

# Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

## 2nd Lecture / 2. Vorlesung

Dr.-Ing. René Marklein

[marklein@uni-kassel.de](mailto:marklein@uni-kassel.de)

<http://www.tet.e-technik.uni-kassel.de>

<http://www.uni-kassel.de/fb16/tet/marklein/index.html>

Universität Kassel  
Fachbereich Elektrotechnik / Informatik  
(FB 16)  
Fachgebiet Theoretische Elektrotechnik  
(FG TET)  
Wilhelmshöher Allee 71  
Büro: Raum 2113 / 2115  
D-34121 Kassel

University of Kassel  
Dept. Electrical Engineering / Computer Science  
(FB 16)  
Electromagnetic Theory  
(FG TET)  
Wilhelmshöher Allee 71  
Office: Room 2113 / 2115  
D-34121 Kassel

## Notation and Field Quantities / Notation und Feldgrößen

**Vector / Vektor:**  
**Electric Field Strength / Elektrische Feldstärke**

$$\underline{\mathbf{E}}(\mathbf{R}, t) = \underline{E}_x(\mathbf{R}, t) \underline{\mathbf{e}}_x + \underline{E}_y(\mathbf{R}, t) \underline{\mathbf{e}}_y + \underline{E}_z(\mathbf{R}, t) \underline{\mathbf{e}}_z$$

3 Vector Components /  
3 Vektorkomponenten

$$= E_x(x, y, z, t) \underline{\mathbf{e}}_x + E_y(x, y, z, t) \underline{\mathbf{e}}_y + E_z(x, y, z, t) \underline{\mathbf{e}}_z$$

mit  $\{x, y, z\} = \{x_1, x_2, x_3\}$

$$= \sum_{i=1}^3 E_{x_i}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i}$$

$$= E_{x_i}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i}$$

**Dyad / Dyade:**  
**Permittivity Dyad / Permittivitätsdyade**

$$\underline{\underline{\epsilon}}(\mathbf{R}, t) = \underline{\epsilon}_{xx}(\mathbf{R}, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_x + \underline{\epsilon}_{xy}(\mathbf{R}, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_y + \underline{\epsilon}_{xz}(\mathbf{R}, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_z$$

$$+ \underline{\epsilon}_{yx}(\mathbf{R}, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_x + \underline{\epsilon}_{yy}(\mathbf{R}, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_y + \underline{\epsilon}_{yz}(\mathbf{R}, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_z$$

$$+ \underline{\epsilon}_{zx}(\mathbf{R}, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_x + \underline{\epsilon}_{zy}(\mathbf{R}, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_y + \underline{\epsilon}_{zz}(\mathbf{R}, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_z$$

9 Dyadic Components /  
9 dyadische Komponenten

$$= \epsilon_{xx}(x, y, z, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_x + \epsilon_{xy}(x, y, z, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_y + \epsilon_{xz}(x, y, z, t) \underline{\mathbf{e}}_x \underline{\mathbf{e}}_z$$

$$+ \epsilon_{yx}(x, y, z, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_x + \epsilon_{yy}(x, y, z, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_y + \epsilon_{yz}(x, y, z, t) \underline{\mathbf{e}}_y \underline{\mathbf{e}}_z$$

$$+ \epsilon_{zx}(x, y, z, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_x + \epsilon_{zy}(x, y, z, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_y + \epsilon_{zz}(x, y, z, t) \underline{\mathbf{e}}_z \underline{\mathbf{e}}_z$$

mit  $\{x, y, z\} = \{x_1, x_2, x_3\}$

$$= \sum_{i=1}^3 \sum_{j=1}^3 \epsilon_{x_i x_j}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i} \underline{\mathbf{e}}_{x_j}$$

$$= \epsilon_{x_i x_j}(x_1, x_2, x_3, t) \underline{\mathbf{e}}_{x_i} \underline{\mathbf{e}}_{x_j}$$

with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

*Einstein's Summation Convention:* If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. /  
*Einsteinsche Summenkonvention:* Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.

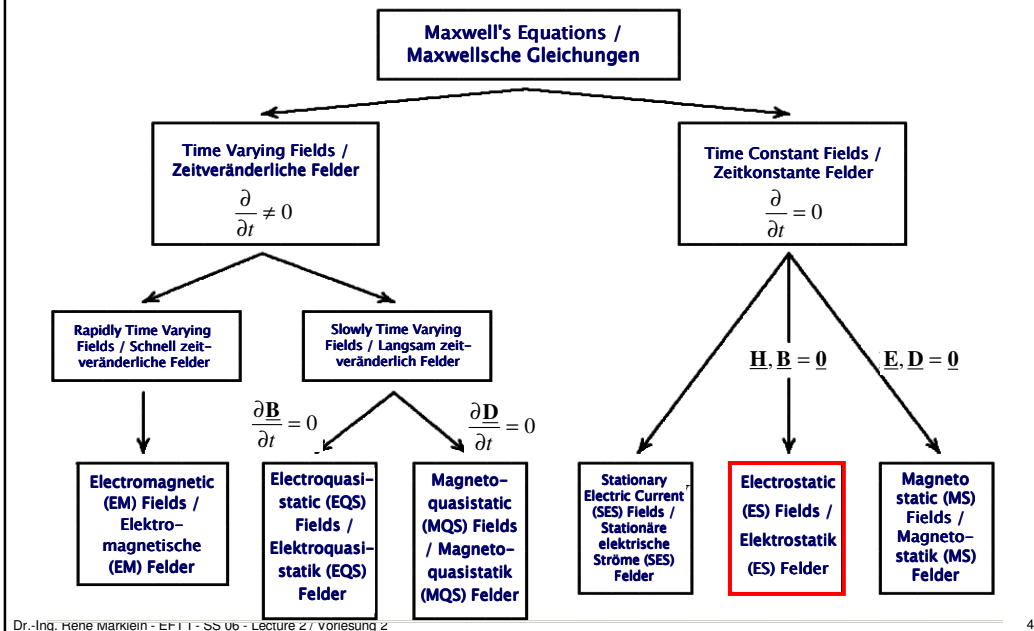
## Governing Equations of Electromagnetic Fields and Waves / Grundgleichungen elektromagnetischer Felder und Wellen

