Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

3rd Lecture / 3. Vorlesung

Dr.-Ing. René Marklein

marklein@uni-kassel.de
http://www.tet.e-technik.uni-kassel.de
http://www.uni-kassel.de/fb16/tet/marklein/index.html

Universität Kassel
Fachbereich Elektrotechnik / Informatik
(FB 16)
Fachgebiet Theoretische Elektrotechnik
(FG TET)
Wilhelmshöher Allee 71
Büro: Raum 2113 / 2115
D-34121 Kassel

University of Kassel
Dept. Electrical Engineering / Computer Science
(FB 16)
Electromagnetic Field Theory
(FG TET)
Wilhelmshöher Allee 71
Office: Room 2113 / 2115
D-34121 Kassel

Math: Requirements & Recommendations / Mathe: Voraussetzungen & Empfehlungen

Analysis / Analysis

Vector Analysis / Vektoranalysis

Algebra / Algebra

Differential Geometry / Differentialgeometrie

Differential Equations / Differentialgleichungen

Special Functions / Spezielle Funktionen

Integral Transforms / Integraltransformationen

Prof. Dr. rer. nat. Karl-Jörg Langenberg

$\Rightarrow$ Mathematical Foundation of Electromagnetic Field Theory I & II / Mathematische Grundlagen der Elektromagnetischen Feldtheorie I & II

Different Coordinate Systems / Verschiedene Koordinatensysteme

- Cartesian (Rectangular) Coordinate System / Kartesisches Koordinatensystem
- Cylindrical Coordinate System / Zylinderkoordinatensystem
- Spherical Coordinate System / Kugelkoordinatensystem

What is the benefit of the Use of a Problem Matched Coordinate Systems ? /
Was ist der Nutzen der Verwendung eines problemangepassten Koordinatensystemen ?

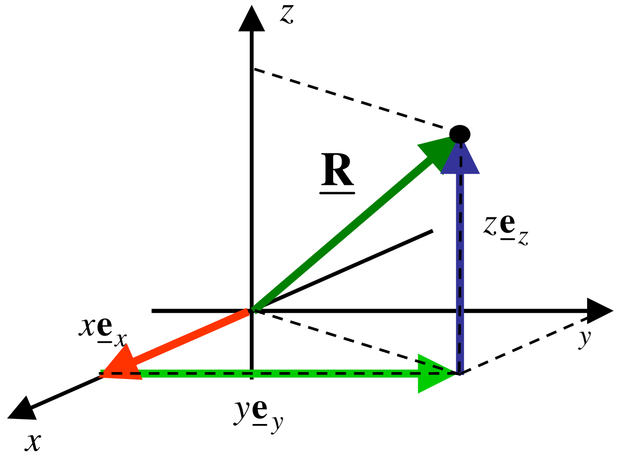
⇓

(Easier) Solution of the Problem under Concern! /
(Einfachere) Lösung des betrachteten Problems?

Position Vector / Ortsvektor (Positionsvektor)

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_x(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_y(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_z(\underline{\mathbf{R}})$$
$$= R_x(\underline{\mathbf{R}})\underline{\mathbf{e}}_x + R_y(\underline{\mathbf{R}})\underline{\mathbf{e}}_y + R_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$
$$= x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$



Coordinates / Koordinaten: $x, y, z; \quad -\infty < x, y, z < \infty$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren: $\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \qquad |\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten:

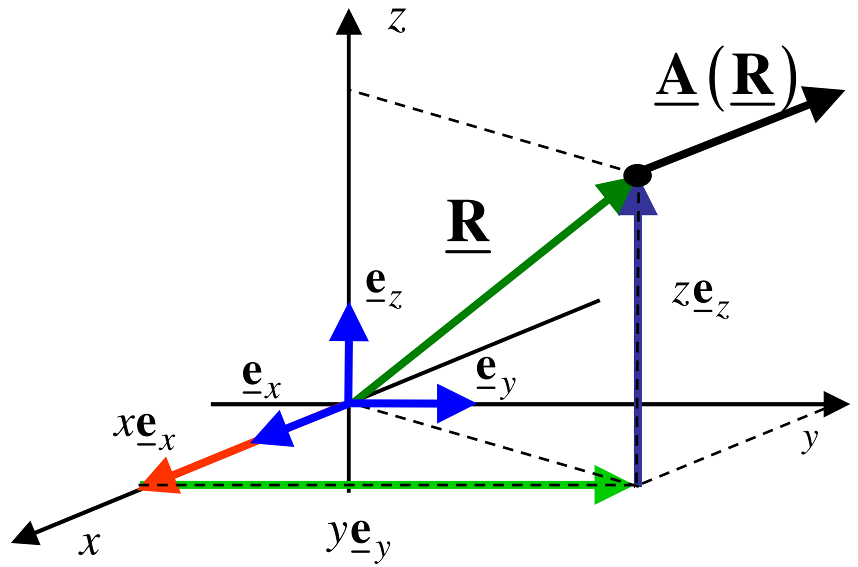
$$R_x(x, y, z) = x$$
$$R_y(x, y, z) = y$$
$$R_z(x, y, z) = z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten:

$$\underline{\mathbf{R}}_x(\underline{\mathbf{R}}) = R_x(x, y, z)\underline{\mathbf{e}}_x = x\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{R}}_y(\underline{\mathbf{R}}) = R_y(x, y, z)\underline{\mathbf{e}}_y = y\underline{\mathbf{e}}_y$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = R_z(x, y, z)\underline{\mathbf{e}}_z = z\underline{\mathbf{e}}_z$$

Field Vector / Feldvektor

Cartesian Coordinate System / Kartesisches Koordinatensystem



Coordinates / Koordinaten x, y, z

Limits / Grenzen

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$|\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$$

$\perp$: Perpendicular / Senkrecht

Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}}) = \underline{\mathbf{A}}_x(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_y(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_z(\underline{\mathbf{R}})$$

$$= \mathrm{A}_x(x, y, z)\underline{\mathbf{e}}_x + \mathrm{A}_y(x, y, z)\underline{\mathbf{e}}_y + \mathrm{A}_z(x, y, z)\underline{\mathbf{e}}_z$$

Notation and Field Quantities / Notation und Feldgrößen

Vector / Vektor: Electric Field Strength / Elektrische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underbrace{\underline{\mathbf{E}}_x(\underline{\mathbf{R}},t) + \underline{\mathbf{E}}_y(\underline{\mathbf{R}},t) + \underline{\mathbf{E}}_z(\underline{\mathbf{R}},t)}_{\text{3 Vector Components / 3 Vektorkomponenten}}$$

$$= \mathrm{E}_x(x,y,z,t)\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\underline{\mathbf{e}}_z$$

mit $\{x,y,z\} = \{x_1,x_2,x_3\}$

$$= \sum_{i=1}^{3} \mathrm{E}_{x_i}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}$$

$$= \mathrm{E}_{x_i}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}$$

Dyad / Dyade: Permittivity Dyad / Permittivitätsdyade

$$\underline{\underline{\varepsilon}}(\underline{\mathbf{R}},t) = \underbrace{\begin{aligned} &\underline{\underline{\varepsilon}}_{xx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{xy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{xz}(\underline{\mathbf{R}},t) \\ &+\underline{\underline{\varepsilon}}_{yx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{yy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{yz}(\underline{\mathbf{R}},t) \\ &+\underline{\underline{\varepsilon}}_{zx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{zy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{zz}(\underline{\mathbf{R}},t) \end{aligned}}_{\text{9 Dyadic Components / 9 dyadische Komponenten}}$$

$$\begin{aligned} =\, &\varepsilon_{xx}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_x + \varepsilon_{xy}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_y + \varepsilon_{xz}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_z \\ &+\varepsilon_{yx}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_x + \varepsilon_{yy}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_y + \varepsilon_{yz}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_z \\ &+\varepsilon_{zx}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_x + \varepsilon_{zy}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_y + \varepsilon_{zz}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_z \end{aligned}$$

mit $\{x,y,z\} = \{x_1,x_2,x_3\}$

$$= \sum_{i=1}^{3}\sum_{j=1}^{3} \varepsilon_{x_i x_j}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\underline{\mathbf{e}}_{x_j}$$

$$= \varepsilon_{x_i x_j}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\underline{\mathbf{e}}_{x_j}$$

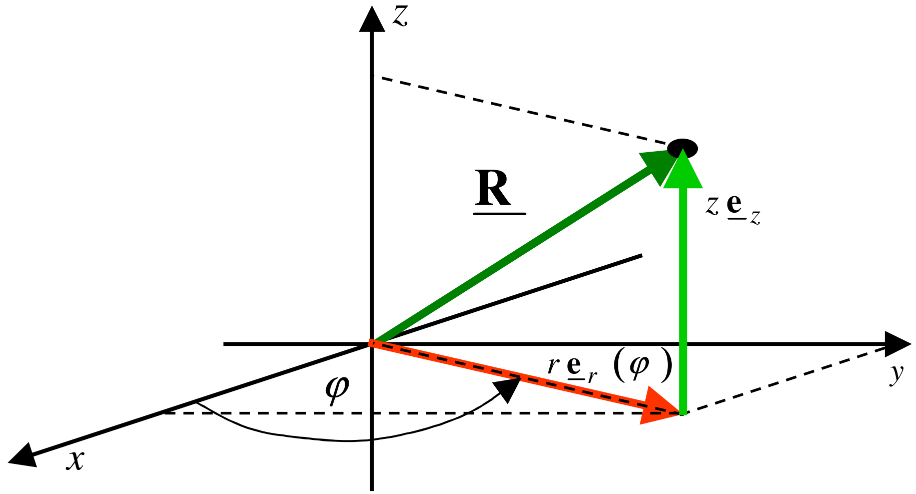
with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

Einstein's Summation Convention: If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. / *Einsteinsche Summenkonvention*: Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.

Position Vector / Ortsvektor (Positionsvektor)

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_r(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_z(\underline{\mathbf{R}})$$
$$= R_r(\underline{\mathbf{R}})\underline{\mathbf{e}}_r(\varphi) + R_\varphi(\underline{\mathbf{R}})\underline{\mathbf{e}}_\varphi(\varphi) + R_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$
$$= r\,\underline{\mathbf{e}}_r(\varphi) + z\,\underline{\mathbf{e}}_z$$



Coordinates / Koordinaten $r, \varphi, z; \quad 0 \le r < \infty, 0 \le \varphi < 2\pi, -\infty < z < \infty$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_r(\varphi), \underline{\mathbf{e}}_\varphi(\varphi), \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_r(\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi) \perp \underline{\mathbf{e}}_z \quad |\underline{\mathbf{e}}_r(\varphi)| = |\underline{\mathbf{e}}_\varphi(\varphi)| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten

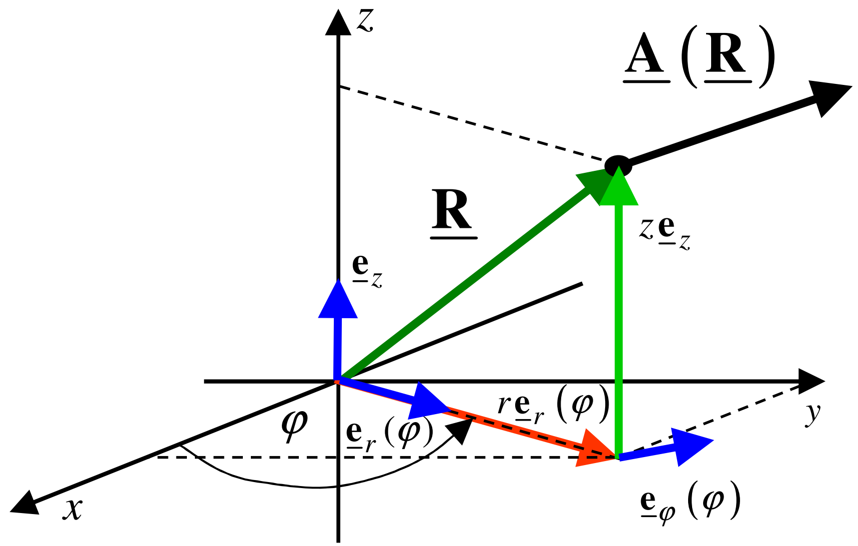
$$R_r(r,\varphi,z) = r\,\underline{\mathbf{e}}_r(\varphi)$$
$$R_\varphi(r,\varphi,z) = 0$$
$$R_z(r,\varphi,z) = z\,\underline{\mathbf{e}}_z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_r(\underline{\mathbf{R}}) = R_r(r)\underline{\mathbf{e}}_r(\varphi) = r\,\underline{\mathbf{e}}_r(\varphi)$$
$$\underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = R_z(z)\underline{\mathbf{e}}_z = z\,\underline{\mathbf{e}}_z$$

Field Vector / Feldvektor

Cylindrical Coordinate System / Zylinderkoordinatensystem



Coordinates / Koordinaten r, φ, z

Limits / Grenzen

$$0 \le r < \infty$$
$$0 \le \varphi < 2\pi$$
$$-\infty < z < \infty$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_r(\varphi), \underline{\mathbf{e}}_\varphi(\varphi), \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_r(\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi) \perp \underline{\mathbf{e}}_z$$

$$\left|\underline{\mathbf{e}}_r(\varphi)\right| = \left|\underline{\mathbf{e}}_\varphi(\varphi)\right| = \left|\underline{\mathbf{e}}_z\right| = 1$$

$\perp$: Perpendicular / Senkrecht

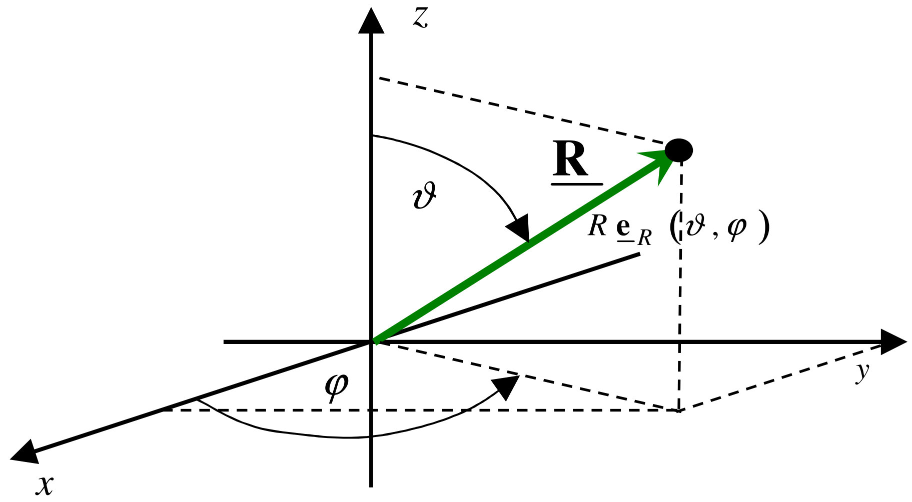
Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}}) = \underline{\mathbf{A}}_r(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_\varphi(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_z(\underline{\mathbf{R}})$$
$$= \mathrm{A}_r(r,\varphi,z)\,\underline{\mathbf{e}}_r(\varphi) + A_\varphi(r,\varphi,z)\,\underline{\mathbf{e}}_\varphi(\varphi) + A_z(r,\varphi,z)\,\underline{\mathbf{e}}_z$$

Position Vector / Ortsvektor (Positionsvektor)

Spherical Coordinate System / Kugelkoordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_R(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\vartheta(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}})$$
$$= R_R(\underline{\mathbf{R}})\underline{\mathbf{e}}_R(\vartheta,\varphi) + R_\vartheta(\underline{\mathbf{R}})\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)$$
$$+ R_\varphi(\underline{\mathbf{R}})\underline{\mathbf{e}}_\varphi(\varphi)$$
$$= R\underline{\mathbf{e}}_R(\vartheta,\varphi)$$



Coordinates / Koordinaten $R, \vartheta, \varphi; \quad 0 \le R < \infty, 0 \le \vartheta \le \pi; 0 \le \varphi < 2\pi$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren $\underline{\mathbf{e}}_R, \underline{\mathbf{e}}_\vartheta, \underline{\mathbf{e}}_\varphi$

$$\underline{\mathbf{e}}_R \perp \underline{\mathbf{e}}_\vartheta \perp \underline{\mathbf{e}}_\varphi \quad |\underline{\mathbf{e}}_R| = |\underline{\mathbf{e}}_\vartheta| = |\underline{\mathbf{e}}_\varphi| = 1$$

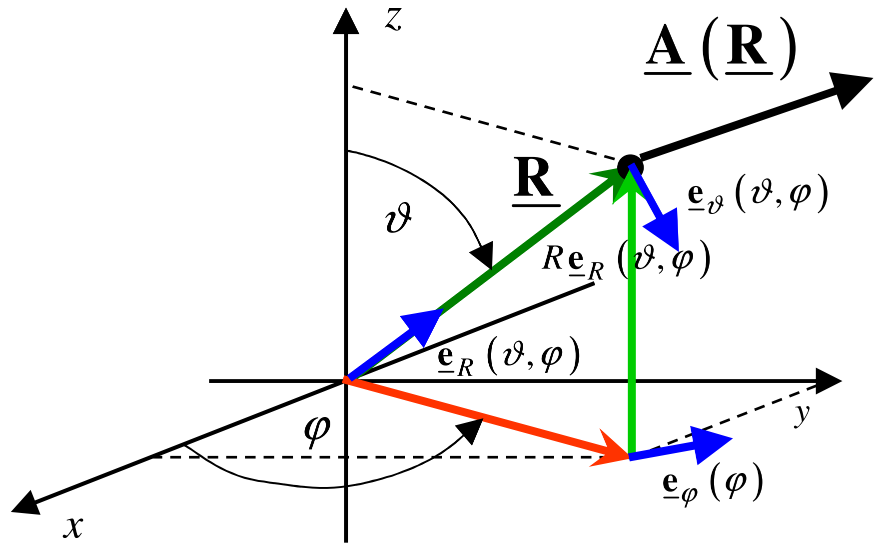
Scalar Vector Components / Skalare Vektorkomponenten $R_R(R,\vartheta,\varphi), R_\vartheta(R,\vartheta,\varphi), R_\varphi(R,\vartheta,\varphi)$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_R(\underline{\mathbf{R}}) = R_R(R,\vartheta,\varphi)\underline{\mathbf{e}}_R(\vartheta,\varphi) = R\underline{\mathbf{e}}_R(\vartheta,\varphi)$$
$$\underline{\mathbf{R}}_\vartheta(\underline{\mathbf{R}}) = R_\vartheta(R,\vartheta,\varphi)\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) = R_\varphi(R,\vartheta,\varphi)\underline{\mathbf{e}}_\varphi(\varphi) = \underline{\mathbf{0}}$$

Field Vector / Feldvektor

Spherical Coordinate System / Kugelkoordinatensystem



Coordinates / Koordinaten R, ϑ, φ

Limits / Grenzen

$$0 \le R < \infty$$
$$0 \le \vartheta \le \pi$$
$$0 \le \varphi < 2\pi$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_R(\vartheta,\varphi), \underline{\mathbf{e}}_\vartheta(\vartheta,\varphi), \underline{\mathbf{e}}_\varphi(\varphi)$$

$$\underline{\mathbf{e}}_R(\vartheta,\varphi) \perp \underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi)$$

$\perp$: Perpendicular / Senkrecht

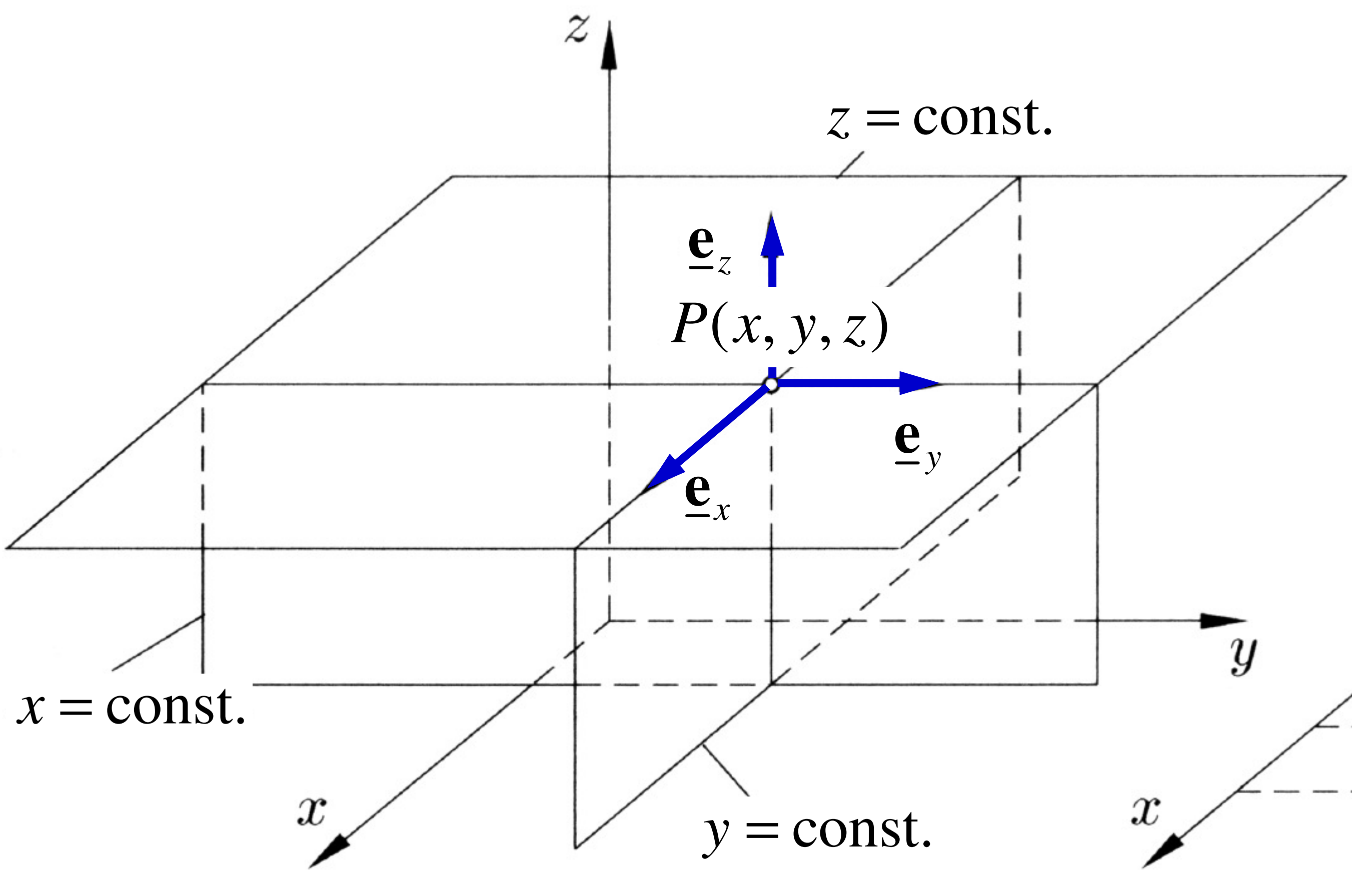
$$|\underline{\mathbf{e}}_R(\vartheta,\varphi)| = |\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)| = |\underline{\mathbf{e}}_\varphi(\varphi)| = 1$$

