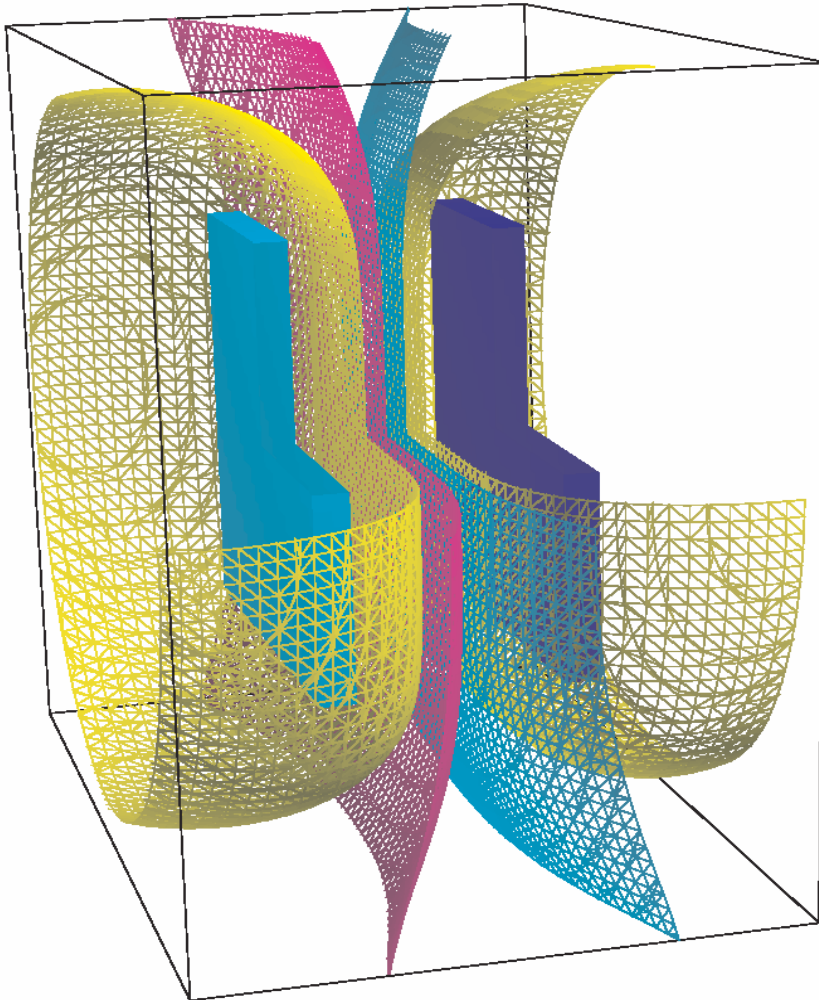


Vector Field: Electrostatic Field Strength /
Vektorfeld: Elektrostatische Feldstärke

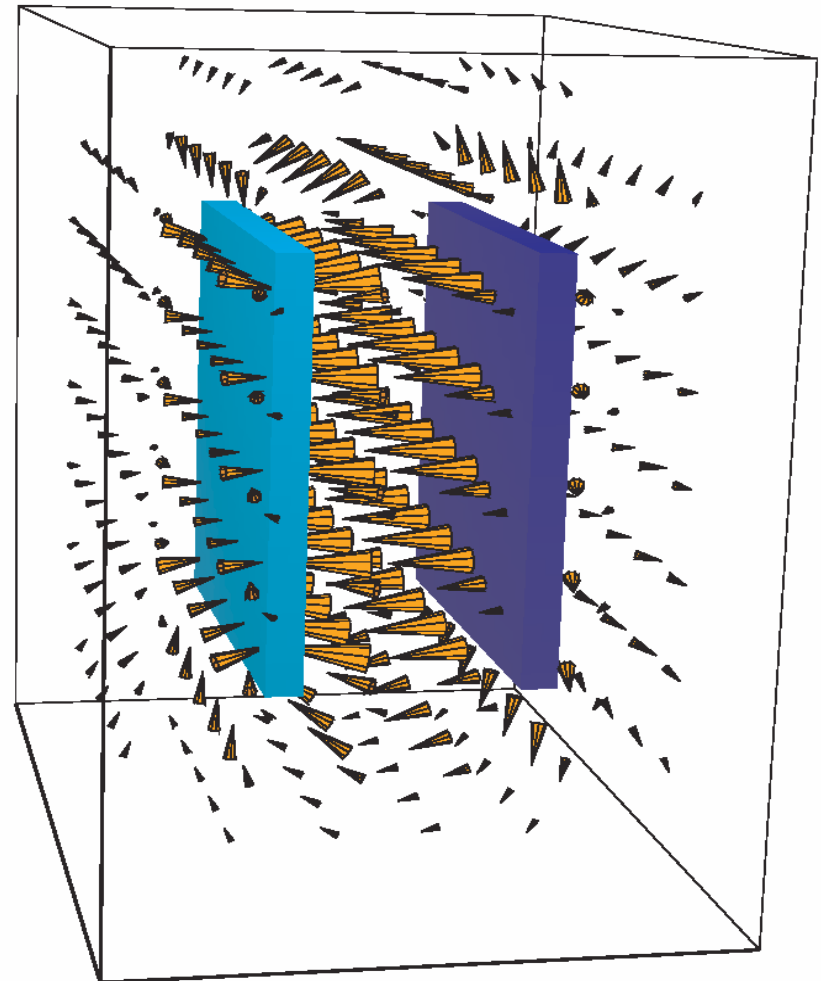


Example: Parallel Plate Capacitor – Electrostatic Field Problem / Beispiel: Paralleler Plattenkondensator – Elektrostatisches Feldproblem

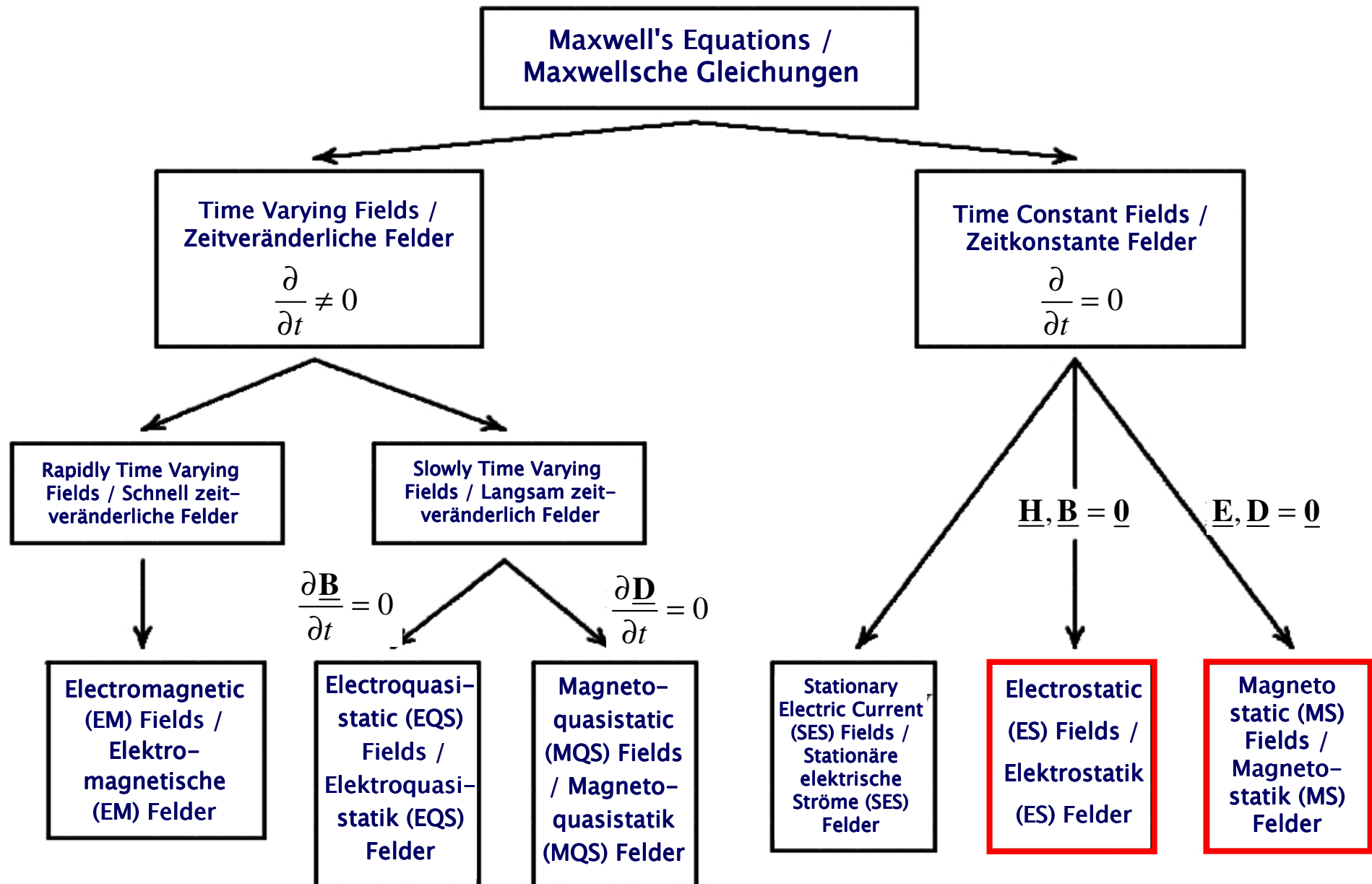
Scalar Field: Electrostatic Potential /
Skalarfeld: Elektrostatisches Potenzial



Vector Field: Electrostatic Field Strength /
Vektorfeld: Elektrostatische Feldstärke



Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen



Example: Spark Plug and Relay / Beispiel: Zündkerze und Relais

Electrostatic (ES) Fields /
Elektrostatische (ES) Felder

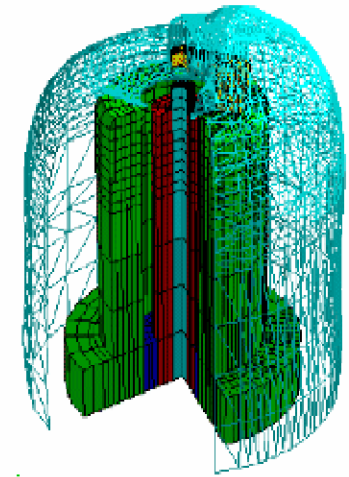
Spark Plug / Zündkerze

Magnetostatic (MS) Fields /
Magnetostatische (MS) Felder

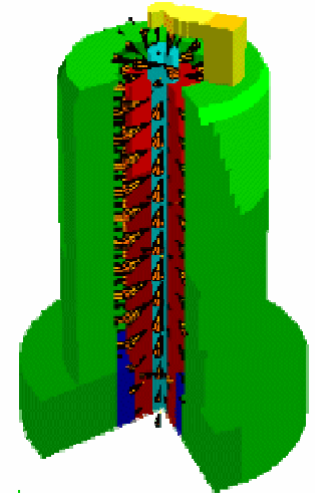
Relay / Relais



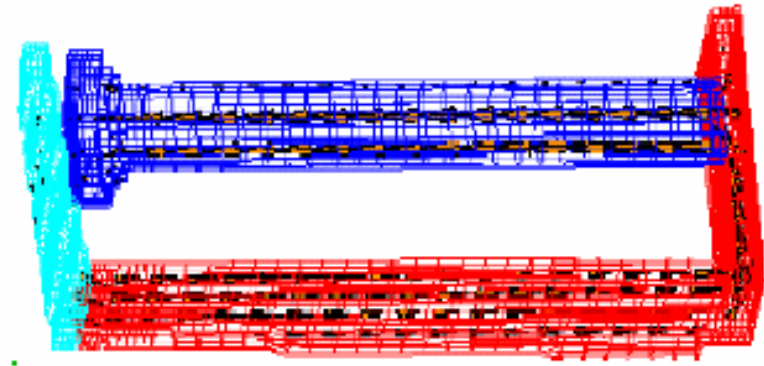
Elektric Field Strength /
Elektrische Feldstärke



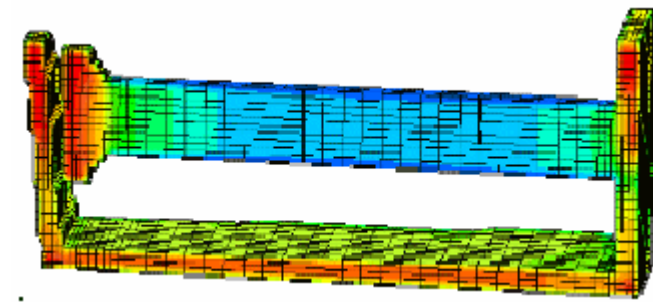
Elektric Potential /
Elektrisches Potenzial



Elektric Field Strength /
Elektrische Feldstärke

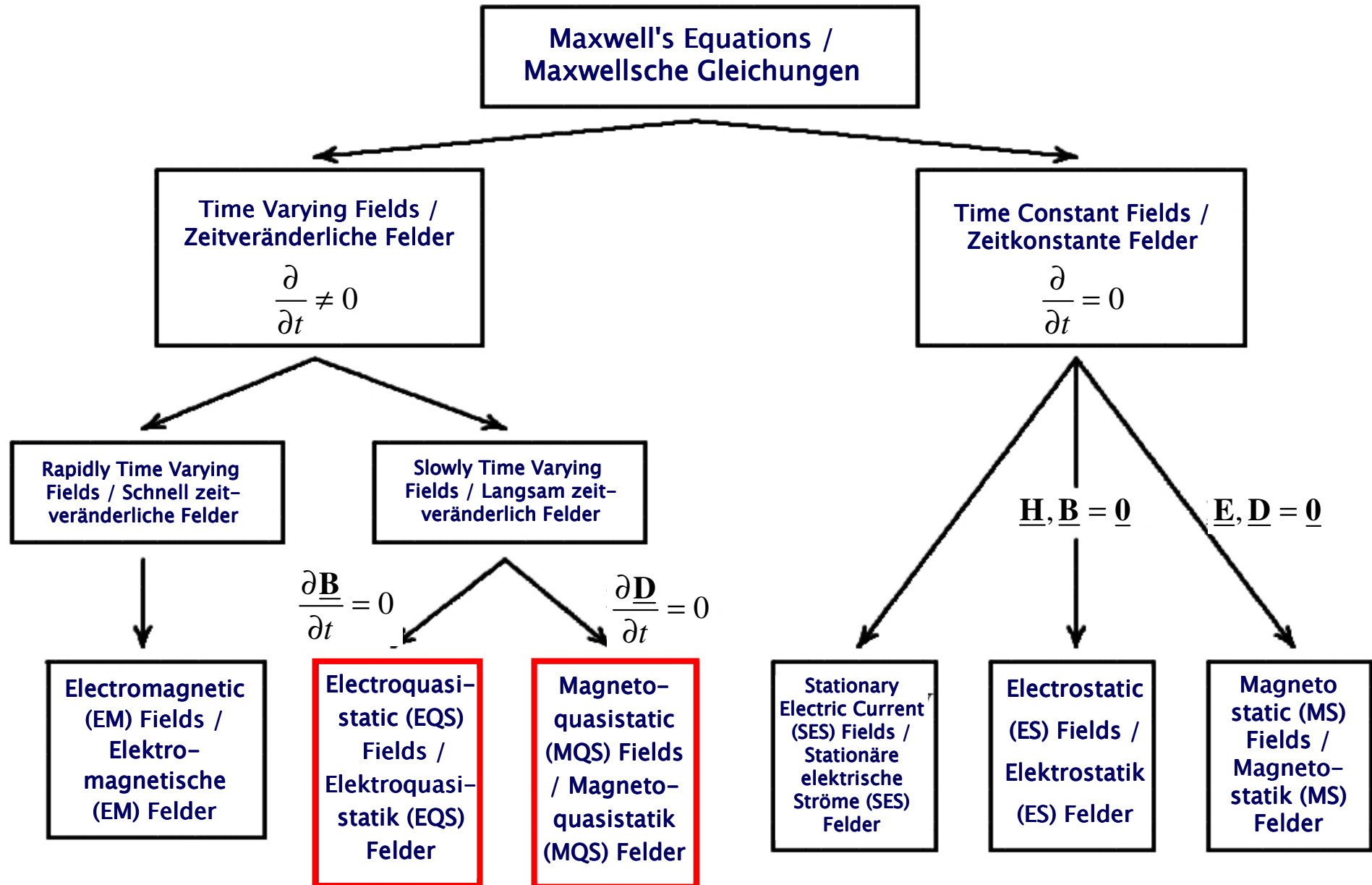


Magnetic Flux Density /
Magnetische Flussdichte



Permeability /
Permeabilität

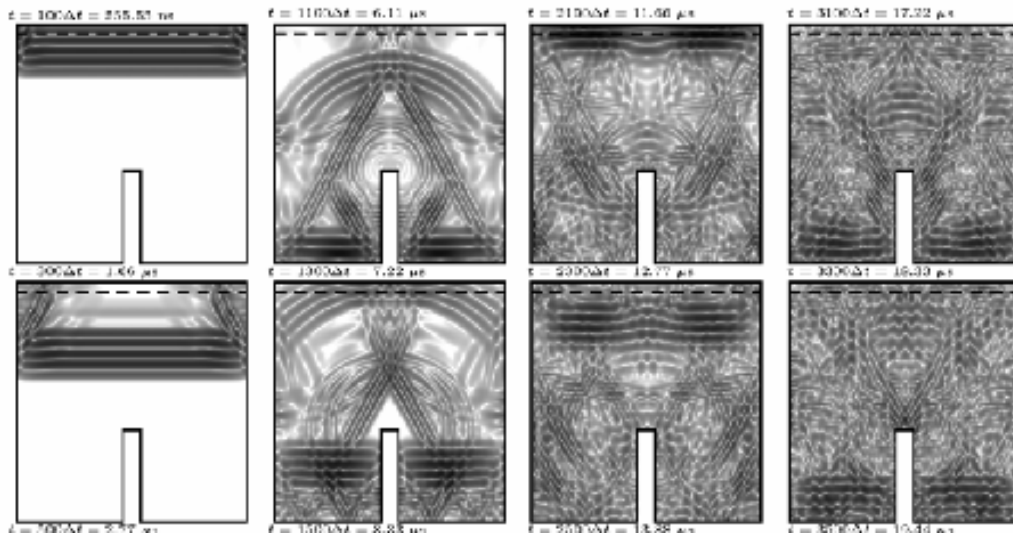
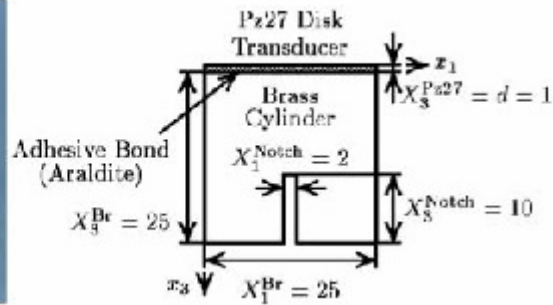
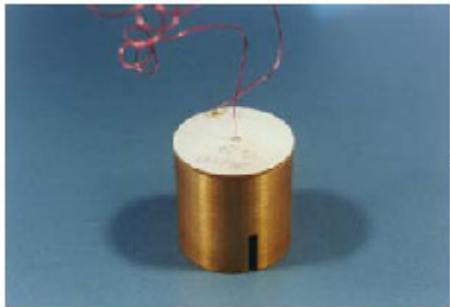
Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen



Example: Piezoelectric Sensor / Beispiel: Piezoelektrischer Sensor

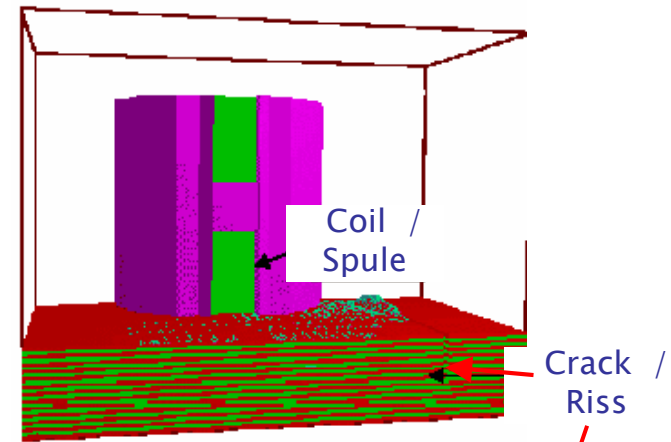
Electroquasistatic (EQS) Fields / Elektroquasistatische (EQS) Felder

Non-Destructive Testing: Piezoelectric Sensor / Zerstörungsfreie Materialprüfung: Piezoelektrischer Sensor