Governing Equations in Differential Form / Grundgleichungen in Differentialform	Governing Equations in Integral Form / Grundgleichungen in Integralform
$\nabla \times \underline{E}(\underline{R}, t) = -\frac{\partial}{\partial t} \underline{B}(\underline{R}, t) - \underline{J}_m(\underline{R}, t)$ $\nabla \times \underline{H}(\underline{R}, t) = \frac{\partial}{\partial t} \underline{D}(\underline{R}, t) + \underline{J}_c(\underline{R}, t)$ $\nabla \cdot \underline{D}(\underline{R}, t) = \rho_c(\underline{R}, t)$ $\nabla \cdot \underline{B}(\underline{R}, t) = \rho_m(\underline{R}, t)$	$\oint_{C=\partial S} \underline{E}(\underline{R}, t) \cdot d\underline{R} = -\iint_S \frac{\partial}{\partial t} \underline{B}(\underline{R}, t) \cdot d\underline{S} - \iint_S \underline{J}_m(\underline{R}, t) \cdot d\underline{S}$ $\oint_{C=\partial S} \underline{H}(\underline{R}, t) \cdot d\underline{R} = \iint_S \frac{\partial}{\partial t} \underline{D}(\underline{R}, t) \cdot d\underline{S} + \iint_S \underline{J}_c(\underline{R}, t) \cdot d\underline{S}$ $\oiint_{S=\partial V} \underline{D}(\underline{R}, t) \cdot d\underline{S} = \iiint_V \rho_c(\underline{R}, t) dV$ $\oiint_{S=\partial V} \underline{B}(\underline{R}, t) \cdot d\underline{S} = \iiint_V \rho_m(\underline{R}, t) dV$
$\nabla \cdot \underline{J}_c(\underline{R}, t) = -\frac{\partial}{\partial t} \rho_c(\underline{R}, t)$ $\nabla \cdot \underline{J}_m(\underline{R}, t) = -\frac{\partial}{\partial t} \rho_m(\underline{R}, t)$	$\oiint_{S=\partial V} \underline{J}_c(\underline{R}, t) \cdot d\underline{S} = -\iiint_V \frac{\partial}{\partial t} \rho_c(\underline{R}, t) dV$ $\oiint_{S=\partial V} \underline{J}_m(\underline{R}, t) \cdot d\underline{S} = -\iiint_V \frac{\partial}{\partial t} \rho_m(\underline{R}, t) dV$

Dr.-Ing. René Marklein - EFT I - SS 06 - Lecture 2 / Vorlesung 2

3

## Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen

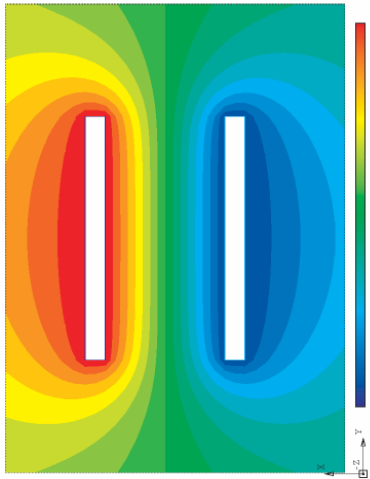


Dr.-Ing. René Marklein - EFT I - SS 06 - Lecture 2 / Vorlesung 2

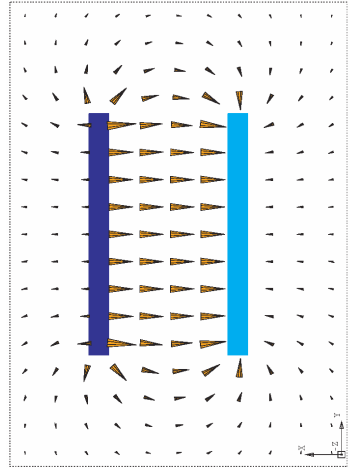
4

**Electrostatic Field Problem – Example: Parallel Plate Capacitor /  
 Elektrostatiches Feldproblem – Beispiel: Paralleler Plattenkondensator**

**Scalar Field: Electrostatic Potential /  
 Skalarfeld: Elektrostatiches Potenzial**



**Vector Field: Electrostatic Field Strength /  
 Vektorfeld: Elektrostatiches Feldstärke**

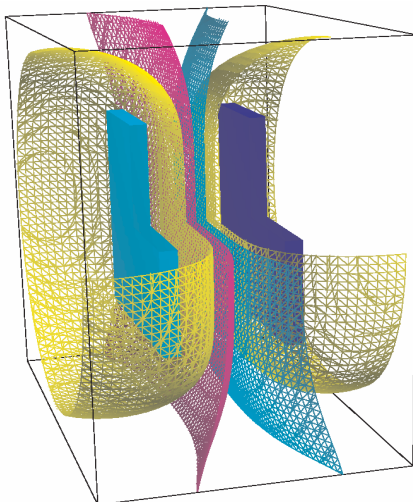


Dr.-Ing. René Marklein - EFT I - SS 06 - Lecture 2 / Vorlesung 2

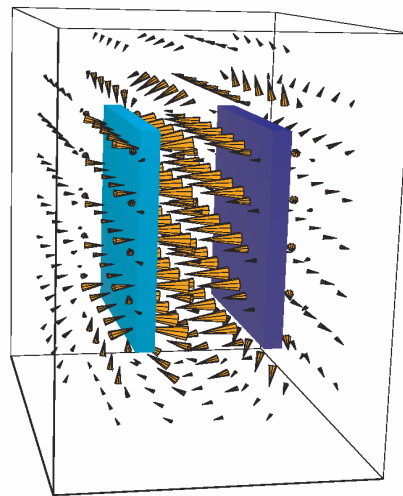
5

**Example: Parallel Plate Capacitor – Electrostatic Field Problem /  
 Beispiel: Paralleler Plattenkondensator – Elektrostatiches Feldproblem**