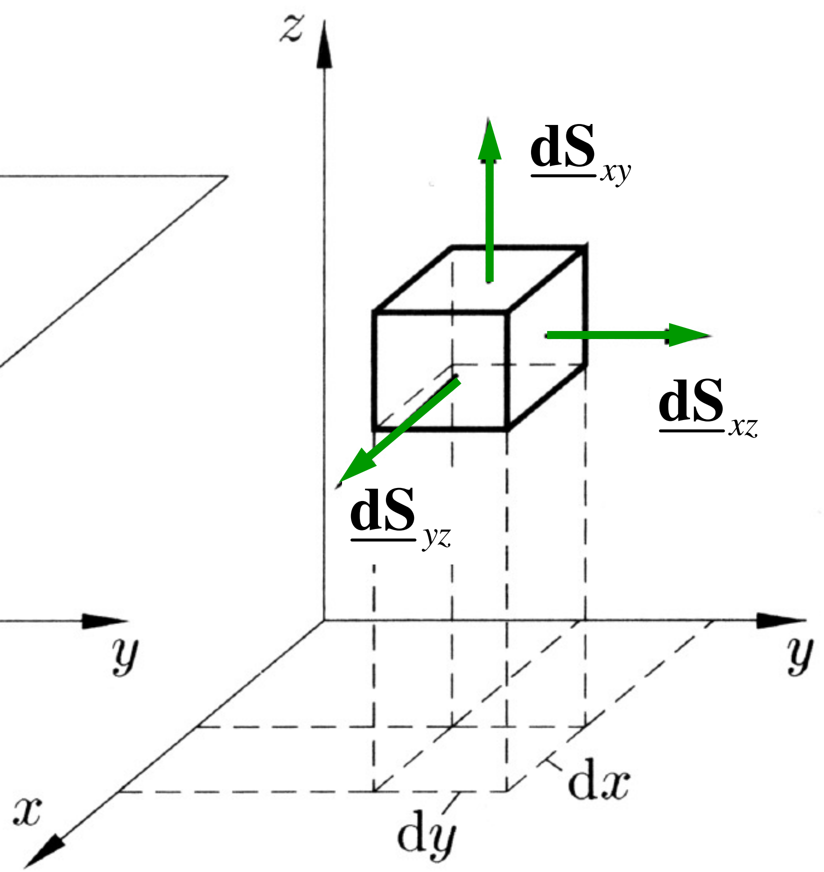
Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}},t) = \underline{\mathbf{A}}_R(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_\vartheta(\underline{\mathbf{R}}) + \underline{\mathbf{A}}_\varphi(\underline{\mathbf{R}})$$

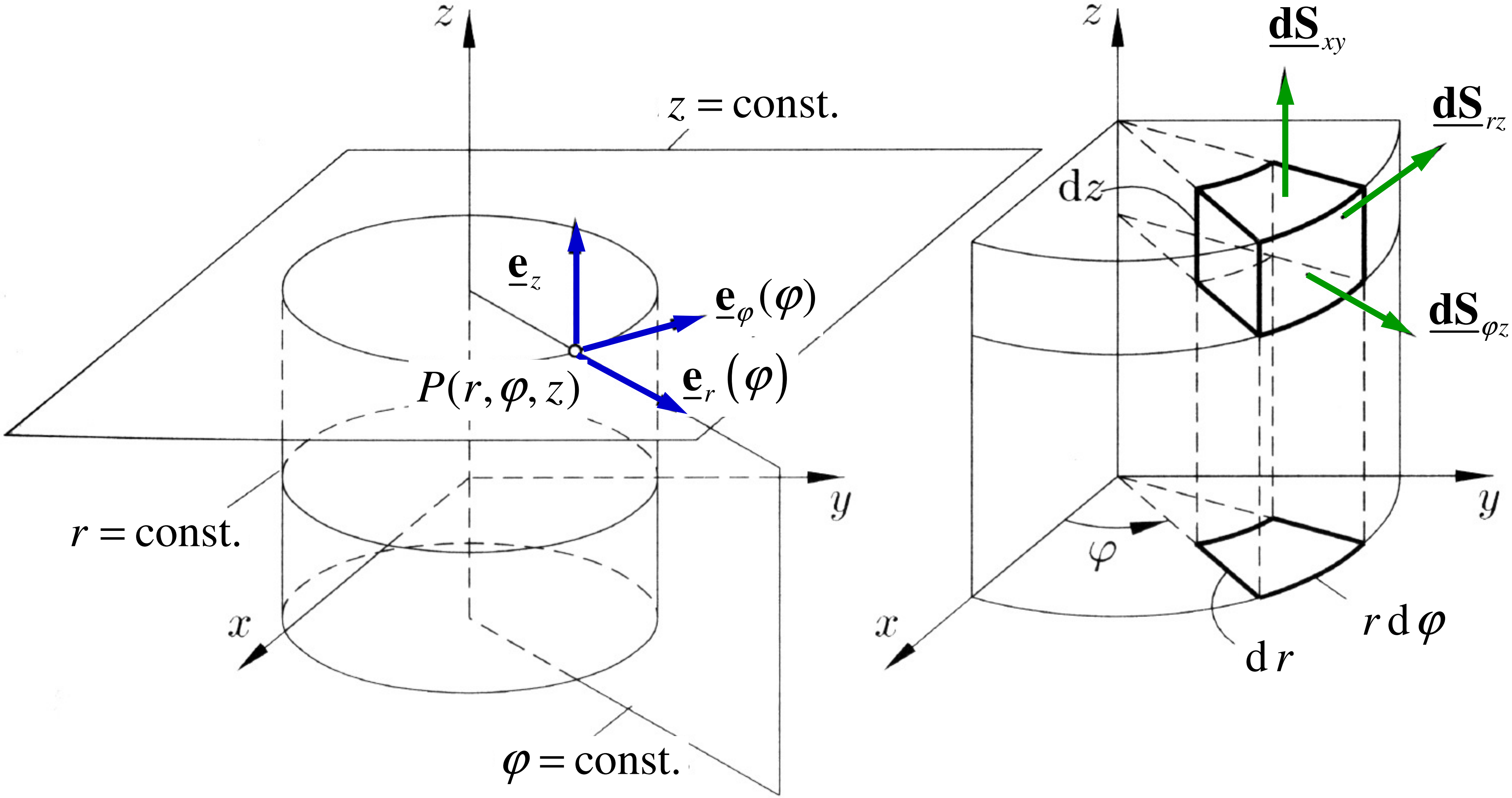
$$= \mathrm{A}_R(R,\vartheta,\varphi)\underline{\mathbf{e}}_R(\vartheta,\varphi) + A_\vartheta(R,\vartheta,\varphi)\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) + A_\varphi(R,\vartheta,\varphi)\underline{\mathbf{e}}_\varphi(\varphi)$$

Cartesian Coordinate System: Coordinate Surfaces, Unit Vectors, Surface Elements and Volume Element / Kartesischen Koordinatensystemen: Koordinatenflächen, Einheitsvektoren, Flächenelemente und Volumenelement