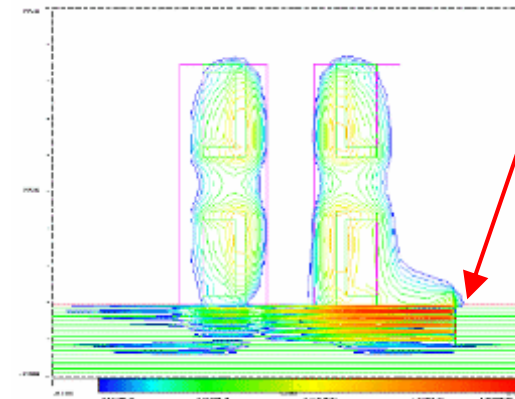


Magnetoquasistatic (MQS) Fields / Magnetoquasistatische (MQS) Felder

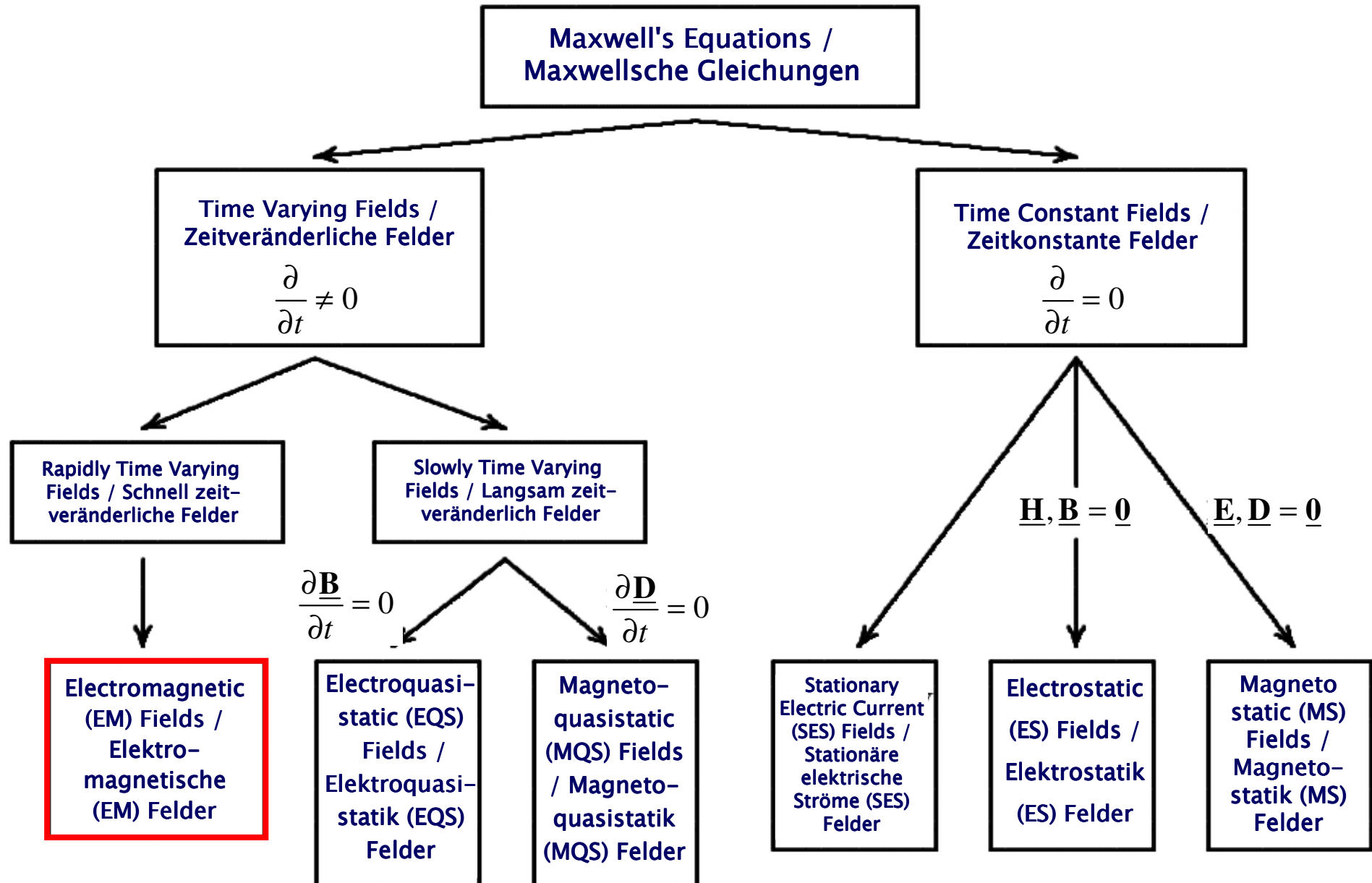
Non-Destructive Testing: Eddy Current Sensor / Zerstörungsfreie Materialprüfung: Wirbelstromsensor



Geometry / Geometrie



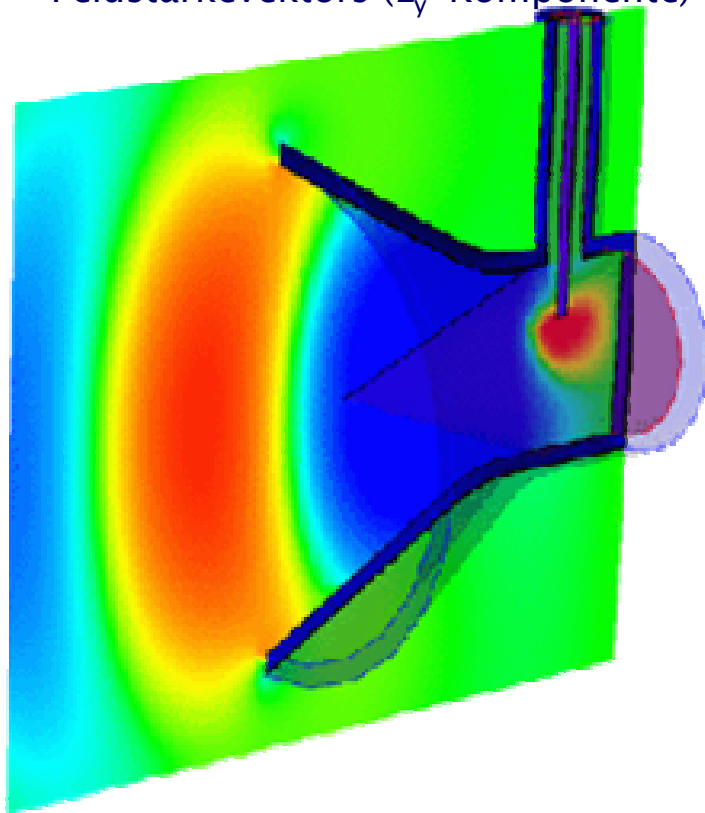
Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen



Examples: Antenna and Human Head Interaction / Beispiele: Antenne und Interaktion mit menschlichem Kopf

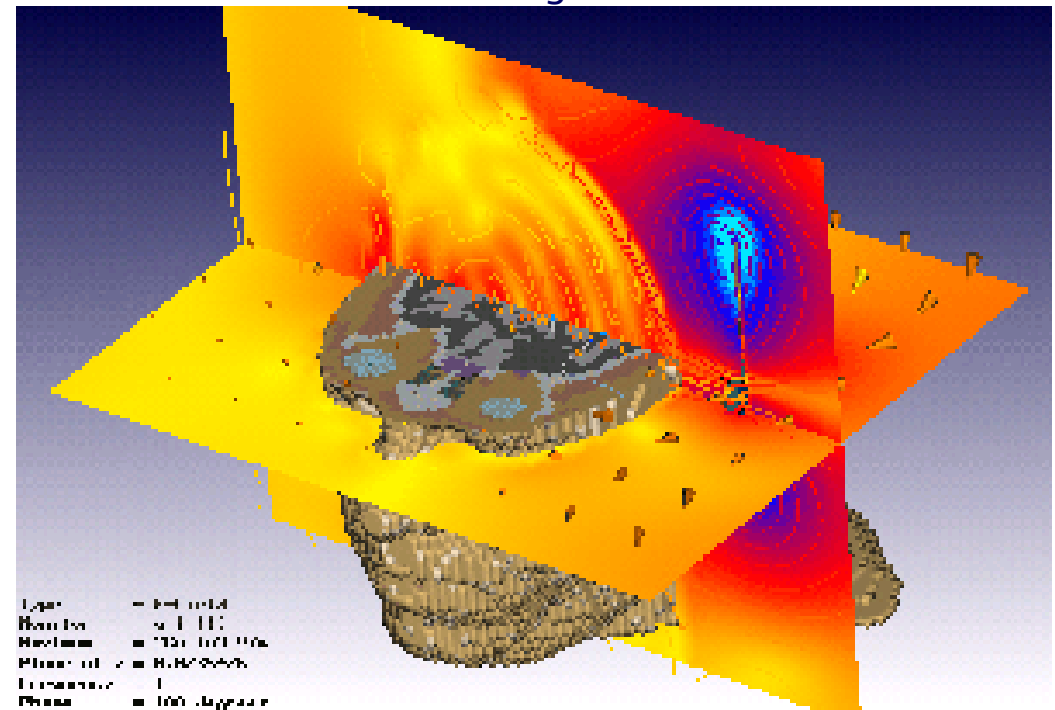
Rapidly Time Varying Electromagnetic (EM) Fields / Zeitlich schnell veränderliche elektromagnetische (EM) Felder

Horn Antenna: Contour Plot of Electric Field Strength Vector (E_y Component) / Hornantenne: Konturdarstellung des elektrischen Feldstärkevektors (E_y -Komponente)



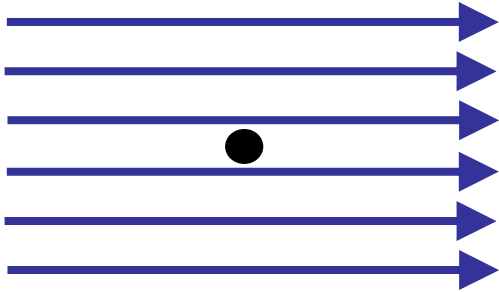
(CST Microwave Studio, www.cst.de)

Biomedical Application: Human head model irradiated by the electromagnetic field of a mobile phone / Biomedizinische Anwendung: Menschliches Kopfmodell bei Bestrahlung durch das elektromagnetische Feld eines



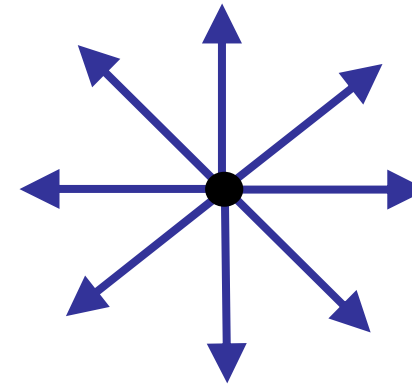
(CST Microwave Studio, www.cst.de)