**Scalar Field: Electrostatic Potential /  
 Skalarfeld: Elektrostatiches Potenzial**



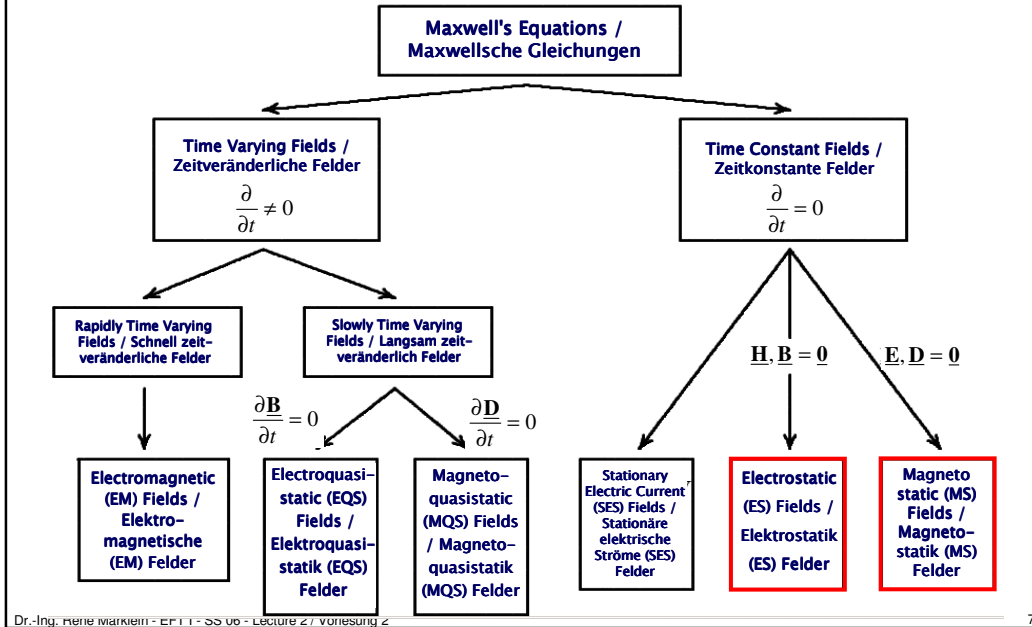
**Vector Field: Electrostatic Field Strength /  
 Vektorfeld: Elektrostatiches Feldstärke**



Dr.-Ing. René Marklein - EFT I - SS 06 - Lecture 2 / Vorlesung 2

6

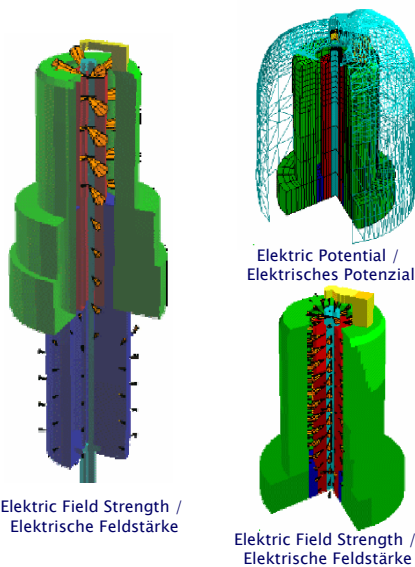
## Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen



### Example: Spark Plug and Relay / Beispiel: Zündkerze und Relais

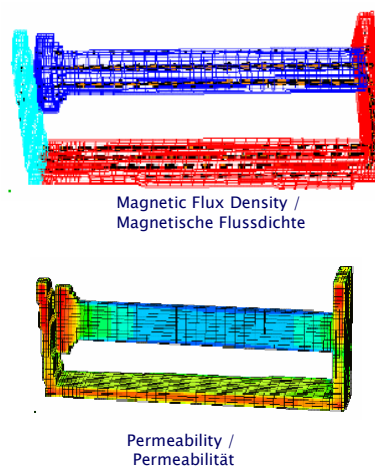
**Electrostatic (ES) Fields /  
Elektrostatistische (ES) Felder**