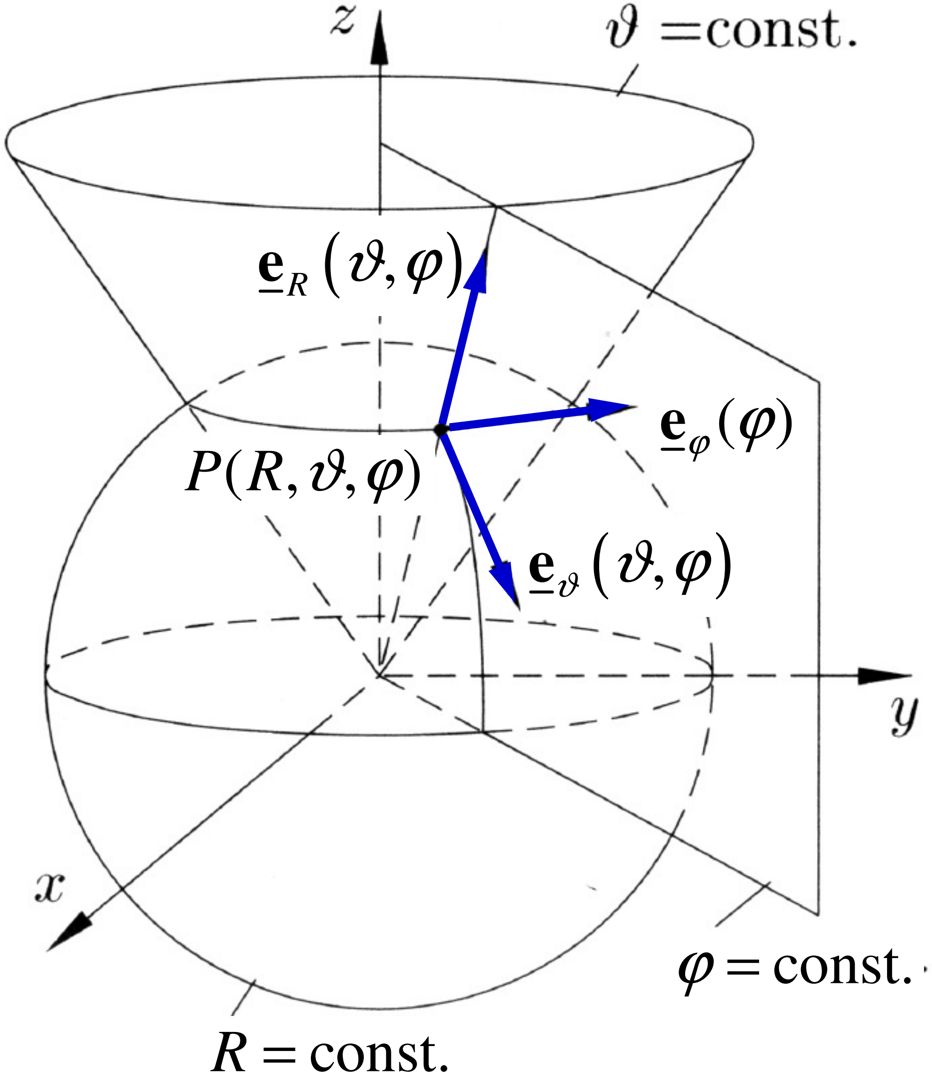


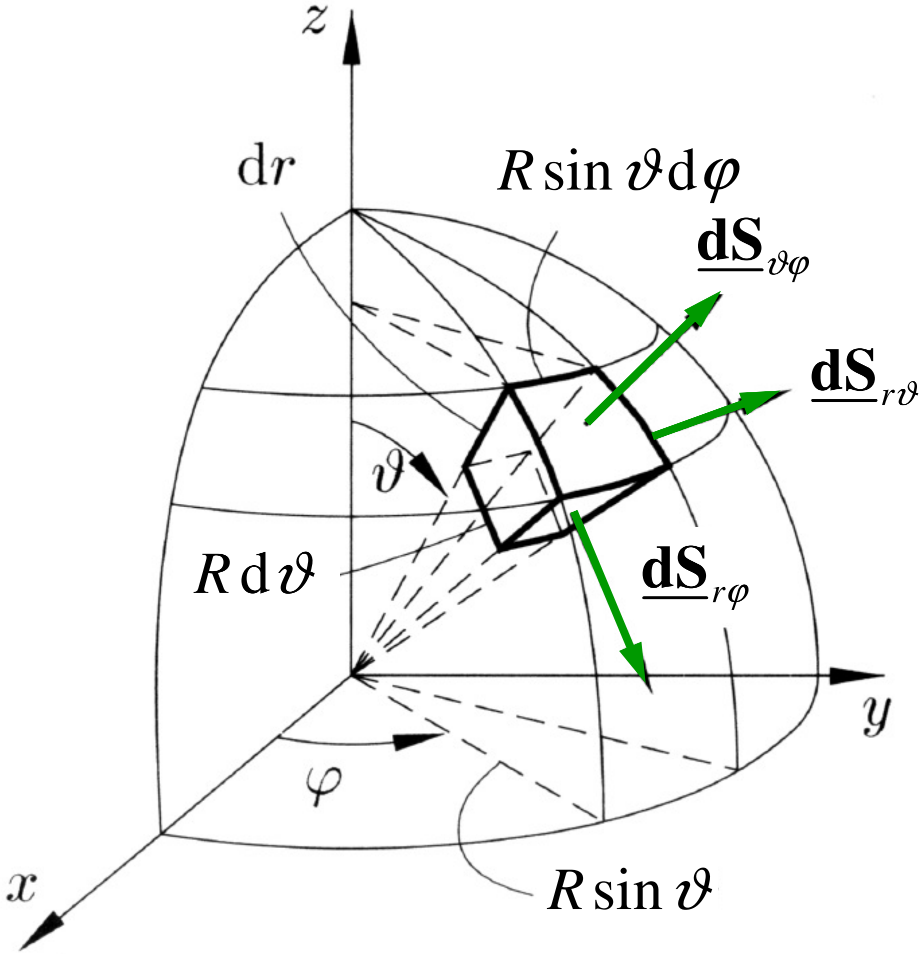


Cylindrical Coordinate System: Coordinate Surfaces, Unit Vectors, Surface Elements and Volume Element / Zylinderkoordinatensystem: Koordinatenflächen, Einheitsvektoren, Flächenelemente und Volumenelement



Spherical Coordinate System: Coordinate Surfaces, Unit Vectors, Surface Elements and Volume Element / Kugelkoordinatensystem: Koordinatenflächen, Einheitsvektoren, Flächenelemente und Volumenelement





Metric Coefficients and Vector Differential Line Elements / Metrische Koeffizienten und vektorielle differentielle Linienelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_x = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_x\, h_x\, \mathrm{d}x = \underline{\mathbf{e}}_x\, \mathrm{d}x$$

$$\underline{\mathbf{dR}}_y = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_y\, h_y\, \mathrm{d}y = \underline{\mathbf{e}}_y\, \mathrm{d}y$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{n}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\, h_z\, \mathrm{d}z = \underline{\mathbf{e}}_z\, \mathrm{d}z$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_r = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_r\, h_r\, \mathrm{d}r = \underline{\mathbf{e}}_r\, \mathrm{d}r$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\, h_\varphi\, \mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\, r\, \mathrm{d}\varphi$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\, h_z\, \mathrm{d}z = \underline{\mathbf{e}}_z\, \mathrm{d}z$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

$$\underline{\mathbf{dR}}_R = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_R\, h_R\, \mathrm{d}R = \underline{\mathbf{e}}_R\, \mathrm{d}R$$

$$\underline{\mathbf{dR}}_\vartheta = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\vartheta\, h_\vartheta\, \mathrm{d}\vartheta = \underline{\mathbf{e}}_\vartheta\, R\, \mathrm{d}\vartheta$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\, h_\varphi\, \mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\, R\sin\vartheta\, \mathrm{d}\varphi$$

Metric Coefficients and Differential Volume and Surface Elements / Metrische Koeffizienten und differentielle Volumen- und Flächenelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\begin{aligned} \mathrm{d}V &= h_x\,\mathrm{d}x\,h_y\,\mathrm{d}y\,h_z\,\mathrm{d}z \\ &= h_x\,h_y\,h_z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ &= \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{yz} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z)\,h_y\,h_z\,\mathrm{d}y\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{xz} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x)\,h_x\,h_z\,\mathrm{d}x\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_y\,\mathrm{d}x\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{xy} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y)\,h_x\,h_y\,\mathrm{d}x\,\mathrm{d}y \\ &= \underline{\mathbf{e}}_z\,\mathrm{d}x\,\mathrm{d}y \end{aligned}$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\begin{aligned} \mathrm{d}V &= h_r\,\mathrm{d}r\,h_\varphi\,\mathrm{d}\varphi\,h_z\,\mathrm{d}z \\ &= h_r\,h_\varphi\,h_z\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}y \\ &= r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{\varphi z} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_z)\,h_\varphi\,h_z\,\mathrm{d}\varphi\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_r\,r\,\mathrm{d}y\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{rz} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_r)\,h_r\,h_z\,\mathrm{d}r\,\mathrm{d}z \\ &= \underline{\mathbf{e}}_\varphi\,\mathrm{d}r\,\mathrm{d}z \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{r\varphi} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_r \times \underline{\mathbf{e}}_\varphi)\,h_r\,h_\varphi\,\mathrm{d}r\,\mathrm{d}\varphi \\ &= \underline{\mathbf{e}}_z\,r\,\mathrm{d}r\,\mathrm{d}\varphi \end{aligned}$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

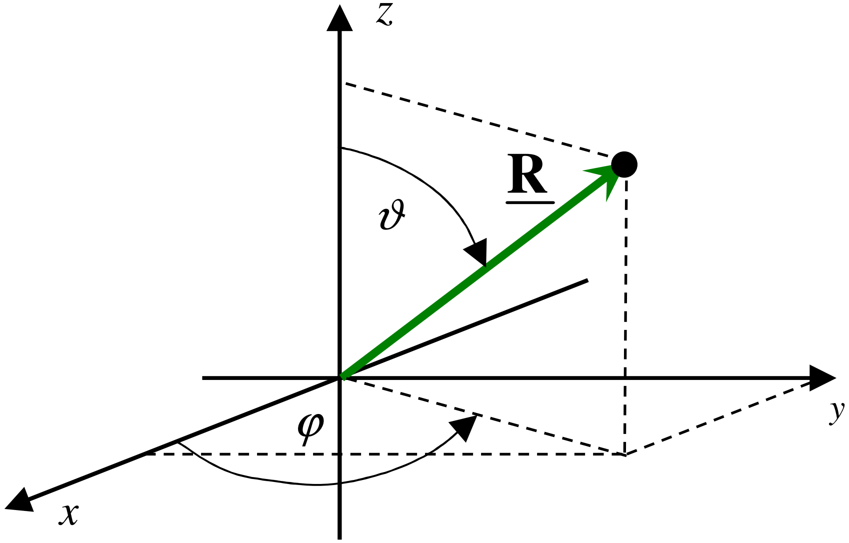
$$\begin{aligned} \mathrm{d}V &= h_R\,\mathrm{d}R\,h_\vartheta\,\mathrm{d}\vartheta\,h_\varphi\,\mathrm{d}\varphi \\ &= h_R\,h_\vartheta\,h_\varphi\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \\ &= R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{\vartheta\varphi} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_\vartheta \times \underline{\mathbf{e}}_\varphi)\,h_\vartheta\,h_\varphi\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \\ &= \underline{\mathbf{e}}_R\,R^2\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{r\varphi} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_R)\,h_R\,h_\varphi\,\mathrm{d}R\,\mathrm{d}\varphi \\ &= \underline{\mathbf{e}}_\vartheta\,R\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{dS}}_{R\vartheta} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= (\underline{\mathbf{e}}_R \times \underline{\mathbf{e}}_\vartheta)\,h_R\,h_\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta \\ &= \underline{\mathbf{e}}_\varphi\,R\,\mathrm{d}R\,\mathrm{d}\vartheta \end{aligned}$$

Coordinates of Different Coordinate Systems / Koordinaten verschiedenen Koordinatensystemen



Transformation Table / Umrechnungstabelle

Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
x y z	$r\cos\varphi$ $r\sin\varphi$ z	$R\sin\vartheta\cos\varphi$ $R\sin\vartheta\sin\varphi$ $R\cos\vartheta$
$\sqrt{x^2+y^2}$ $\arctan\frac{y}{x}$ z	r φ z	$R\sin\vartheta$ φ $R\cos\vartheta$
$\sqrt{x^2+y^2+z^2}$ $\arctan\frac{\sqrt{x^2+y^2}}{z}$ $\arctan\frac{y}{x}$	$\sqrt{r^2+z^2}$ $\arctan\frac{r}{z}$ φ	R ϑ φ

Examples / Beispiele

1. Formulate *x* as a function of the cylinder and spherical coordinates. / Formuliere *x* als Funktion der Zylinder- und Kugelkoordinaten.

$$x = r\cos\varphi = R\sin\vartheta\cos\varphi$$

2. Formulate *r* as a function of the Cartesian and spherical coordinates. / Formuliere *r* als Funktion der Kartesischen und Kugelkoordinaten.

$$r = \sqrt{x^2 + y^2} = R\sin\vartheta$$

3. Formulate $\sqrt{x^2 + y^2}$ as a function of the cylinder coordinates. / Formuliere $\sqrt{x^2 + y^2}$ als Funktion der Zylinderkoordinaten.

$$\sqrt{x^2 + y^2} = \sqrt{(r\cos\varphi)^2 + (r\sin\varphi)^2} = r\sqrt{\underbrace{\cos^2\varphi + \sin^2\varphi}_{=1}} = r$$

Scalar Vector Components in Different Coordinate Systems / Skalare Vektorkomponenten in verschiedenen Koordinatensystemen

Transformation Table / Umrechnungstabelle

Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\underline{\mathbf{A}} = A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z$	$\underline{\mathbf{A}} = A_r \underline{\mathbf{e}}_r + A_\varphi \underline{\mathbf{e}}_\varphi + A_z \underline{\mathbf{e}}_z$	$\underline{\mathbf{A}} = A_R \underline{\mathbf{e}}_R + A_\vartheta \underline{\mathbf{e}}_\vartheta + A_\varphi \underline{\mathbf{e}}_\varphi$
A_x A_y A_z	$A_r \cos\varphi - A_\varphi \sin\varphi$ $A_r \sin\varphi + A_\varphi \cos\varphi$ A_z	$A_R \sin\vartheta \cos\varphi + A_\vartheta \cos\vartheta \cos\varphi - A_\varphi \sin\varphi$ $A_R \sin\vartheta \sin\varphi + A_\vartheta \cos\vartheta \sin\varphi + A_\varphi \cos\varphi$ $A_R \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \cos\varphi + A_y \sin\varphi$ $-A_x \sin\varphi + A_y \cos\varphi$ A_z	A_r A_φ A_z	$A_R \sin\vartheta + A_\vartheta \cos\vartheta$ A_φ $A_R \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \sin\vartheta \cos\varphi + A_y \sin\vartheta \sin\varphi + A_z \cos\vartheta$ $A_x \cos\vartheta \cos\varphi + A_y \cos\vartheta \sin\varphi - A_z \sin\vartheta$ $-A_x \sin\varphi + A_y \cos\varphi$	$A_r \sin\vartheta + A_z \cos\vartheta$ $A_r \cos\vartheta - A_z \sin\vartheta$ A_φ	A_R A_ϑ A_φ

Example: Coordinate Transformation of the Position Vector / Beispiel: Koordinatentransformation des Ortsvektor

Position Vector in the Cartesian Coordinate System / Ortsvektor im Kartesischen Koordinatensystem

$$\underline{\mathbf{R}} = \underbrace{x}_{R_x(x,y,z)} \underline{\mathbf{e}}_x + \underbrace{y}_{R_y(x,y,z)} \underline{\mathbf{e}}_y + \underbrace{z}_{R_z(x,y,z)} \underline{\mathbf{e}}_z$$