Examples: Div and Curl / Beispiele: Div und Rot



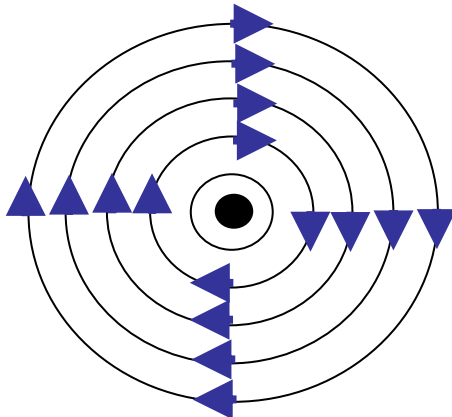
$$\operatorname{div} \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} = 0$$

$$\operatorname{curl} / \operatorname{rot} \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



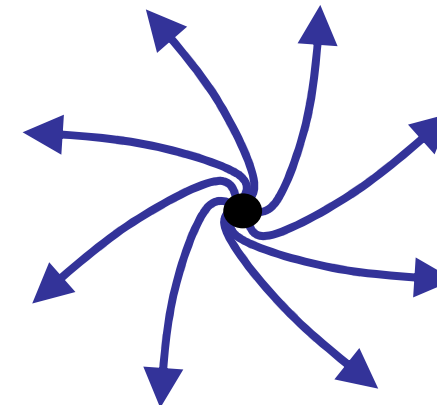
$$\operatorname{div} \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} \neq 0$$

$$\operatorname{curl} / \operatorname{rot} \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



$$\operatorname{div} \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} = 0$$

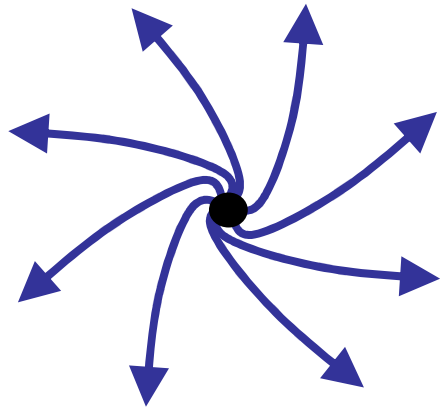
$$\operatorname{curl} / \operatorname{rot} \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$



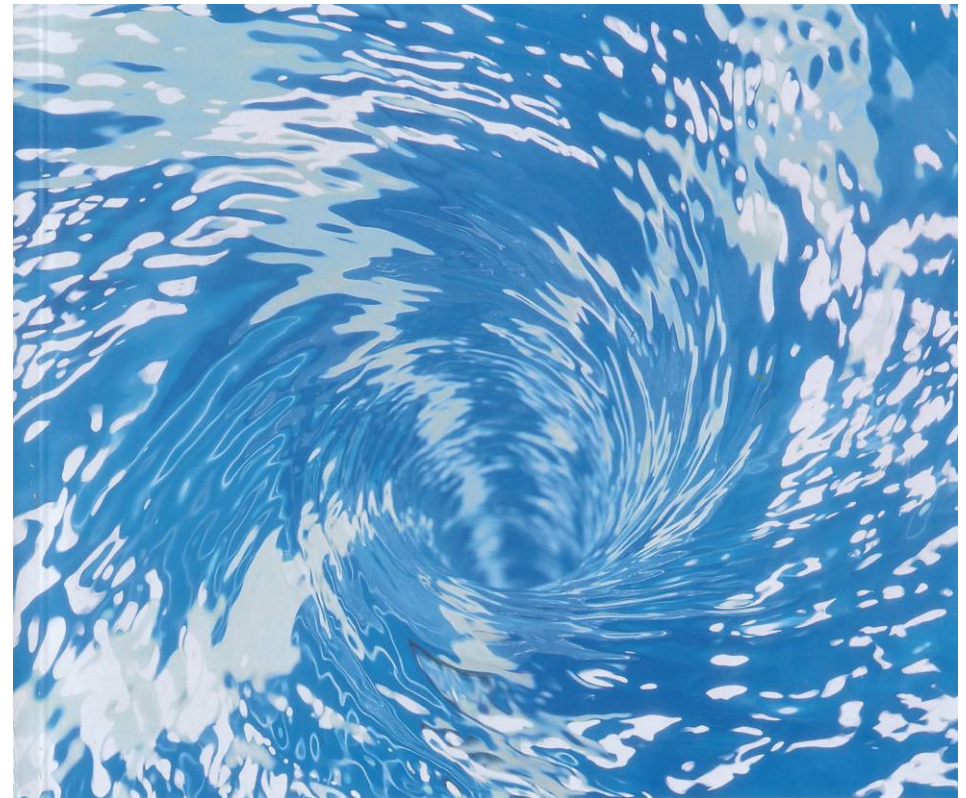
$$\operatorname{div} \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} \neq 0$$

$$\operatorname{curl} / \operatorname{rot} \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$

Examples: Div and Curl / Beispiele: Div und Rot



$$\operatorname{div} \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} \neq 0$$
$$\operatorname{curl} / \operatorname{rot} \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$

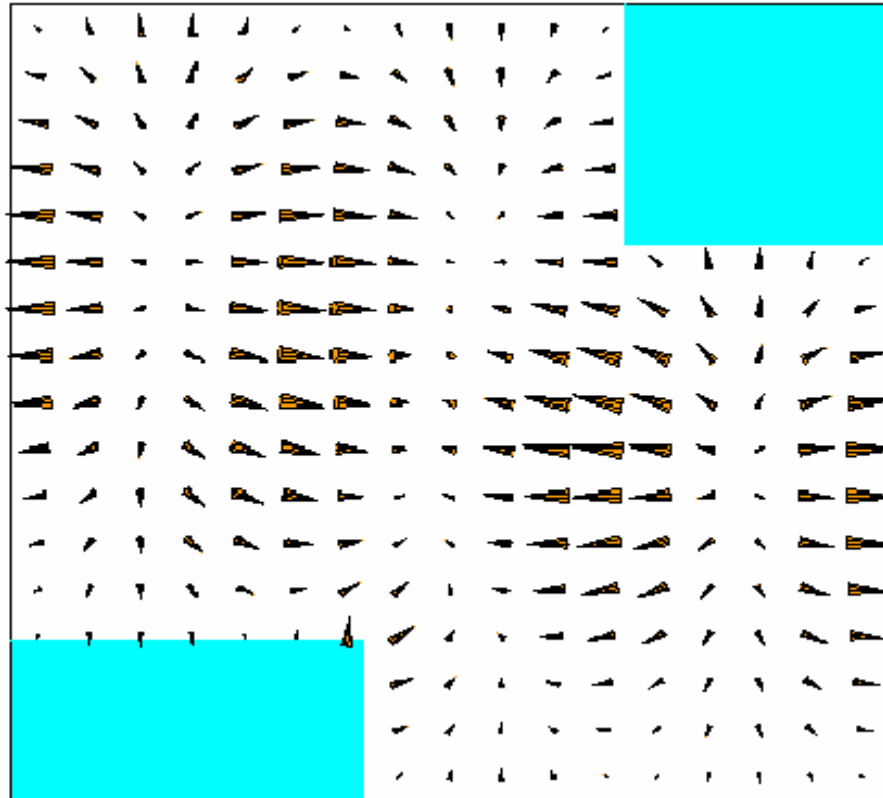


Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

Vector Field / Vektorfeld

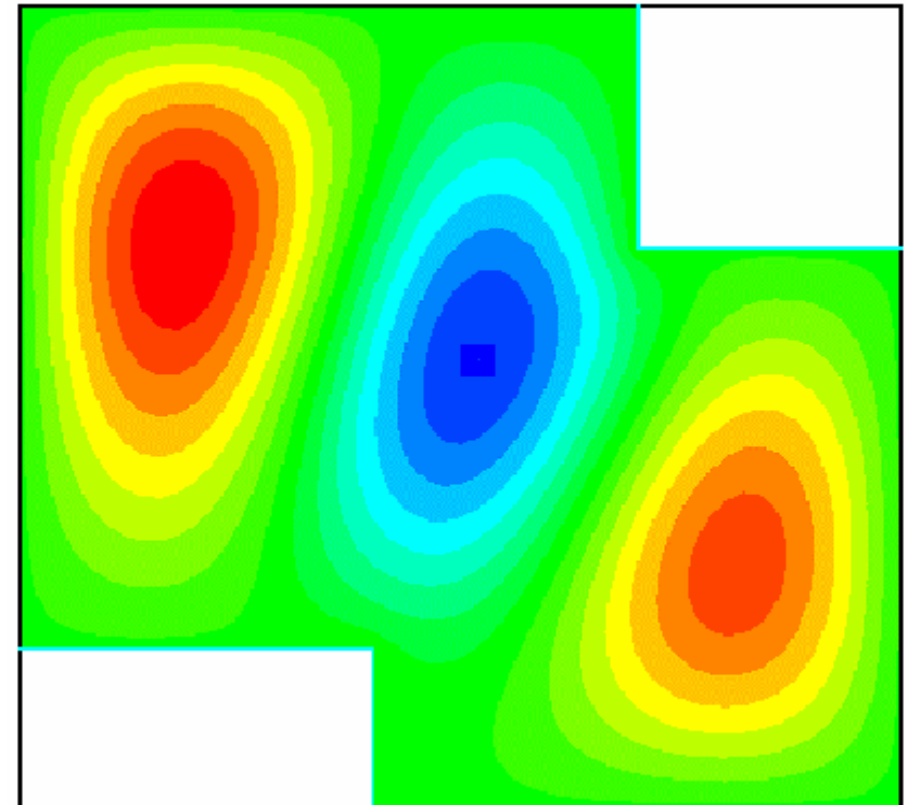
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$

$$\operatorname{div} \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \frac{\partial}{\partial x} D_x(x, y, z) + \frac{\partial}{\partial y} D_y(x, y, z) + \frac{\partial}{\partial z} D_z(x, y, z)$$



Scalar Field / Skalarfeld

$$\rho_e(\underline{\mathbf{R}}) = \operatorname{div} \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}})$$



Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

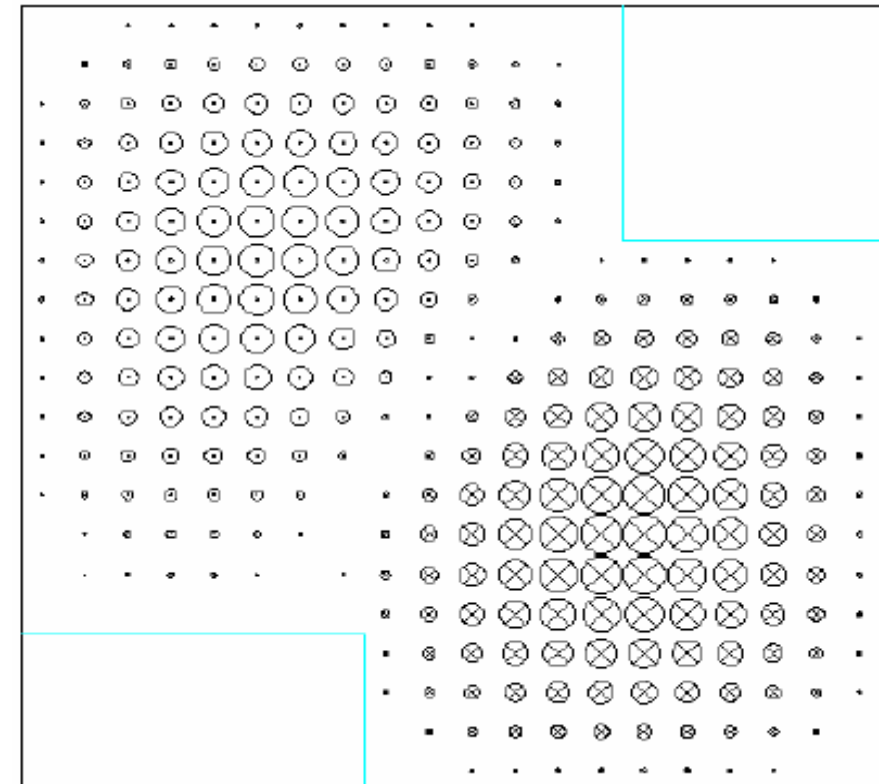
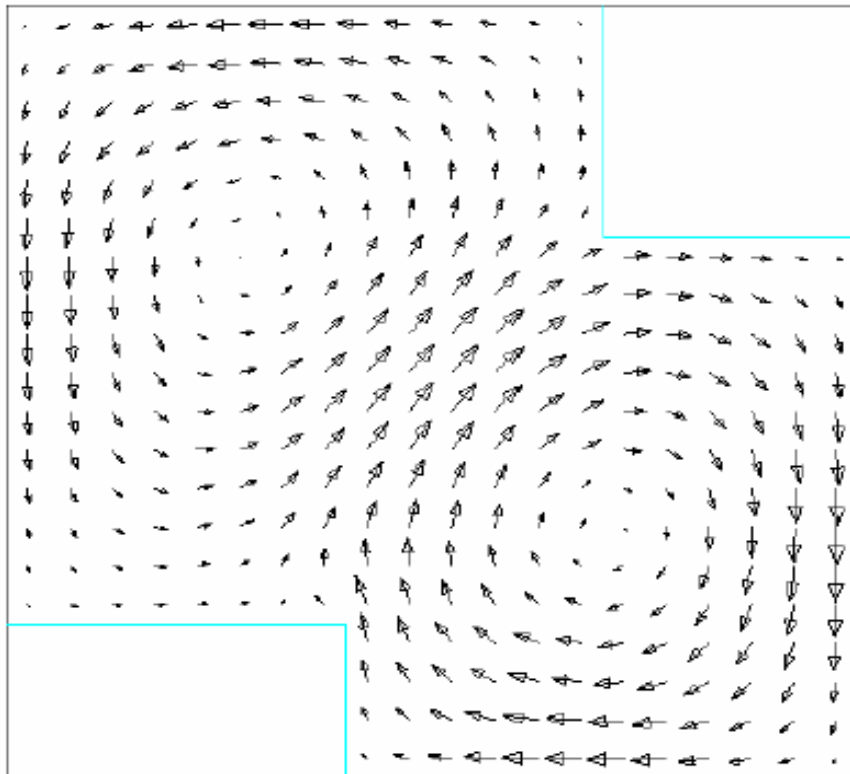
Vector Field / Vektorfeld

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}, t)$$

Vector Field / Vektorfeld

$$\text{curl/rot } \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t)$$


$$\text{curl/rot } \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = \left(\mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z} \right) \times \underline{\mathbf{E}}(x, y, z, t)$$



Hertzian Dipole Antenna in Free-Space / Hertzische Dipolantenne im Freiraum (1)

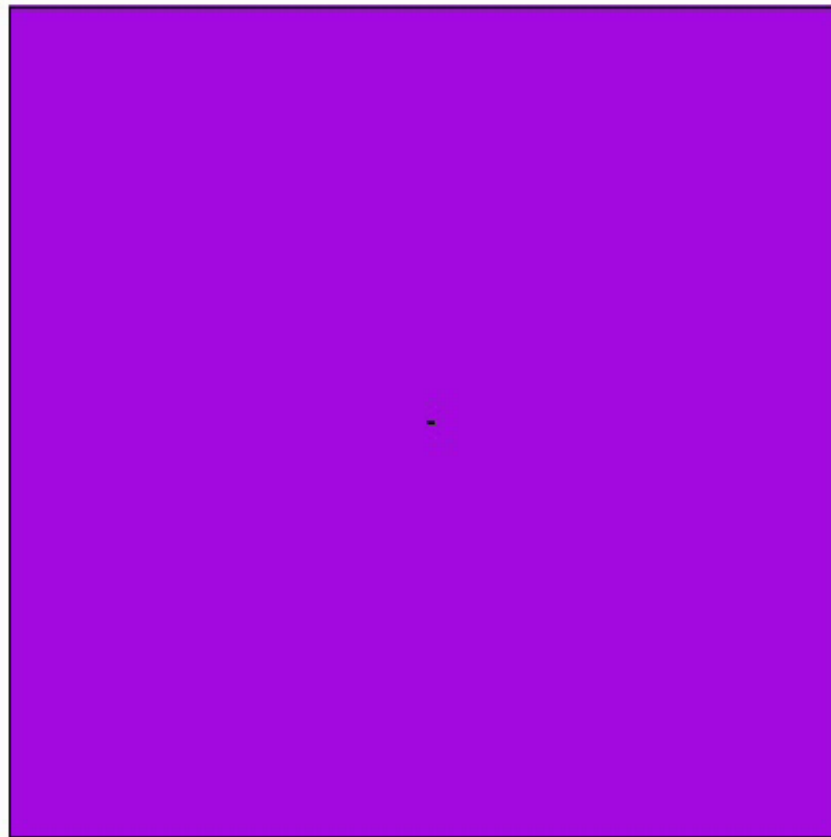
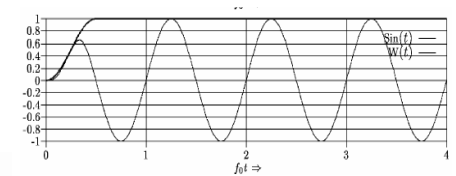
Excitation: Point-like Transient Electric Current Density /
Anregung: Punktförmige transiente elektrische Stromdichte

$$\underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) = \begin{cases} 0 & t < 0 \\ I_0 \sin(\omega_0 t) \delta(\underline{\mathbf{R}}) \underline{\mathbf{e}}_z & t \geq 0 \end{cases}$$


$$\underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) \sim I_0 \sin(\omega_0 t) \underline{\mathbf{e}}_z$$

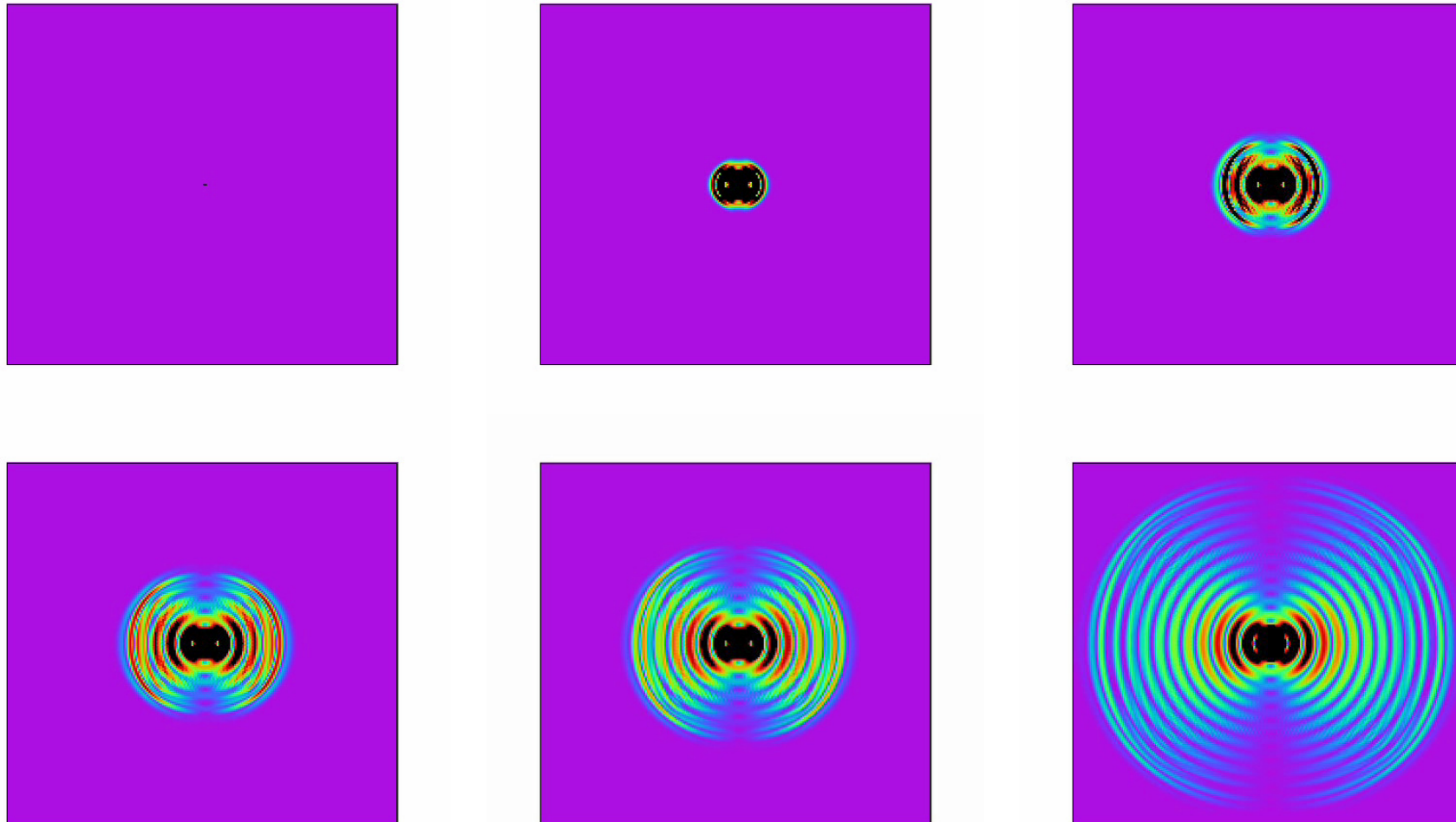
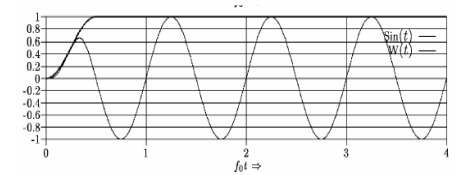
Hertzian Dipole Antenna in Free-Space / Hertzische Dipolantenne im Freiraum (2)