**Spark Plug / Zündkerze**

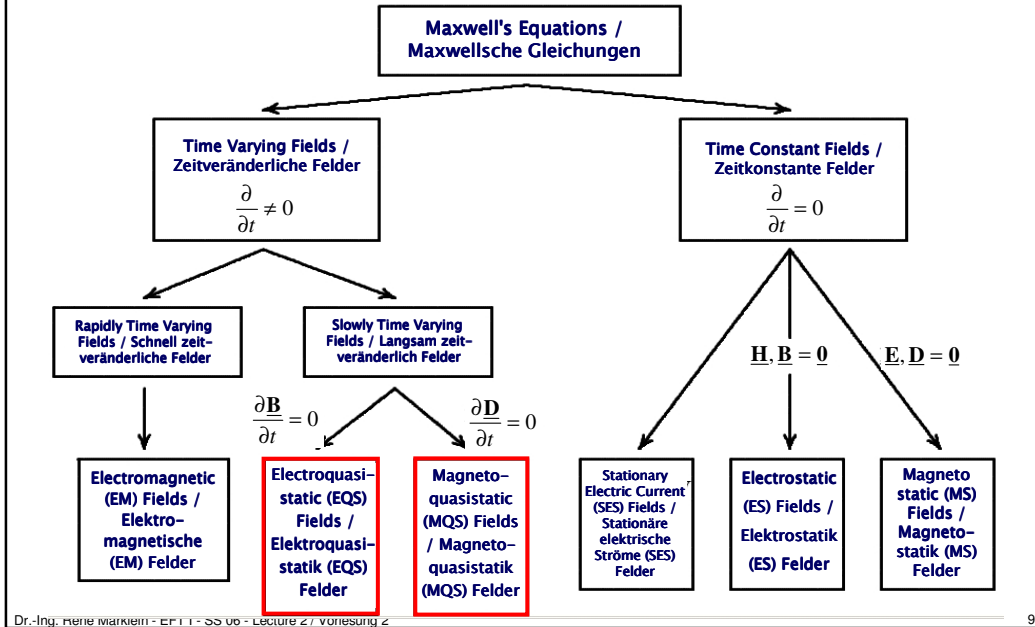


**Magnetostatic (MS) Fields /  
Magnetostatische (MS) Felder**

**Relay / Relais**



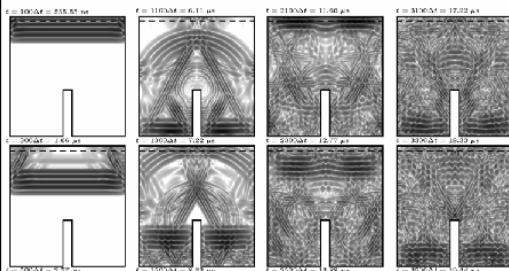
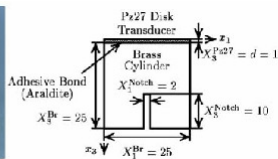
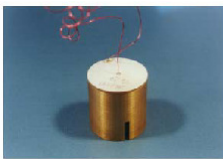
## Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen



### Example: Piezoelectric Sensor / Beispiel: Piezoelektrischer Sensor

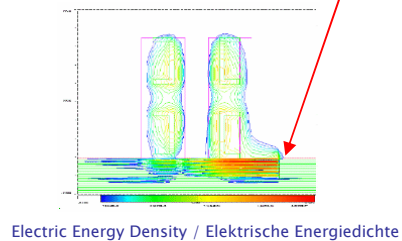
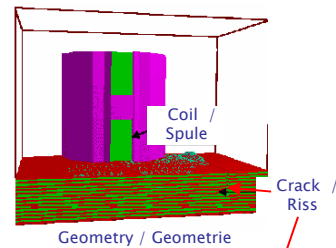
#### Electroquasistatic (EQS) Fields / Elektroquasistatische (EQS) Felder

Non-Destructive Testing: Piezoelectric Sensor /  
Zerstörungsfreie Materialprüfung: Piezoelektrischer Sensor

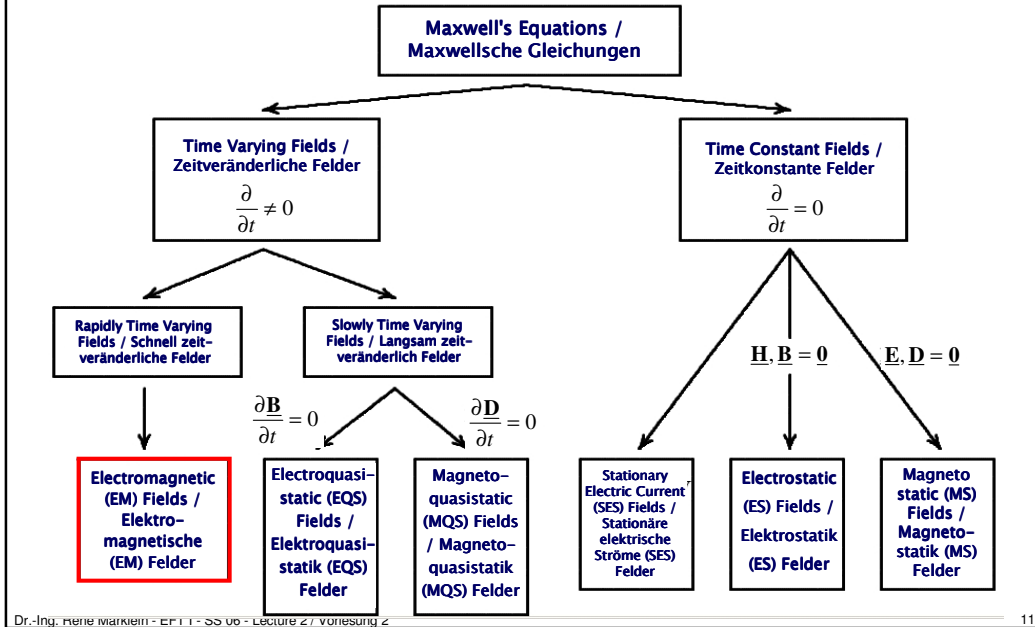


#### Magnetoquasistatic (MQS) Fields / Magnetoquasistatische (MQS) Felder

Non-Destructive Testing: Eddy Current Sensor /  
Zerstörungsfreie Materialprüfung: Wirbelstromsensor



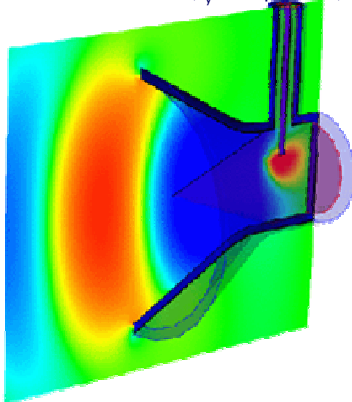
## Classification of Maxwell's Equations / Klassifikation der Maxwell'schen Gleichungen