Transformation of the Coordinates / Transformation der Koordinaten

$$R_x(r,\varphi,z) = x(r,\varphi,z) = r\cos\varphi$$
$$R_y(r,\varphi,z) = y(r,\varphi,z) = r\sin\varphi$$
$$R_z(r,\varphi,z) = z(r,\varphi,z) = z$$

Position Vector in the Cylinder Coordinate System / Ortsvektor im Zylinderkoordinatensystem ?

$$\underline{\mathbf{R}}(r,\varphi,y,R_r,R_\varphi,R_z)$$
$$= R_r(r,\varphi,y)\underline{\mathbf{e}}_r(\varphi) + R_\varphi(r,\varphi,y)\underline{\mathbf{e}}_\varphi(\varphi) + R_z(r,\varphi,y)\underline{\mathbf{e}}_z$$

Position Vector in the Cartesian Coordinate System as a Function of Cylinder Coordinates / Ortsvektor im Kartesischen Koordinatensystem als Funktion der Zylinderkoordinaten

$$\underline{\mathbf{R}} = \underbrace{r\cos\varphi}_{R_x(r,\varphi,z)} \underline{\mathbf{e}}_x + \underbrace{r\sin\varphi}_{R_y(r,\varphi,z)} \underline{\mathbf{e}}_y + \underbrace{z}_{R_z(r,\varphi,z)} \underline{\mathbf{e}}_z$$

Transformation of the Scalar Vector Components / Transformation der skalaren Vektorkomponenten

$$R_r(r,\varphi,z,R_x,R_y,R_z) = R_x\cos\varphi + R_y\sin\varphi$$
$$R_\varphi(r,\varphi,z,R_x,R_y,R_z) = -R_x\sin\varphi + R_y\cos\varphi$$
$$R_z(r,\varphi,z,R_x,R_y,R_z) = R_z$$

$$\downarrow$$

$$R_r = r\cos\varphi\cos\varphi + r\sin\varphi\sin\varphi$$
$$= r\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{=1} = r$$
$$R_\varphi = -r\cos\varphi\sin\varphi + r\sin\varphi\cos\varphi$$
$$= 0$$
$$R_z = R_z$$

Position Vector in the Cylinder Coordinate System / Ortsvektor in dem Zylinderkoordinatensystem

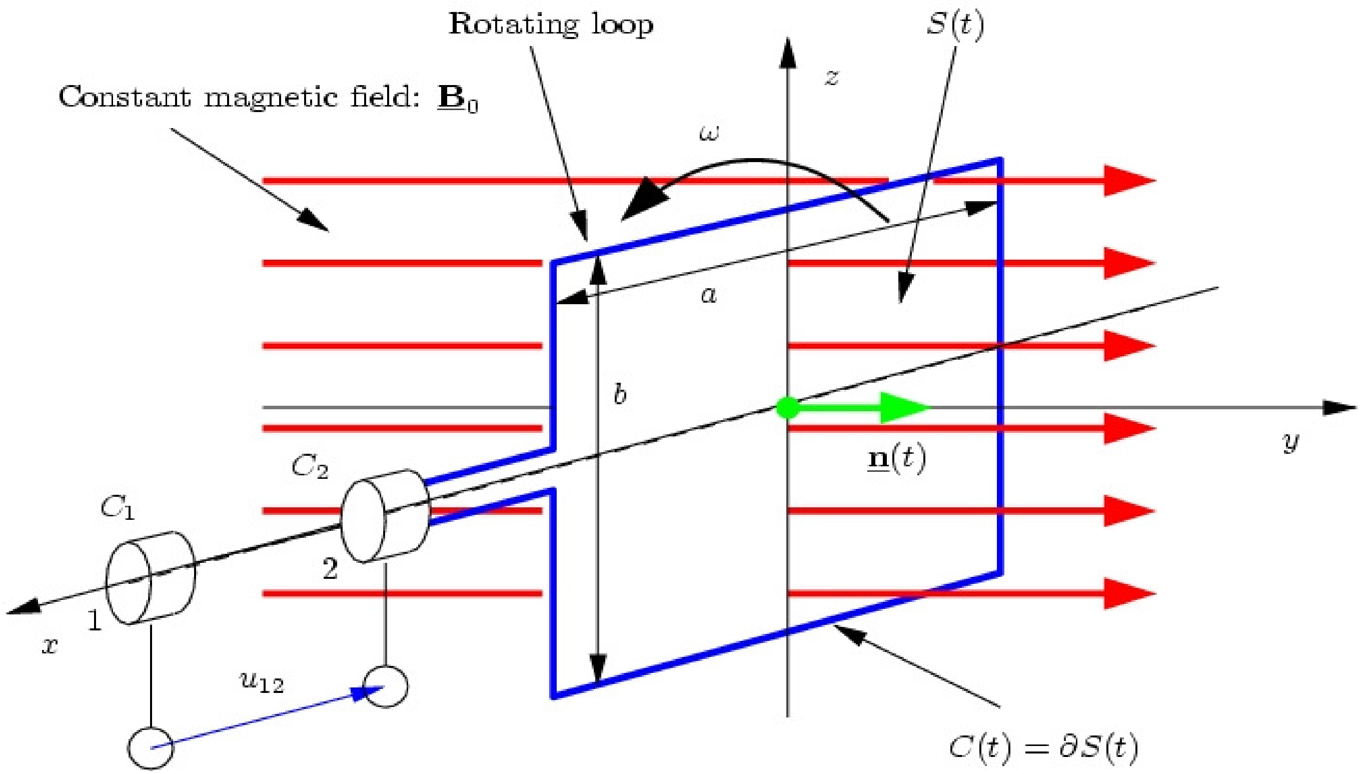
$$\underline{\mathbf{R}} = \underbrace{r}_{R_r} \underline{\mathbf{e}}_r(\varphi) + \underbrace{z}_{R_z} \underline{\mathbf{e}}_z$$

Faraday‘s Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (1)

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Time Dependent Surface / Zeitabhängige Fläche $\quad S(t) \quad C(t)=\partial S(t) \quad$ Time Dependent Contour / Zeitabhängige Kontur



Faraday‘s Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (2)

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$\oint_{C(t)=\partial S(t)}[\circ]\bullet\underline{\mathbf{dR}}$	[m]	Closed Contour Integral / Geschlossenes Kurvenintegral
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$	[V/m]	Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{dR}}$	[m]	Vectorial Differential Line Element / Vektorielles differentielles Linienelement
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}}$	[V]	Scalar Product of E and dR = tangential projection of E onto dR / Skalarprodukt von E auf dR = Tangentialprojektion von E auf dR

Vectorial Differential Line Element / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}} = \underline{\mathbf{s}}\ \mathrm{d}R$$

Tangential Unit Vector / Tangentialer Einheitsvektor

Scalar Differential Line Element / Skalares differentielles Linienelement

Different Products / Verschiedene Produkte

Scalar Product / Skalarprodukt $$C = \underline{\mathbf{A}} \bullet \underline{\mathbf{B}}$$

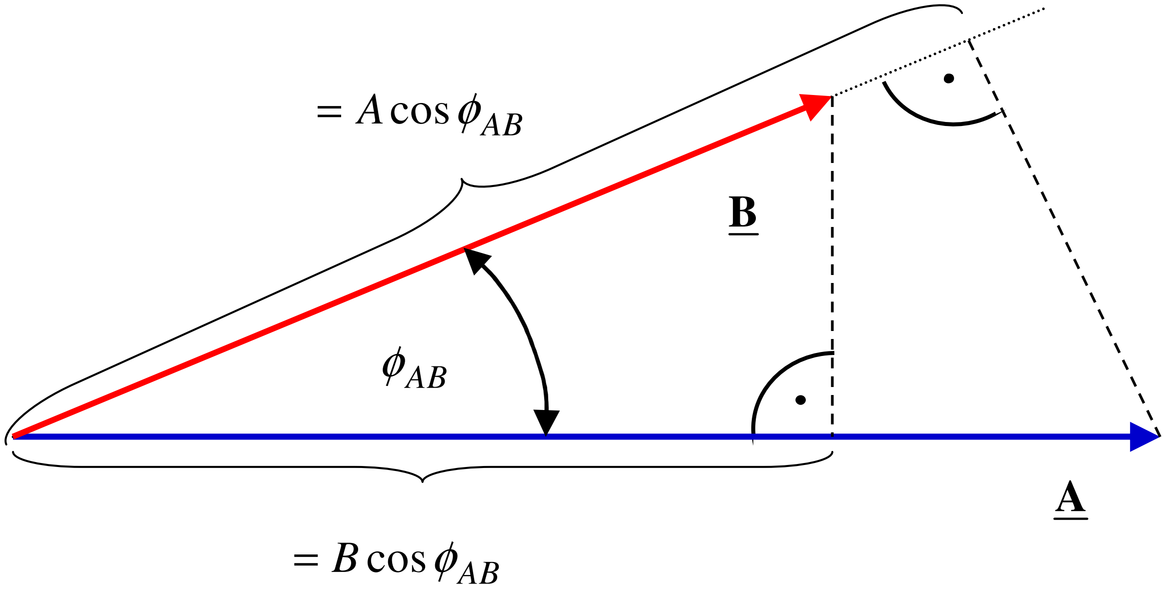
Vector Product / Vektorprodukt $$\underline{\mathbf{C}} = \underline{\mathbf{A}} \times \underline{\mathbf{B}}$$

Dyadic Product / Dyadisches Produkt $$\underline{\underline{\mathbf{C}}} = \underline{\mathbf{A}}\,\underline{\mathbf{B}}$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (1)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = |\underline{\mathbf{A}}||\underline{\mathbf{B}}| \cos \underbrace{\angle(\underline{\mathbf{A}}, \underline{\mathbf{B}})}_{\phi_{AB}}$$

$$= AB \cos \phi_{AB}$$



Enclosed Angle / Eingeschlossener Winkel ϕ_{AB}

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = \underline{\mathbf{B}} \bullet \underline{\mathbf{A}}$$

$$= BA \cos \phi_{BA}$$

$$= AB \cos \phi_{AB}$$

$$\cos \phi_{AB} = \frac{\underline{\mathbf{A}} \bullet \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}$$

$$\cos(\phi_{AB}) = \cos(-\phi_{AB})$$

$$\phi_{AB} = \arccos\left(\frac{\underline{\mathbf{A}} \bullet \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}\right)$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (2)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0}$$

$$+ A_y B_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0}$$

$$+ A_z B_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1}$$

$$= A_x B_x + A_y B_y + A_z B_z$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\begin{array}{lll} \underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x = 1 & \underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x = 0 & \underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x = 0 \\ \underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y = 0 & \underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y = 1 & \underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y = 0 \\ \underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z = 0 & \underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z = 0 & \underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z = 1 \end{array}$$

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$= (A_{x_1} \underline{\mathbf{e}}_{x_1} + A_{x_2} \underline{\mathbf{e}}_{x_2} + A_{x_3} \underline{\mathbf{e}}_{x_3}) \bullet (B_{x_1} \underline{\mathbf{e}}_{x_1} + B_{x_2} \underline{\mathbf{e}}_{x_2} + B_{x_3} \underline{\mathbf{e}}_{x_3})$$

$$= A_{x_1} B_{x_1} + A_{x_2} B_{x_2} + A_{x_3} B_{x_3}$$

$$= \sum_{i=1}^{3} A_{x_i} B_{x_i}$$

Cartesian Coordinates / Kartesische Koordinaten

$$x = x_1$$

$$y = x_2$$

$$z = x_3$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (3)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= \sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet B_{x_j} \underline{\mathbf{e}}_{x_j}$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}$$

$$= A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}$$

$$= A_{x_i} \underbrace{B_{x_j} \delta_{ij}}_{=B_{x_i}} \left(\text{or/oder} \quad \underbrace{\underbrace{A_{x_i} \delta_{ij}}_{=A_{x_j}} B_{x_j}}_{=A_{x_j} B_{x_j}} \right)$$

$$= A_{x_i} B_{x_i}$$

Kronecker Delta / Kronecker-Delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

> *Einstein's Summation Convention*: If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. / *Einsteinsche Summenkonvention*: Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.