Monofrequent/Monochromatic Excitation /
Monofrequente/Monochromatische Anregung $\mathbf{J}_{ez}(\underline{\mathbf{R}}, t) \sim \sin(\omega_0 t)$



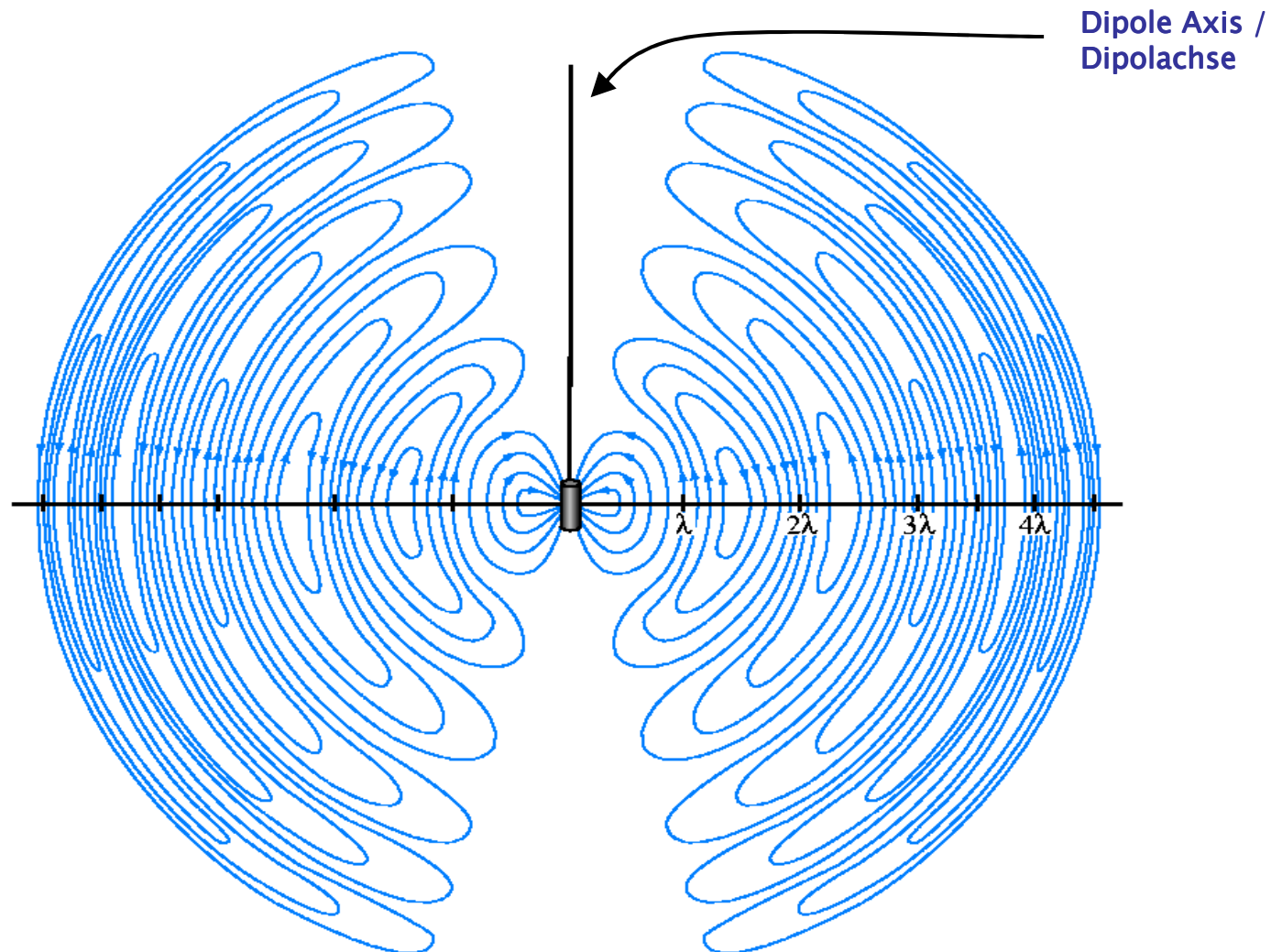
Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (3)

Monofrequent/Monochromatic Excitation /
Monofrequente/Monochromatische Anregung $J_{ez}(\underline{\mathbf{R}}, t) \sim \sin(\omega_0 t)$



Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (4)

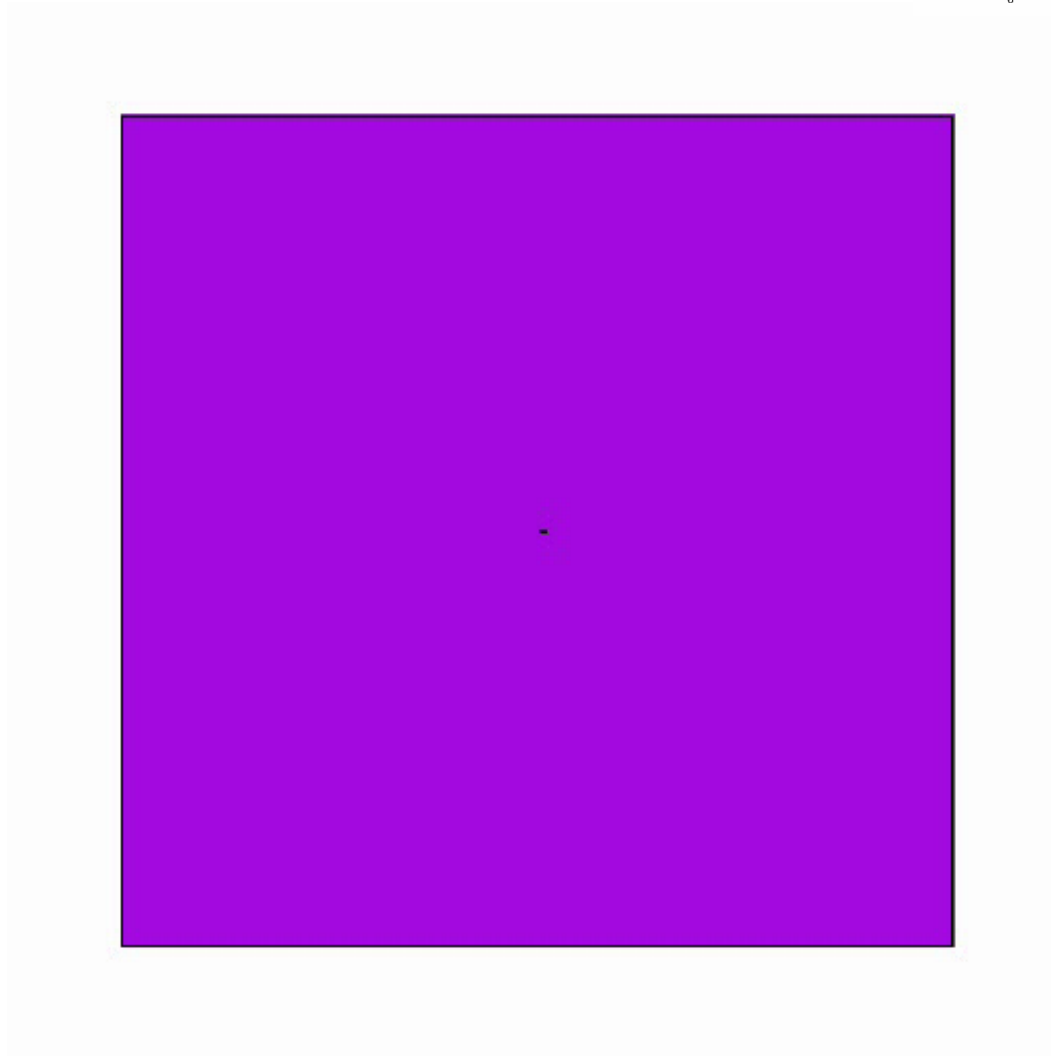
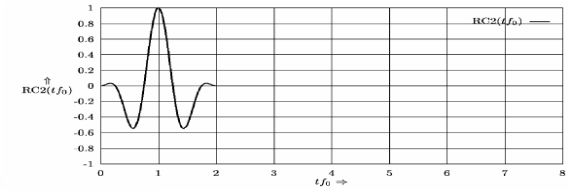
Electric Field Lines Surrounding an Oscillating Dipole at a Given Instant /
Elektrische Feldlinien, die einen oszillierenden Dipol zu einem festen Zeitpunkt umgeben



Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (5)

Broadband Pulse Excitation /
Breitbandige Impulsanregung:

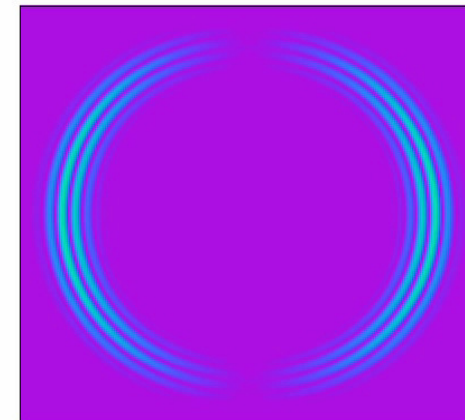
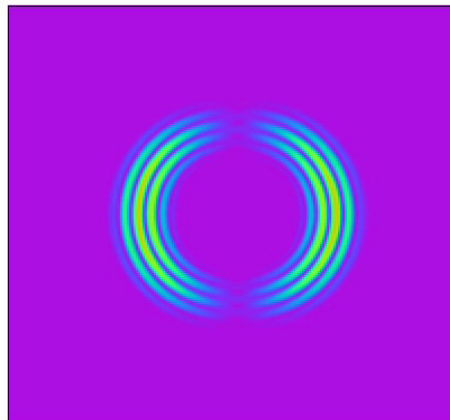
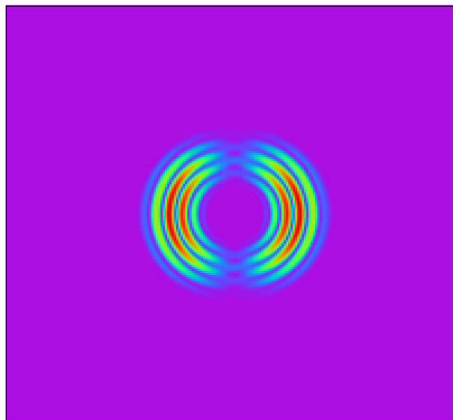
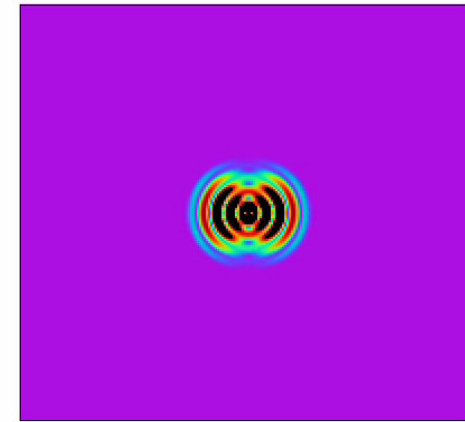
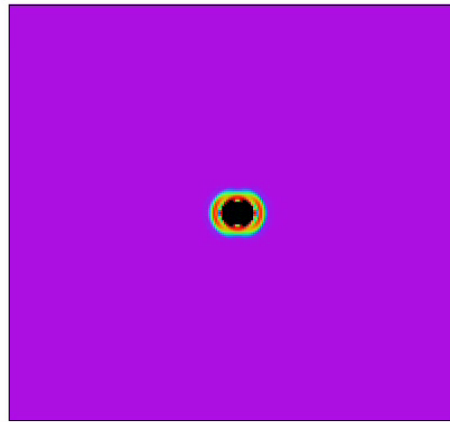
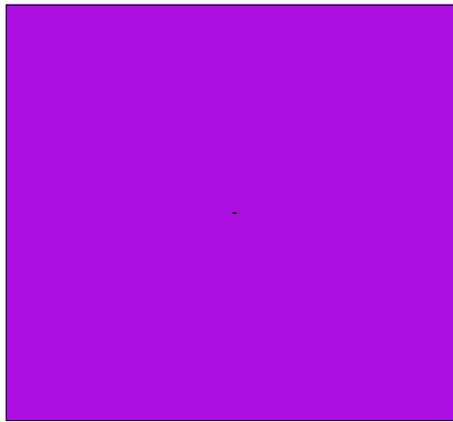
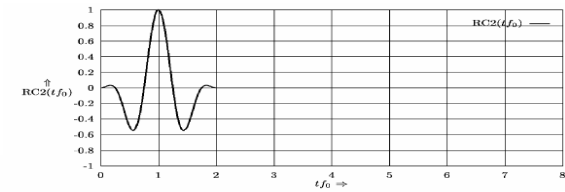
$$J_{ez}(\underline{\mathbf{R}}, t) \sim RC_2(t)$$



Hertzian Dipole Antenna in Free-Space / Hertzische Dipolantenne im Freiraum (6)

Broadband Pulse Excitation /
Breitbandige Impulsanregung:

$$J_{ez}(\underline{\mathbf{R}}, t) \sim RC_2(t)$$

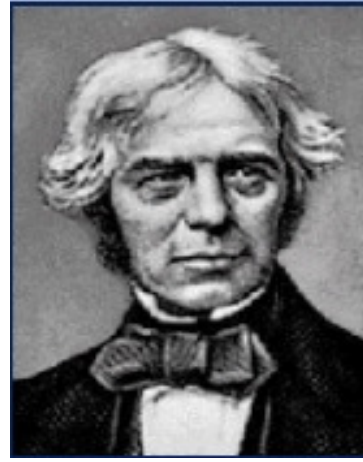


Maxwell's Equations / Maxwell'sche Gleichungen

1.



2.



3.



4.



1. **André Marie Ampère (1775–1836)** 1827: Ampère presented the first mathematical theory of electrodynamics and discovered the magnetic effect of electric currents. / Ampère stellte die erste mathematisch fundierte elektrodynamische Theorie vor und entdeckte die magnetische Wirkung elektrischer Ströme.

2. **Michael Faraday (1791–1867)** 1831: Faraday discovers electromagnetic induction. / Faraday entdeckt die elektromagnetische Induktion.

3. **James Clerk Maxwell (1831–1879)** 1864: Maxwell presents his theory of electromagnetism. / Maxwell präsentiert seine Theorie des Elektromagnetismus.

4. **Heinrich Rudolf Hertz (1857–1894)** 1885: Hertz demonstrates the electromagnetic wave propagation in a series of experiments in a period through 1887. / Hertz demonstriert in einer Periode bis 1887 die Ausbreitung von elektromagnetischen Wellen.

Heinrich Rudolf Hertz

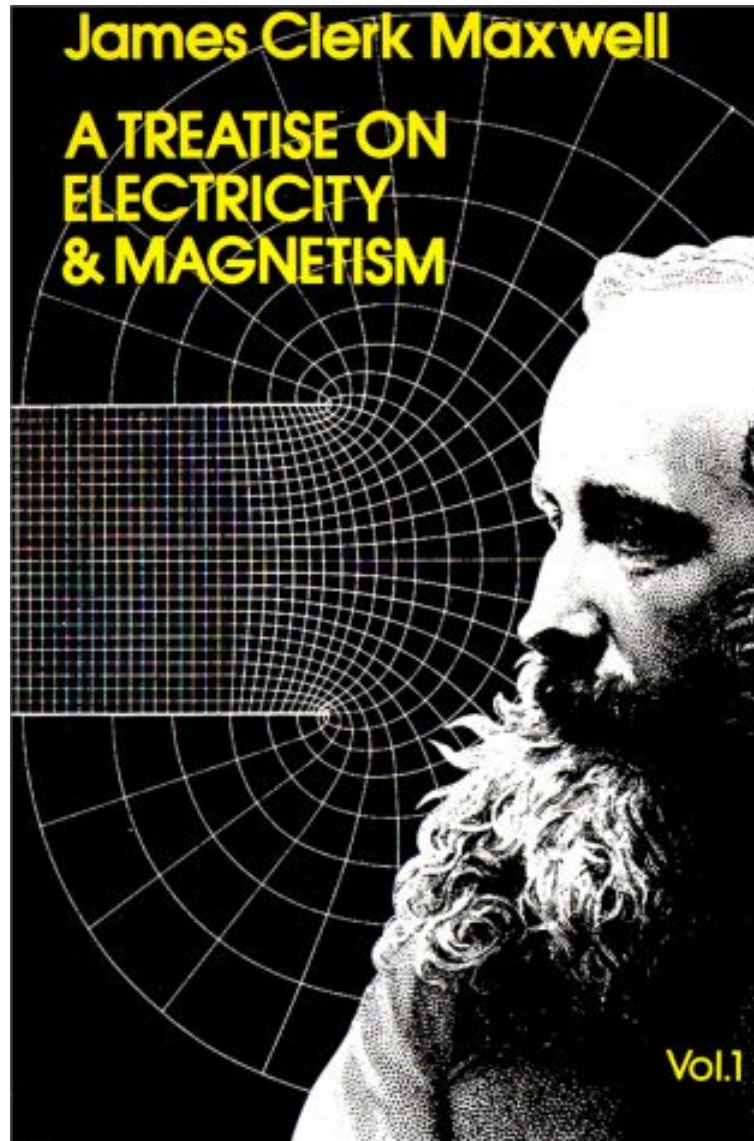


Heinrich Rudolf Hertz (* [22. Februar 1857](#) in [Hamburg](#), † [1. Januar 1894](#) in [Bonn](#)) war ein deutscher [Physiker](#).

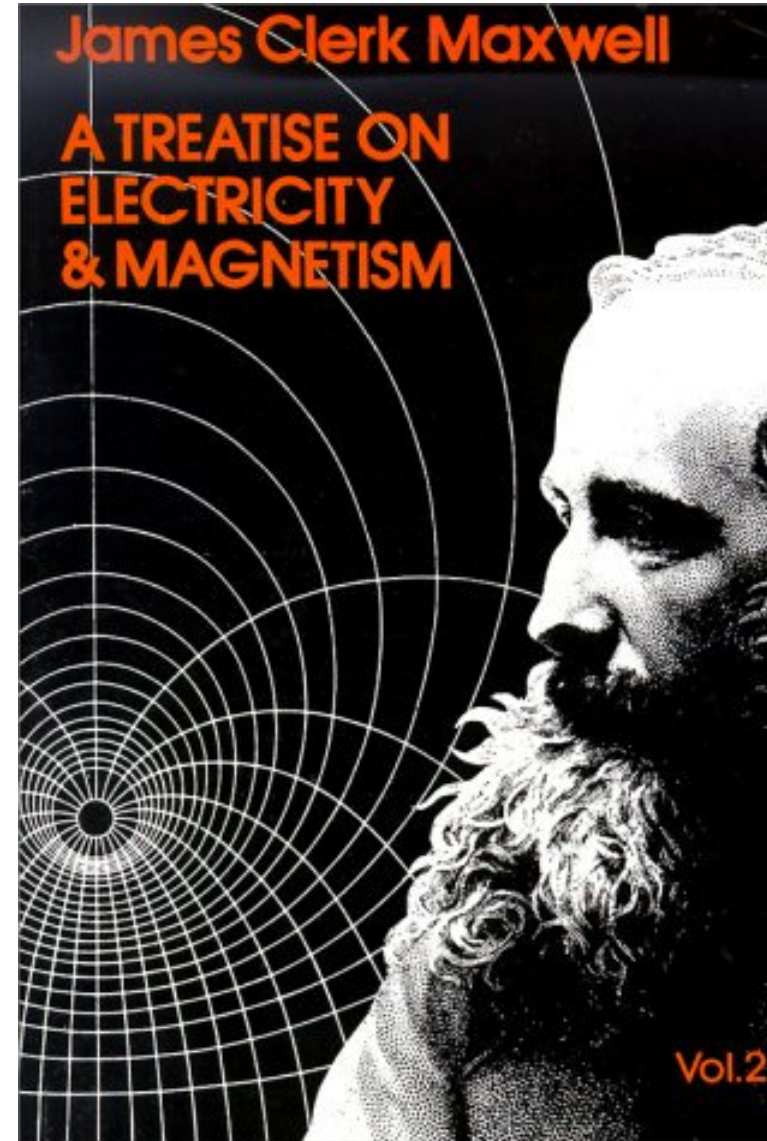
Er studierte an der Universität [Berlin](#). Von [1885](#) bis [1889](#) lehrte er als Professor für Physik an der [technischen Hochschule in Karlsruhe](#). Ab 1889 war er Professor für Physik an der [Universität in Bonn](#). Sein Hauptverdienst lag im experimentellen Nachweis von [James Clerk Maxwells](#) elektromagnetischer Theorie des [Lichts](#) von [1884](#). Hertz entdeckte in Karlsruhe die Existenz der [elektromagnetischen Wellen](#). Er wies nach, dass sie sich auf die gleiche Art und mit der gleichen Geschwindigkeit ausbreiten wie Lichtwellen (siehe: [Brechung](#), [Polarisation](#) und [Reflexion](#)). Seine Ergebnisse lieferten die Grundlage für die Entwicklung der drahtlosen [Telegraphie](#) und des [Radios](#). Die Einheit der [Frequenz](#), eine Schwingung pro Sekunde = 1 [Hertz](#) (Abk. 1 Hz), ist nach ihm benannt und seit [1933](#) im internationalen [metrischen System](#) verankert. Ebenfalls wurde der Hamburger Fernsehturm ([Heinrich-Hertz-Turm](#)) und das zur [Fraunhofer-Gesellschaft](#) gehörige Berliner Heinrich-Hertz-Institut nach ihm benannt.