## Examples: Antenna and Human Head Interaction / Beispiele: Antenne und Interaktion mit menschlichem Kopf

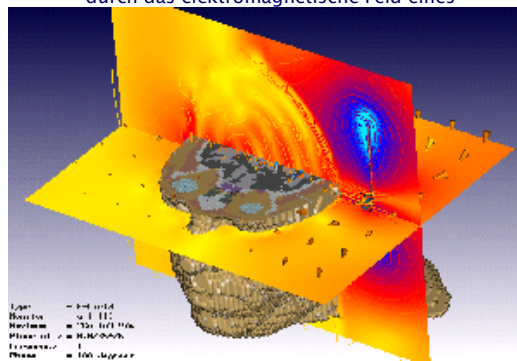
### Rapidly Time Varying Electromagnetic (EM) Fields / Zeitlich schnell veränderliche elektromagnetische (EM) Felder

Horn Antenna: Contour Plot of Electric Field Strength Vector ( $E_y$  Component) /  
Hornantenne: Konturdarstellung des elektrischen Feldstärkevektors ( $E_y$ -Komponente)



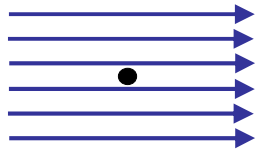
(CST Microwave Studio, www.cst.de)

Biomedical Application: Human head model irradiated by the electromagnetic field of a mobile phone /  
Biomedizinische Anwendung: Menschliches Kopfmodell bei Bestrahlung durch das elektromagnetische Feld eines



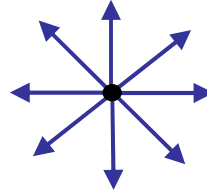
(CST Microwave Studio, www.cst.de)

### Examples: Div and Curl / Beispiele: Div und Rot



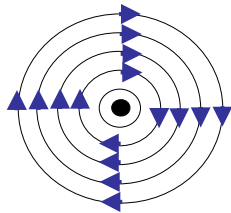
$$\text{div } \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} = 0$$

$$\text{curl/rot } \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



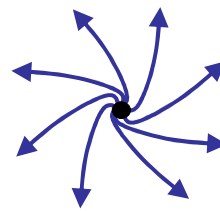
$$\text{div } \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} \neq 0$$

$$\text{curl/rot } \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



$$\text{div } \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} = 0$$

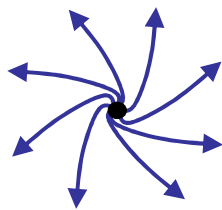
$$\text{curl/rot } \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$



$$\text{div } \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} \neq 0$$

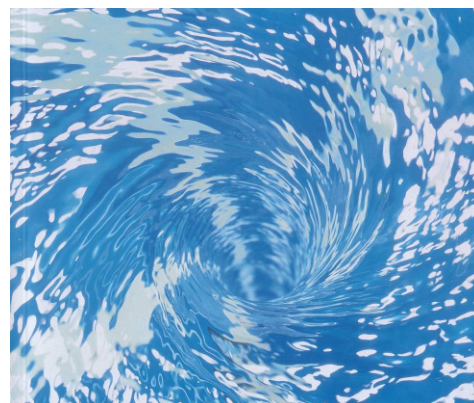
$$\text{curl/rot } \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$

### Examples: Div and Curl / Beispiele: Div und Rot



$$\text{div } \underline{\mathbf{A}} = \nabla \cdot \underline{\mathbf{A}} \neq 0$$

$$\text{curl/rot } \underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$

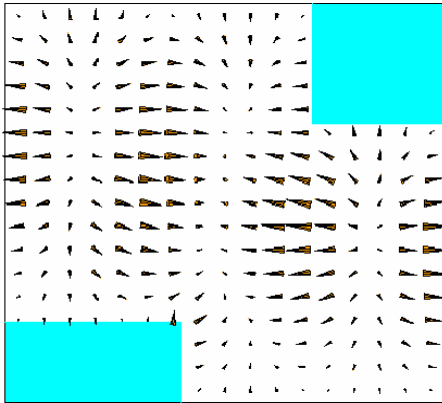


## Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

Vector Field / Vektorfeld

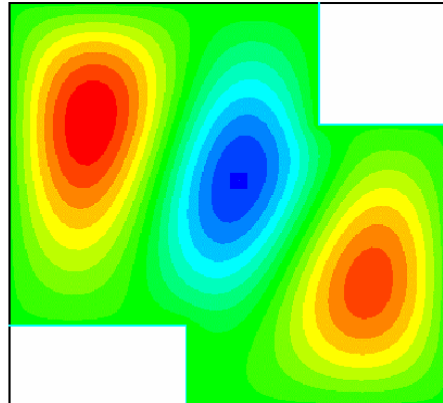
$\underline{D}(\mathbf{R})$

$$\operatorname{div} \underline{D}(\mathbf{R}) = \nabla \cdot \underline{D}(\mathbf{R}) = \frac{\partial}{\partial x} D_x(x, y, z) + \frac{\partial}{\partial y} D_y(x, y, z) + \frac{\partial}{\partial z} D_z(x, y, z)$$



Scalar Field / Skalarfeld

$$\rho_e(\mathbf{R}) = \operatorname{div} \underline{D}(\mathbf{R}) = \nabla \cdot \underline{D}(\mathbf{R})$$