Magnitude of a Vector / Betrag eines Vektors

$$
\begin{aligned}
|\underline{\mathbf{A}}| &= \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}} \\
&= \sqrt{(\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z) \bullet (\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z)} \\
&= \Bigg(\mathrm{A}_x \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + \mathrm{A}_x \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_x \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0} \\
&\quad + \mathrm{A}_y \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_y \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + \mathrm{A}_y \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0} \\
&\quad + \mathrm{A}_z \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_z \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_z \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1} \Bigg)^{\frac{1}{2}} \\
&= \sqrt{\mathrm{A}_x \mathrm{A}_x + \mathrm{A}_y \mathrm{A}_y + \mathrm{A}_z \mathrm{A}_z} \\
&= \sqrt{\mathrm{A}_x^2 + \mathrm{A}_y^2 + \mathrm{A}_z^2} \\
&= \mathrm{A}
\end{aligned}
$$

$$
\begin{aligned}
|\underline{\mathbf{A}}| &= \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}} \\
&= \sqrt{\sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}} \\
&= \sqrt{A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet A_{x_j} \underline{\mathbf{e}}_{x_j}} \\
&= \sqrt{A_{x_i} A_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}} \\
&= \sqrt{A_{x_i}^2}
\end{aligned}
$$

Example: Position Vector and Electric Field Strength Vector / Beispiel: Ortsvektor und elektrischer Feldstärkevektor

Cartesian Coordinate System / Kartesisches Koordinatensystem

Position Vector / Ortsvektor

$$\underline{\mathbf{R}}(x,y,z) = \mathrm{R}_x(x,y,z)\underline{\mathbf{e}}_x + \mathrm{R}_y(x,y,z)\underline{\mathbf{e}}_y + \mathrm{R}_z(x,y,z)\underline{\mathbf{e}}_z = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

Electric Field Strength Vector / Elektrische Feldstärkevektor

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}(x,y,z,t) = \mathrm{E}_x(x,y,z,t)\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\underline{\mathbf{e}}_z$$

Magnitude of the Position Vector (Distance) / Betrag des Ortsvektor (Abstand)

$$\left|\underline{\mathbf{R}}(x,y,z)\right| = \sqrt{\underline{\mathbf{R}}(x,y,z)\bullet\underline{\mathbf{R}}(x,y,z)} = \sqrt{\left(x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z\right)\bullet\left(x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z\right)} = \sqrt{x^2+y^2+z^2}$$

Magnitude of the Electric Field Strength Vector (Strength) / Betrag des elektrische Feldstärkevektors (Stärke)

$$\left|\underline{\mathbf{E}}(x,y,z)\right| = \sqrt{\underline{\mathbf{E}}(x,y,z)\bullet\underline{\mathbf{E}}(x,y,z)} = \sqrt{\left(\mathrm{E}_x\underline{\mathbf{e}}_x + \mathrm{E}_y\underline{\mathbf{e}}_y + \mathrm{E}_z\underline{\mathbf{e}}_z\right)\bullet\left(\mathrm{E}_x\underline{\mathbf{e}}_x + \mathrm{E}_y\underline{\mathbf{e}}_y + \mathrm{E}_z\underline{\mathbf{e}}_z\right)} = \sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}$$

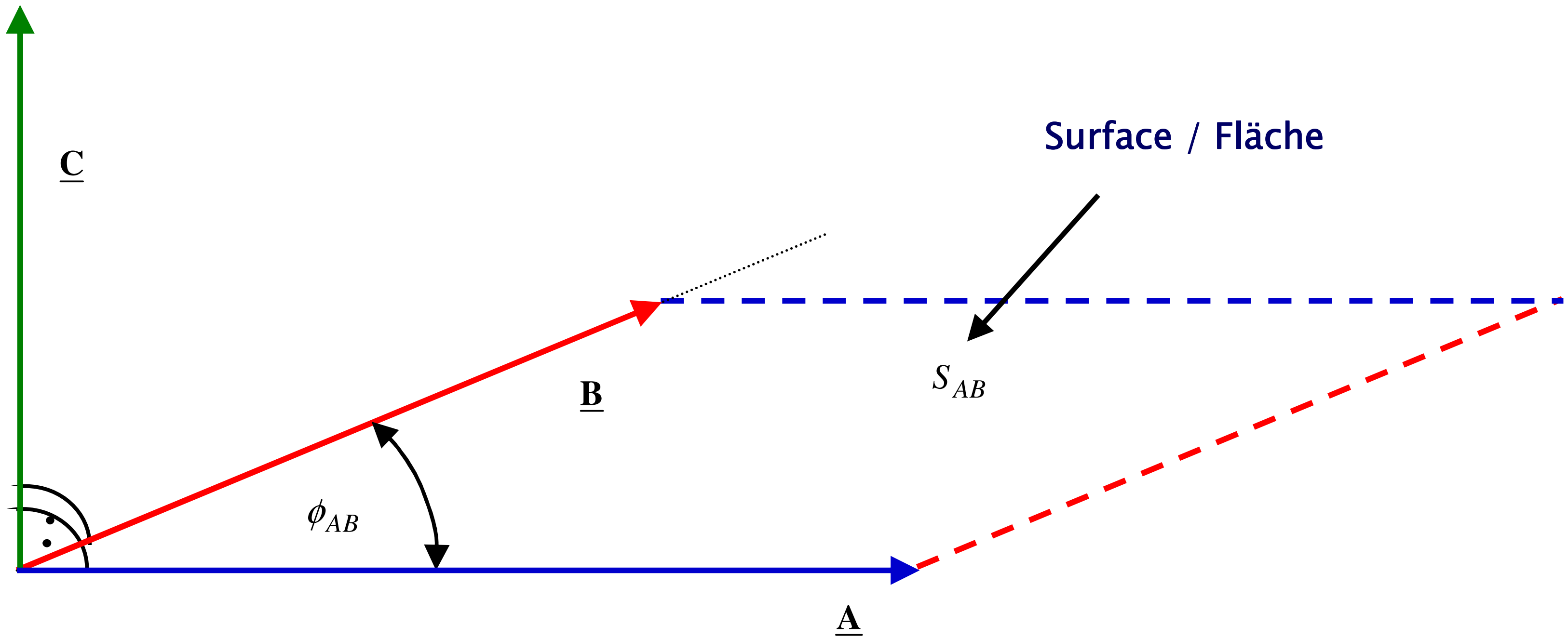
Position Unit Vector (Direction) / Ortseinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{R}}}(x,y,z) = \frac{\underline{\mathbf{R}}(x,y,z)}{\left|\underline{\mathbf{R}}(x,y,z)\right|} = \frac{x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z}{\sqrt{x^2+y^2+z^2}}$$

Electric Field Strength Unit Vector (Direction) / Elektrische Feldstärkeeinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{E}}}(x,y,z) = \frac{\underline{\mathbf{E}}(x,y,z)}{\left|\underline{\mathbf{E}}(x,y,z)\right|} = \frac{\mathrm{E}_x\underline{\mathbf{e}}_x + \mathrm{E}_y\underline{\mathbf{e}}_y + \mathrm{E}_z\underline{\mathbf{e}}_z}{\sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}}$$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (1)



$$\underline{\mathbf{C}} = \underline{\mathbf{A}} \times \underline{\mathbf{B}}$$

$$C = |\underline{\mathbf{A}}||\underline{\mathbf{B}}| \sin \underbrace{\angle(\underline{\mathbf{A}}, \underline{\mathbf{B}})}_{\phi_{AB}}$$

$$= AB \sin \phi_{AB}$$

$$= S_{AB}$$

$\underline{\mathbf{C}} \perp \underline{\mathbf{A}}$ and / und $\underline{\mathbf{C}} \perp \underline{\mathbf{B}}$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (2)

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \times (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x}_{=0} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y}_{=\underline{\mathbf{e}}_z} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z}_{=-\underline{\mathbf{e}}_y}$$

$$+ A_y B_x \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x}_{=-\underline{\mathbf{e}}_z} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y}_{=0} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z}_{=\underline{\mathbf{e}}_x}$$

$$+ A_z B_x \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x}_{=\underline{\mathbf{e}}_y} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y}_{=-\underline{\mathbf{e}}_x} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z}_{=0}$$

$$= (A_y B_z \underline{\mathbf{e}}_x - A_z B_y) \underline{\mathbf{e}}_x + (A_z B_x - A_x B_z) \underline{\mathbf{e}}_y + (A_x B_y - A_y B_x) \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = -\underline{\mathbf{B}} \times \underline{\mathbf{A}} \qquad \underline{\mathbf{A}} \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x = \underline{\mathbf{0}}$$
$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y = \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z = -\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x = -\underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y = \underline{\mathbf{0}}$$
$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_x$$

$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x = \underline{\mathbf{e}}_y$$
$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y = -\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z = \underline{\mathbf{0}}$$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (3)

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Add the first two Columns / Addiere die beiden ersten Spalten

$$= \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \begin{matrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y \\ A_x & A_y \\ B_x & B_y \end{matrix}$$

Sarrus Law / Regel von Sarrus

[Pierre Frédéric Sarrus, 1831]

http://de.wikipedia.org/wiki/Regel_von_Sarrus

$$= (A_y B_z - A_z B_y)\underline{\mathbf{e}}_x$$

$$+ (A_z B_x - A_x B_z)\underline{\mathbf{e}}_y$$

$$+ (A_x B_y - A_y B_x)\underline{\mathbf{e}}_z$$

Dyadic Product / Dyadisches Produkt

$$\underline{\mathbf{A}}\,\underline{\mathbf{B}} = \sum_{i=1}^{3} A_{x_i}\,\underline{\mathbf{e}}_{x_i} \sum_{j=1}^{3} B_{x_j}\,\underline{\mathbf{e}}_{x_j}$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i}\,\underline{\mathbf{e}}_{x_i}\,B_{x_j}\,\underline{\mathbf{e}}_{x_j}$$

$$= A_{x_i}\,\underline{\mathbf{e}}_{x_i}\,B_{x_j}\,\underline{\mathbf{e}}_{x_j}$$

$$= \underbrace{A_{x_i}\,B_{x_j}}_{=D_{x_i x_j}}\,\underline{\mathbf{e}}_{x_i}\,\underline{\mathbf{e}}_{x_j}$$