Heinrich Rudolf Hertz ([February 22, 1857](#) - [January 1, 1894](#)), was the German physicist for whom the [hertz](#), the [SI](#) unit of [frequency](#), is named. In [1888](#), he was the first to demonstrate the existence of [electromagnetic radiation](#) by building apparatus to produce [radio](#) waves. Hertz was born in [Hamburg, Germany](#), to a [Jewish](#) family that had converted to [Christianity](#). His father was an advocate in [Hamburg](#), his mother the daughter of a doctor. While at school, he showed an aptitude for sciences as well as languages, learning [Arabic](#) and [Sanskrit](#). He studied sciences and engineering in the German cities of [Dresden](#), [Munich](#) and [Berlin](#). He was a student of [Gustav R. Kirchhoff](#) and [Hermann von Helmholtz](#). He obtained his PhD in [1880](#), and remained a pupil of Helmholtz until 1883 when he took a post as a lecturer in theoretical physics at the [University of Kiel](#). In 1885 he became a full professor at the [University of Karlsruhe](#) where he discovered electromagnetic waves. Following [Michelson's 1881](#) experiment (precursor to the [1887 Michelson-Morley experiment](#)) which disproved the existence of [luminiferous aether](#), he reformulated [Maxwell's equations](#) to take the new discovery into account. Through experimentation, he proved that electric signals can travel through open air, as had been predicted by [James Clerk Maxwell](#) and [Michael Faraday](#), and which is the basis for the invention of [radio](#). He also discovered the [photoelectric effect](#) (which was later explained by [Albert Einstein](#)) when he noticed that a [charged](#) object loses its charge more readily when illuminated by ultraviolet light. He died in [Bonn](#), Germany. His nephew [Gustav Ludwig Hertz](#) was a [Nobel Prize](#) winner, and Gustav's son [Carl Hellmuth Hertz](#) invented [medical ultrasonography](#).

Maxwell's Equations / Maxwell'sche Gleichungen



<http://www.amazon.de>



<http://www.amazon.de>

Today's Vector Notation of Maxwell's Equations / Heutige Vektornotation der Maxwell'schen Gleichungen

Governing Equations in
Differential Form /
Grundgleichungen in
Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = \frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) + \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

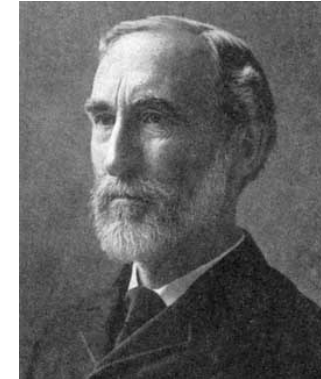
$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \rho_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = \rho_m(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \rho_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \rho_m(\underline{\mathbf{R}}, t)$$

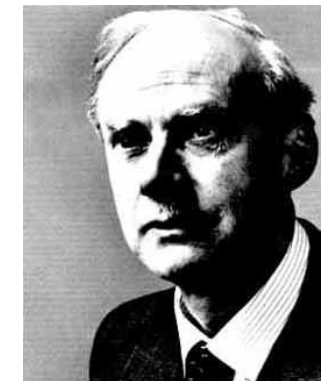
Josiah Willard Gibbs
(1839-1903)



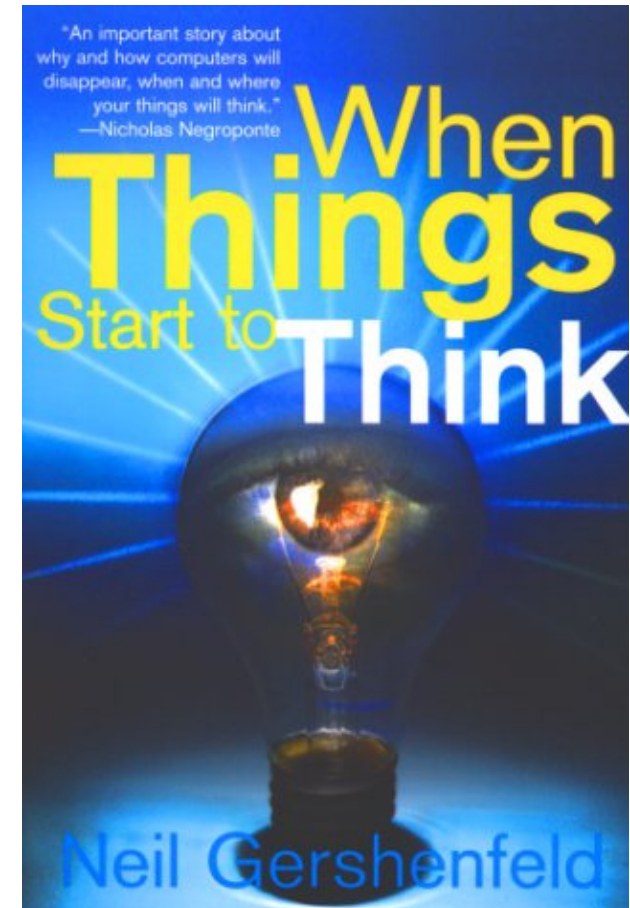
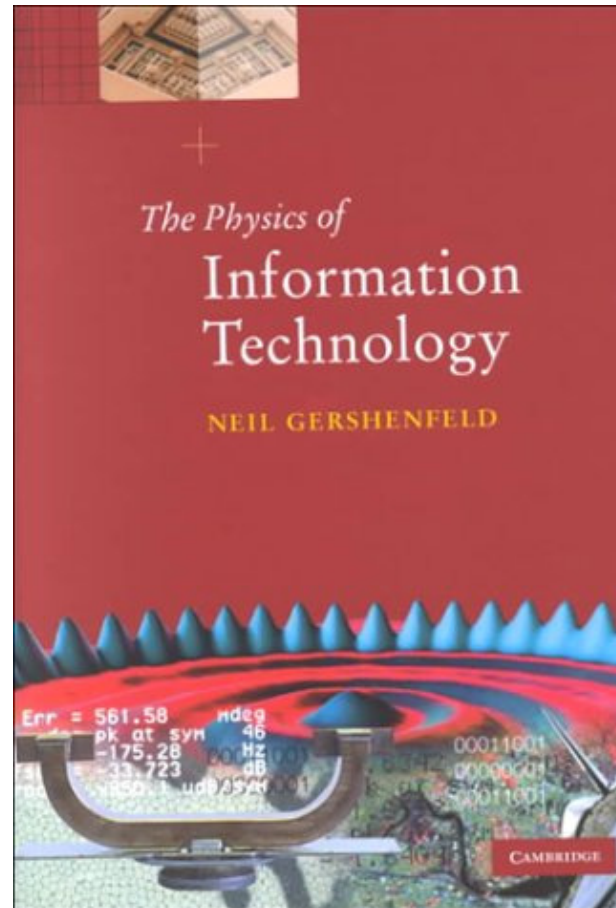
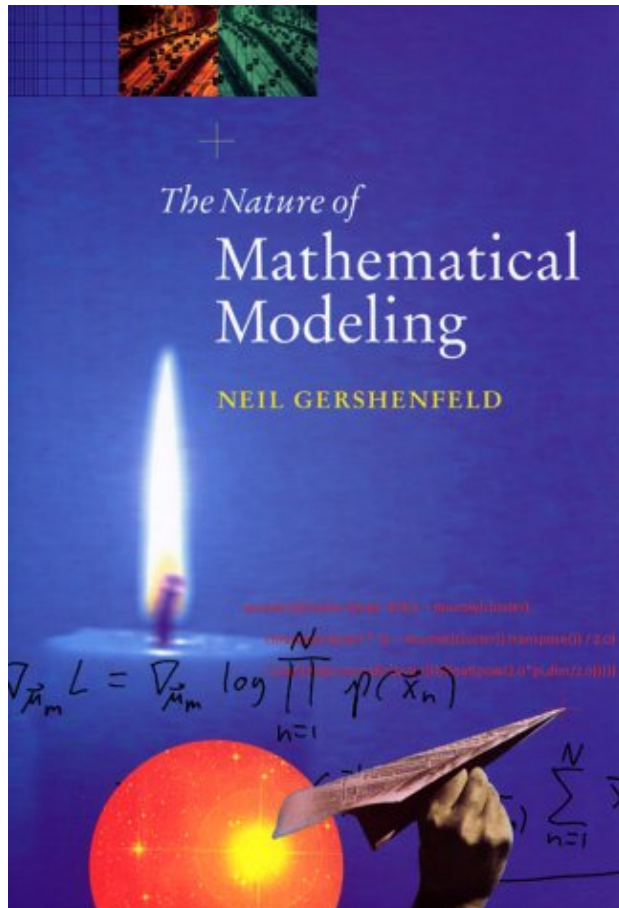
Oliver Heaviside
(1850-1925)



Paul Adrien Maurice Dirac
(1902-1984)



... Other Books / ... andere Bücher



End of Lecture 2 /
Ende der 2. Vorlesung