Dr.-Ing. René Marklein - EFT I - SS 06 - Lecture 2 / Vorlesung 2

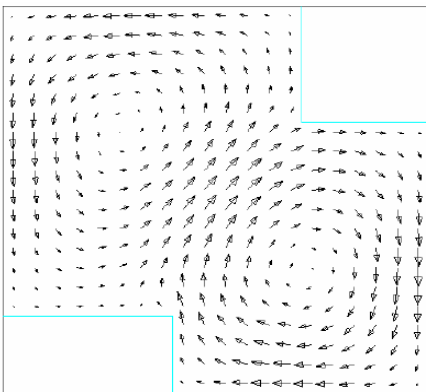
15

## Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

Vector Field / Vektorfeld

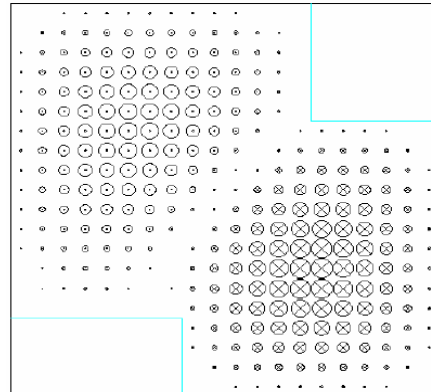
$\underline{E}(\mathbf{R}, t)$

$$\operatorname{curl} / \operatorname{rot} \underline{E}(\mathbf{R}, t) = \nabla \times \underline{E}(\mathbf{R}, t) = \left( \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z} \right) \times \underline{E}(x, y, z, t)$$



Vector Field / Vektorfeld

$$\operatorname{curl} / \operatorname{rot} \underline{E}(\mathbf{R}, t) = \nabla \times \underline{E}(\mathbf{R}, t) = -\frac{\partial}{\partial t} \underline{B}(\mathbf{R}, t)$$



Dr.-Ing. René Marklein - EFT I - SS 06 - Lecture 2 / Vorlesung 2


16



## Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (1)

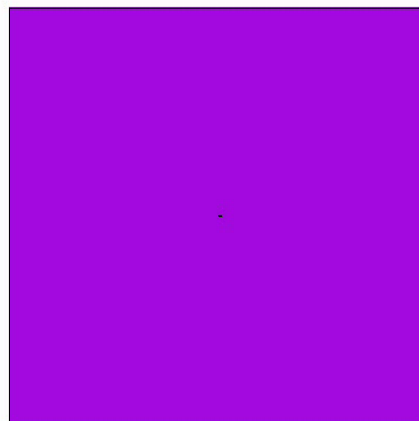
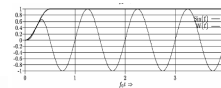
Excitation: Point-like Transient Electric Current Density /  
Anregung: Punktförmige transiente elektrische Stromdichte

$$\underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) = \begin{cases} 0 & t < 0 \\ I_0 \sin(\omega_0 t) \delta(\underline{\mathbf{R}}) \underline{\mathbf{e}}_z & t \geq 0 \end{cases}$$


 $\underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) \sim I_0 \sin(\omega_0 t) \underline{\mathbf{e}}_z$

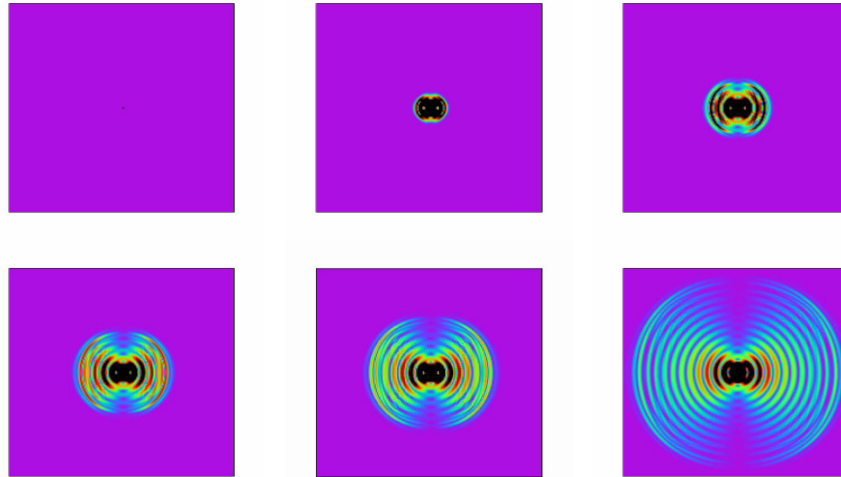
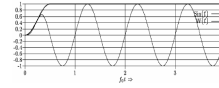
## Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (2)

Monofrequent/Monochromatic Excitation /  
Monofrequente/Monochromatische Anregung  $\underline{\mathbf{J}}_{ez}(\underline{\mathbf{R}}, t) \sim \sin(\omega_0 t)$



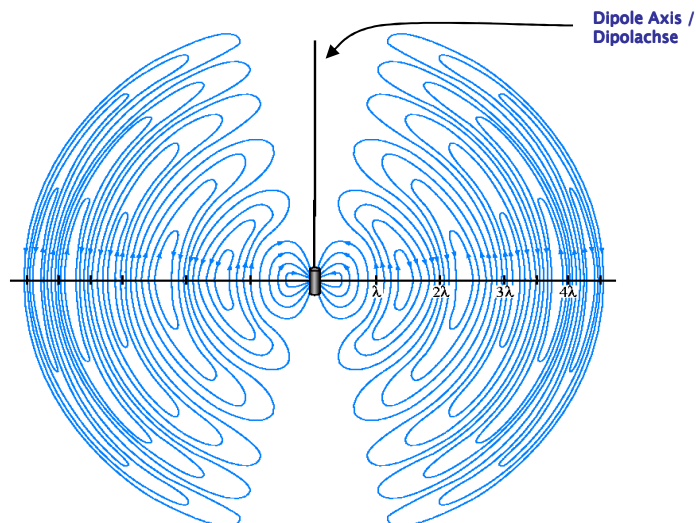
### Hertzian Dipole Antenna in Free-Space / Hertzische Dipolantenne im Freiraum (3)