$$= D_{x_i x_j}\,\underline{\mathbf{e}}_{x_i}\,\underline{\mathbf{e}}_{x_j}$$

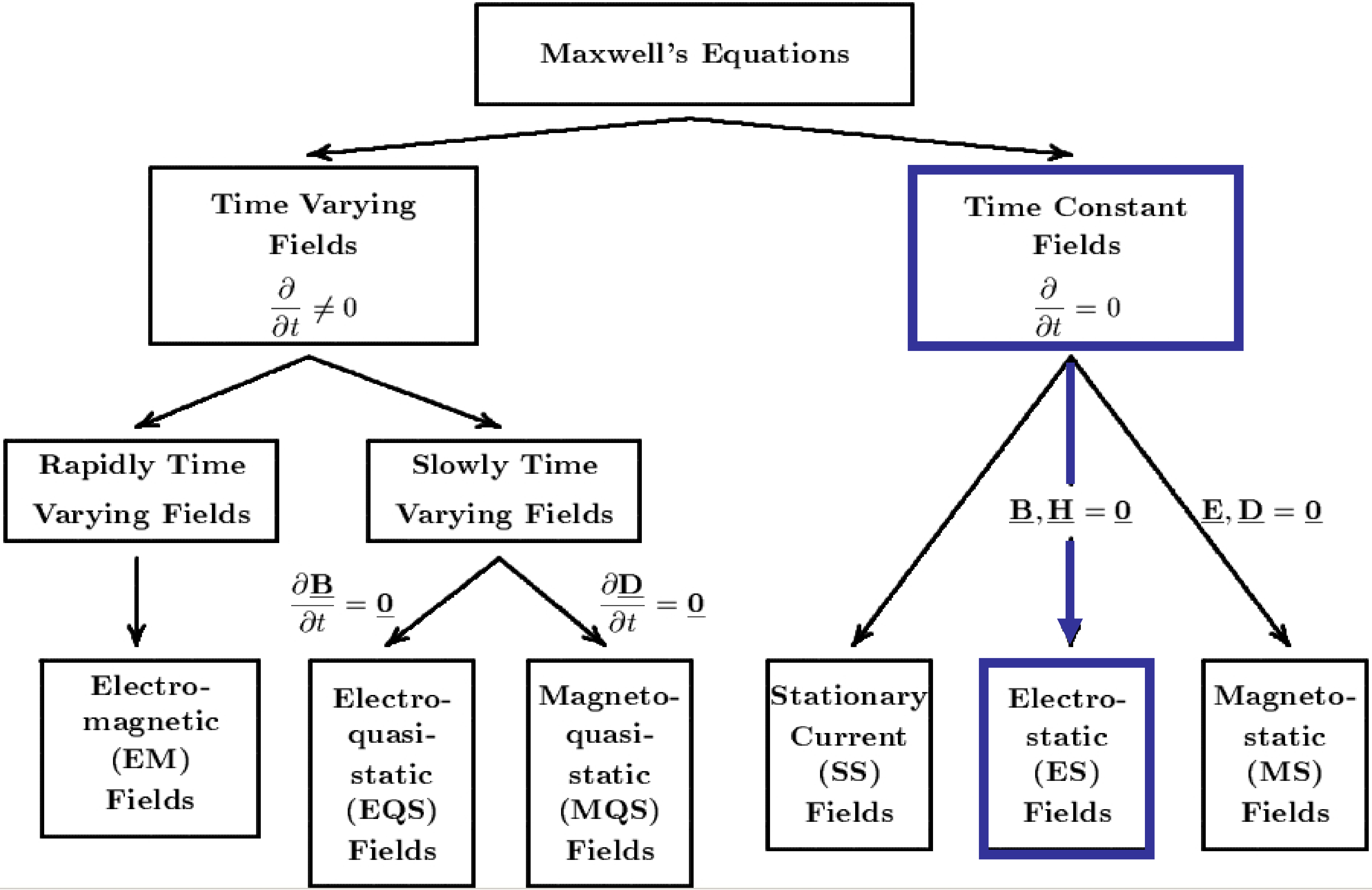
$$= \underline{\underline{\mathbf{D}}}$$

$$\underline{\mathbf{B}}\,\underline{\mathbf{A}} \neq \underline{\mathbf{A}}\,\underline{\mathbf{B}}$$

$$\underline{\mathbf{D}} = \underline{\underline{\boldsymbol{\varepsilon}}} \bullet \underline{\mathbf{E}}$$

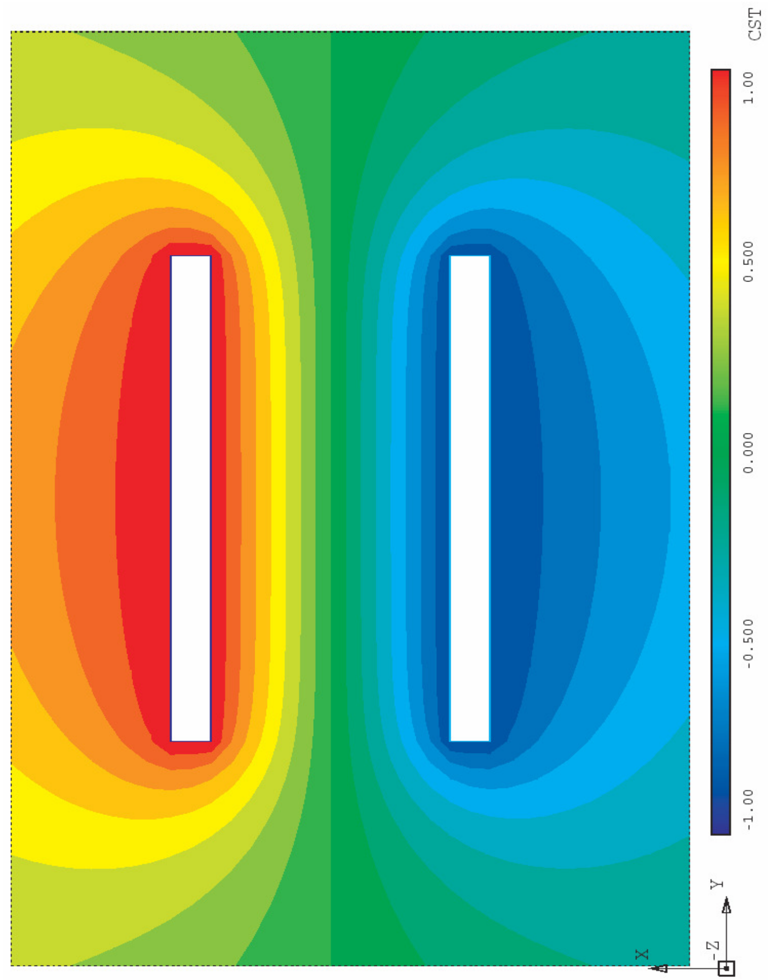
$$\underline{\mathbf{B}} = \underline{\underline{\boldsymbol{\mu}}} \bullet \underline{\mathbf{H}}$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

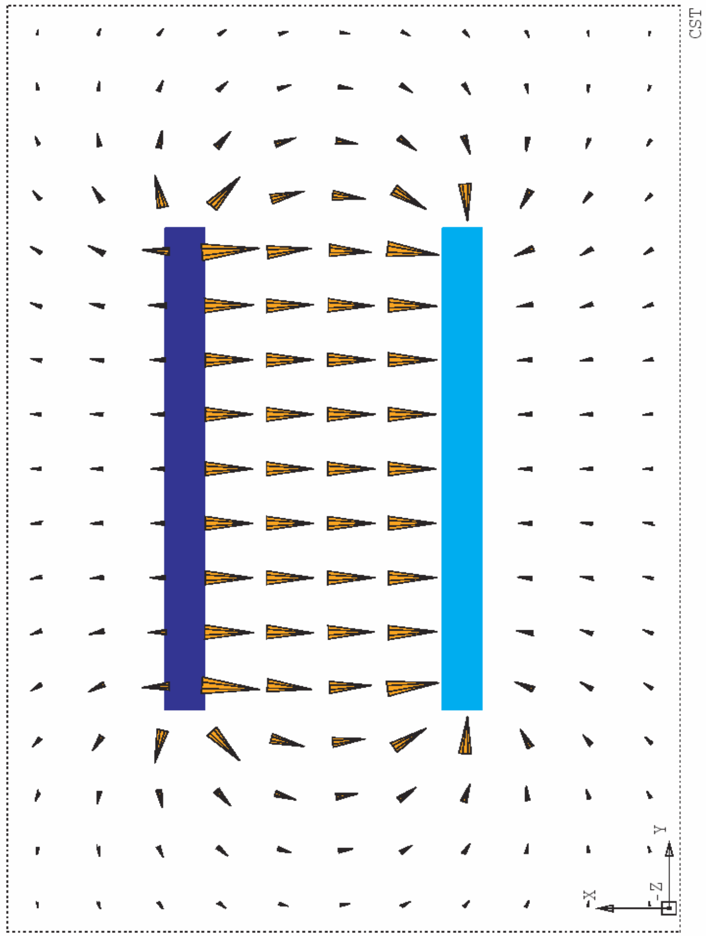


Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

Scalar Field: Electrostatic Potential / Skalarfeld: Elektrostatisches Potenzial



Vector Field: Electrostatic Field Strength / Vektorfeld: Elektrostatische Feldstärke



Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Electrostatic / Elektrostatik $\quad \frac{\partial}{\partial t} \equiv 0 \quad$ No Time Dependence and No Magnetic Field Quantities / Keine Zeitabhängigkeit und keine magnetischen Feldgrößen

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$: Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$: Electric Flux Density / Elektrische Flussdichte
$\rho_{\mathrm{e}}(\underline{\mathbf{R}})$: Electric Charge Density / Elektrische Raumladungsdichte

Integral Form / Integralform	Differential Form / Differentialform
$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dR}} = 0$	$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$
$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}})\,\mathrm{d}V$	$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_{\mathrm{e}}(\underline{\mathbf{R}})$

Curl-Free E-Field / Rotationsfreies E-Feld

Divergence of D Represents Electric Charge Density / Quellstärke von D entspricht der elektrischen Raumladungsdichte

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$= Q_\mathrm{e}$$

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$ [V/m = Newton /Coulomb = N/C]

$\underline{\mathbf{D}}(\underline{\mathbf{R}})$ [As/ m^2]

$\rho_\mathrm{e}(\underline{\mathbf{R}})$ [As/m^3]

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Electric Field Constant / Elektrische Feldkonstante (IEEE, VDE)
Permittivity of Free Space / Permittivität des Freiraumes

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$$

Side Remark: In some Cases / Nebenbemerkung: In einigen Fällen

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \varepsilon_\mathrm{r} \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Permittivity / Permittivität

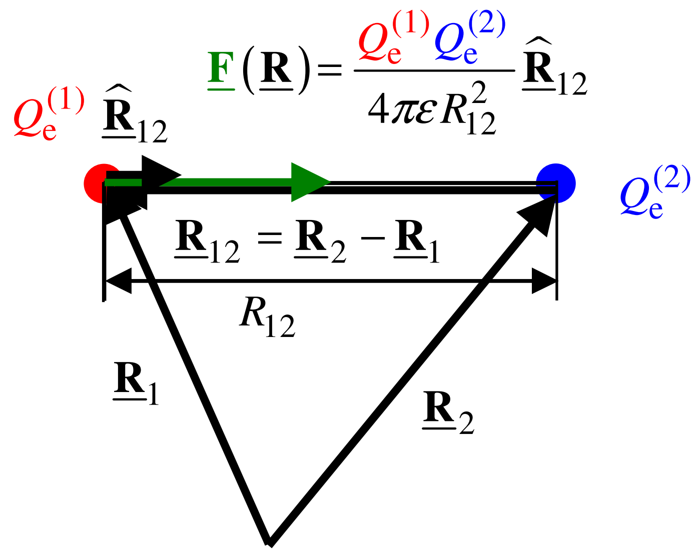
Material	ε_r
Air / Luft	1.006
Paper / Papier	2...4
Wet Earth / Nasse Erde	5...15
Gallium Arsenide / Gallium Arsenid	13
Seawater / Seewasser	70

Coulomb's Law / Coulombsches Gesetz

Charles Augustin de Coulomb (1736 – 1806)

$$\underline{\mathbf{F}}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon}\frac{Q_e^{(1)}Q_e^{(2)}}{R_{12}^2}\underline{\hat{\mathbf{R}}}_{12} \qquad [\mathrm{N}]$$