Monofrequent/Monochromatic Excitation /  
Monofrequente/Monochromatische Anregung  $J_{ez}(\mathbf{R}, t) \sim \sin(\omega_0 t)$



### Hertzian Dipole Antenna in Free-Space / Hertzische Dipolantenne im Freiraum (4)

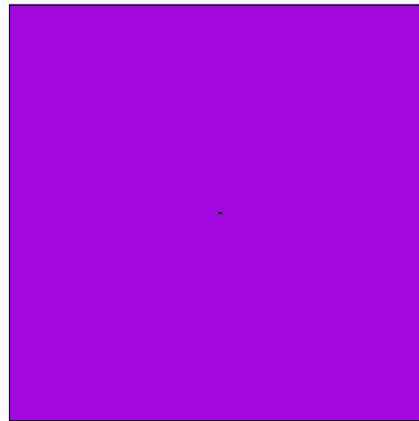
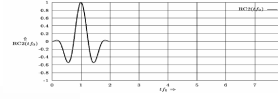
Electric Field Lines Surrounding an Oscillating Dipole at a Given Instant /  
Elektrische Feldlinien, die einen oszillierenden Dipol zu einem festen Zeitpunkt umgeben



### Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (5)

Broadband Pulse Excitation /  
Breitbandige Impulsanregung:

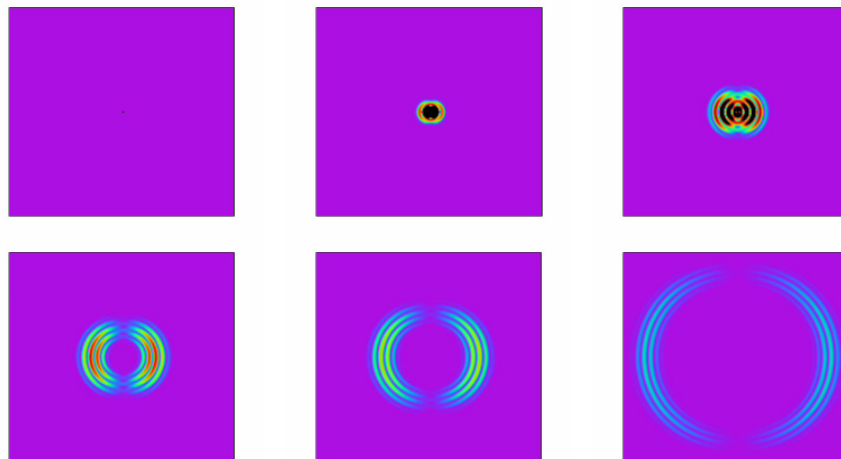
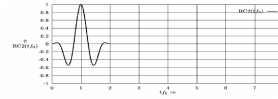
$$J_{ez}(\underline{\mathbf{R}}, t) \sim RC_2(t)$$



### Hertzian Dipole Antenna in Free-Space / Hertzsche Dipolantenne im Freiraum (6)

Broadband Pulse Excitation /  
Breitbandige Impulsanregung:

$$J_{ez}(\underline{\mathbf{R}}, t) \sim RC_2(t)$$



## Maxwell's Equations / Maxwell'sche Gleichungen



1. **André Marie Ampère (1775–1836)** 1827: Ampère presented the first mathematical theory of electrodynamics and discovered the magnetic effect of electric currents. / Ampère stellte die erste mathematisch fundierte elektrodynamische Theorie vor und entdeckte die magnetische Wirkung elektrischer Ströme.
2. **Michael Faraday (1791–1867)** 1831: Faraday discovers electromagnetic induction. / Faraday entdeckt die elektromagnetische Induktion.
3. **James Clerk Maxwell (1831–1879)** 1864: Maxwell presents his theory of electromagnetism. / Maxwell präsentiert seine Theorie des Elektromagnetismus.
4. **Heinrich Rudolf Hertz (1857–1894)** 1885: Hertz demonstrates the electromagnetic wave propagation in a series of experiments in a period through 1887. / Hertz demonstriert in einer Periode bis 1887 die Ausbreitung von elektromagnetischen Wellen.

## Heinrich Rudolf Hertz

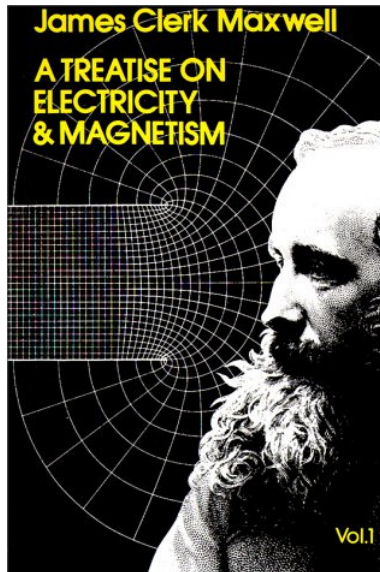


**Heinrich Rudolf Hertz** (\* 22. Februar 1857 in Hamburg, † 1. Januar 1894 in Bonn) war ein deutscher Physiker.

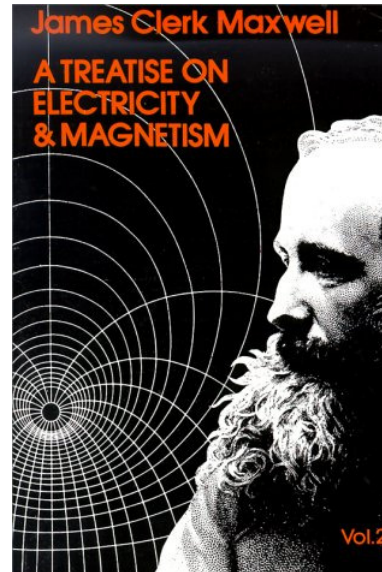
Er studierte an der Universität Berlin. Von 1885 bis 1889 lehrte er als Professor für Physik an der [technischen Hochschule in Karlsruhe](#). Ab 1889 war er Professor für Physik an der [Universität in Bonn](#). Sein Hauptverdienst lag im experimentellen Nachweis von [James Clerk Maxwell's](#) elektromagnetischer Theorie des [Lichts](#) von 1884. Hertz entdeckte in Karlsruhe die Existenz der [elektromagnetischen Wellen](#). Er wies nach, dass sie sich auf die gleiche Art und mit der gleichen Geschwindigkeit ausbreiten wie Lichtwellen (siehe: [Brechung](#), [Polarisation](#) und [Reflexion](#)). Seine Ergebnisse lieferten die Grundlage für die Entwicklung der drahtlosen [Telegraphie](#) und des [Radios](#). Die Einheit der [Frequenz](#), eine Schwingung pro Sekunde = 1 [Hertz](#) (Abk. 1 Hz), ist nach ihm benannt und seit 1933 im internationalen [metrischen System](#) verankert. Ebenfalls wurde der Hamburger Fernsehturm ([Heinrich-Hertz-Turm](#)) und das zur [Fraunhofer-Gesellschaft](#) gehörige Berliner Heinrich-Hertz-Institut nach ihm benannt.

**Heinrich Rudolf Hertz** (February 22, 1857 - January 1, 1894), was the German physicist for whom the [hertz](#), the SI unit of [frequency](#), is named. In 1888, he was the first to demonstrate the existence of [electromagnetic radiation](#) by building apparatus to produce [radio](#) waves. Hertz was born in [Hamburg, Germany](#), to a [Jewish](#) family that had converted to [Christianity](#). His father was an advocate in [Hamburg](#), his mother the daughter of a doctor. While at school, he showed an aptitude for sciences as well as languages, learning [Arabic](#) and [Sanskrit](#). He studied sciences and engineering in the German cities of [Dresden](#), [Munich](#) and [Berlin](#). He was a student of [Gustav R. Kirchhoff](#) and [Hermann von Helmholtz](#). He obtained his PhD in 1880, and remained a pupil of Helmholtz until 1883 when he took a post as a lecturer in theoretical physics at the [University of Kiel](#). In 1885 he became a full professor at the [University of Karlsruhe](#) where he discovered electromagnetic waves. Following [Michelson's 1881](#) experiment (precursor to the [1887 Michelson-Morley experiment](#)) which disproved the existence of [luminiferous aether](#), he reformulated [Maxwell's equations](#) to take the new discovery into account. Through experimentation, he proved that electric signals can travel through open air, as had been predicted by [James Clerk Maxwell](#) and [Michael Faraday](#), and which is the basis for the invention of [radio](#). He also discovered the [photoelectric effect](#) (which was later explained by [Albert Einstein](#)) when he noticed that a [charged](#) object loses its charge more readily when illuminated by ultraviolet light. He died in [Bonn](#), Germany. His nephew [Gustav Ludwig Hertz](#) was a [Nobel Prize](#) winner, and Gustav's son [Carl Hellmuth Hertz](#) invented [medical ultrasonography](#).

## Maxwell's Equations / Maxwell'sche Gleichungen



<http://www.amazon.de>



<http://www.amazon.de>

## Today's Vector Notation of Maxwell's Equations / Heutige Vektornotation der Maxwell'schen Gleichungen

**Governing Equations in  
Differential Form /  
Grundgleichungen in  
Differentialform**

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) - \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t)$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, t) = \frac{\partial}{\partial t} \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) + \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t)$$

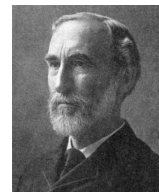
$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}, t) = \rho_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{B}}(\underline{\mathbf{R}}, t) = \rho_m(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{J}}_e(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \rho_e(\underline{\mathbf{R}}, t)$$

$$\nabla \cdot \underline{\mathbf{J}}_m(\underline{\mathbf{R}}, t) = -\frac{\partial}{\partial t} \rho_m(\underline{\mathbf{R}}, t)$$

**Josiah Willard Gibbs  
(1839-1903)**



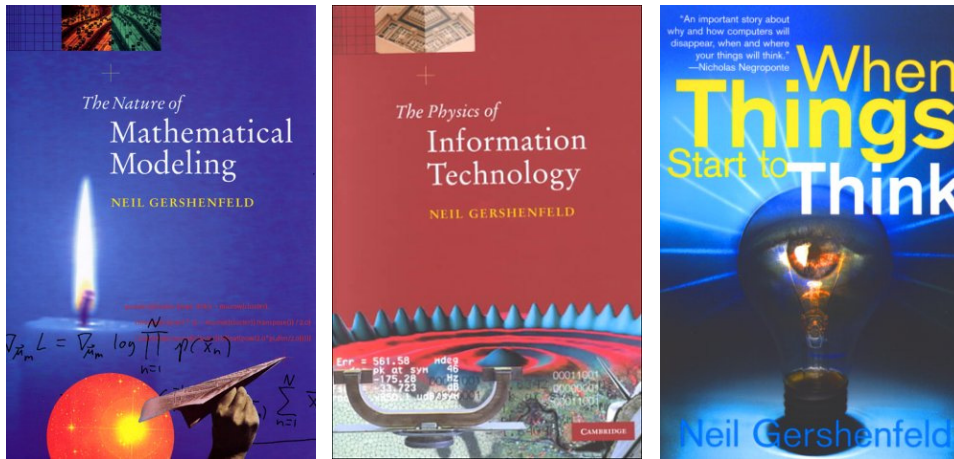
**Oliver Heaviside  
(1850-1925)**



**Paul Adrien Maurice Dirac  
(1902-1984)**



## ... Other Books / ... andere Bücher



End of Lecture 2 /  
Ende der 2. Vorlesung