Force / Kraft	$\underline{\mathbf{F}}(\underline{\mathbf{R}})$	[N]
Electric Point Charge / Elektrische Punktladung	$Q_e^{(1)}$	[As]
Electric Point Charge / Elektrische PunktLadung	$Q_e^{(2)}$	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\underline{\hat{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



$$\underline{\hat{\mathbf{R}}} = \frac{\underline{\mathbf{R}}}{|\underline{\mathbf{R}}|} = \frac{\underline{\mathbf{R}}}{R} \qquad [1] \qquad R = |\underline{\mathbf{R}}| = \sqrt{\underline{\mathbf{R}}\cdot\underline{\mathbf{R}}} \quad [\mathrm{m}]$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

$$R = \sqrt{\underline{\mathbf{R}}\cdot\underline{\mathbf{R}}} = \sqrt{x^2 + y^2 + z^2}$$

$$\underline{\hat{\mathbf{R}}} = \frac{x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z}{\sqrt{x^2 + y^2 + z^2}} = \underline{\mathbf{e}}_R(\vartheta,\varphi)$$

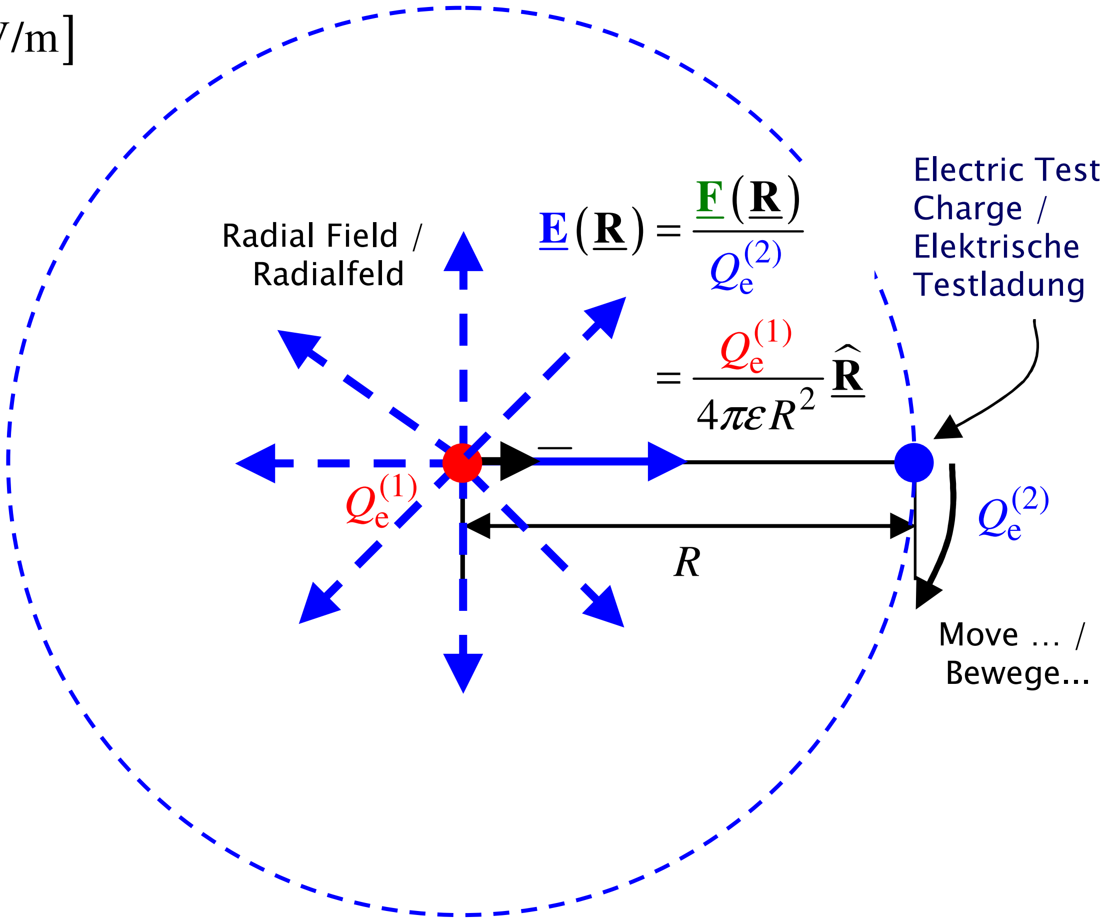
ES Fields – Electric Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Ladung und elektrische Feldstärke – Coulombsches Gesetz

Electric Field Strength: Force Per Unit Charge / Elektrische Feldstärke: Kraft pro Einheitsladung

$Q_e^{(2)}$ Electric Test Charge / Elektrische Testladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{\underline{\mathbf{F}}(\underline{\mathbf{R}})}{Q_e^{(2)}} = \frac{Q_e^{(1)}}{4\pi\varepsilon R^2}\underline{\hat{\mathbf{R}}} \qquad [\text{N/C or V/m}]$$

Electric Field Strength / Elektische Feldstärke	$\underline{\mathbf{E}}(\underline{\mathbf{R}})$	[V/m]
Force / Kraft	$\underline{\mathbf{F}}(\underline{\mathbf{R}})$	[N]
Electric Charge / Elektrische Ladung	$Q_e^{(1)}$	[As]
Electric Test Charge / Elektrische Testladung	$Q_e^{(2)}$	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\underline{\hat{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{\underline{\mathbf{F}}(\underline{\mathbf{R}})}{Q_e^{(2)}} = \frac{Q_e^{(1)}}{4\pi\varepsilon R^2}\underline{\hat{\mathbf{R}}}$$

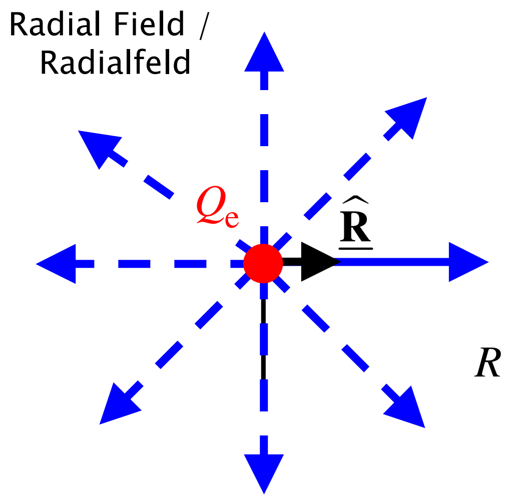
ES Fields – Electric Charge and Electric Field Strength – Coulomb's Law / ES Felder – Elektrische Ladung und elektrische Feldstärke – Coulombsches Gesetz

Electric Field Strength: Force Per Unit Charge / Elektrische Feldstärke: Kraft pro Einheitsladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon R^2}\underline{\widehat{\mathbf{R}}} = \frac{Q_e}{4\pi\varepsilon R}\underline{\mathbf{R}} \qquad [\mathrm{V/m}]$$

$$\underline{\mathbf{R}} = R\underline{\widehat{\mathbf{R}}}$$

Electric Field Strength / Elektische Feldstärke	$\underline{\mathbf{E}}(\underline{\mathbf{R}})$	[V/m]
Electric Charge / Elektrische Ladung	Q_e	[As]
Distance / Abstand	R	[m]
Distance Unit Vector / Abstandseinheitsvektor	$\underline{\widehat{\mathbf{R}}}$	[1]
Permittivity of Free-Space / Permittivität des Freiraumes	ε	[As/Vm]



$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon R^2}\underline{\widehat{\mathbf{R}}} = \frac{Q_e}{4\pi\varepsilon R}\underline{\mathbf{R}}$$

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Electrostatic / Elektrostatik $\frac{\partial}{\partial t} \equiv 0$ No Time Dependence and No Magnetic Field Quantities / Keine Zeitabhängigkeit und keine magnetischen Feldgrößen

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$\underline{\mathbf{D}}(\underline{\mathbf{R}})$: Electric Flux Density / Elektrische Flussdichte

$\rho_{\mathrm{e}}(\underline{\mathbf{R}})$: Electric Charge Density / Elektrische Raumladungsdichte

Integral Form / Integralform	Differential Form / Differentialform
$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$	$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$
$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}})\, \mathrm{d}V$	$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_{\mathrm{e}}(\underline{\mathbf{R}})$

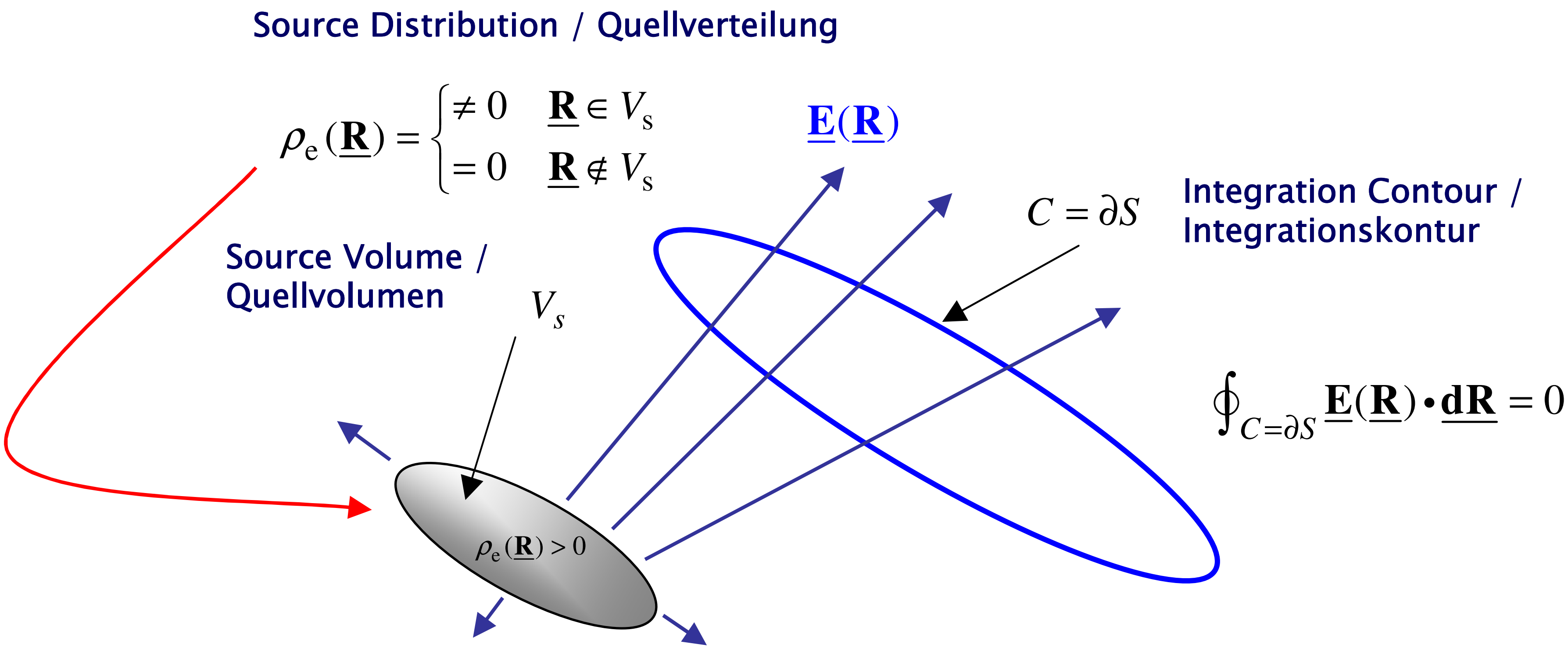
Curl-Free E-Field / Rotationsfreies E-Feld

Divergence of D Represents Electric Charge Density / Quellstärke von D entspricht der elektrischen Raumladungsdichte

⬇

Method of Gauss' Electric Law / Methode des Gaußschen elektrischen Gesetzes

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes



ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes

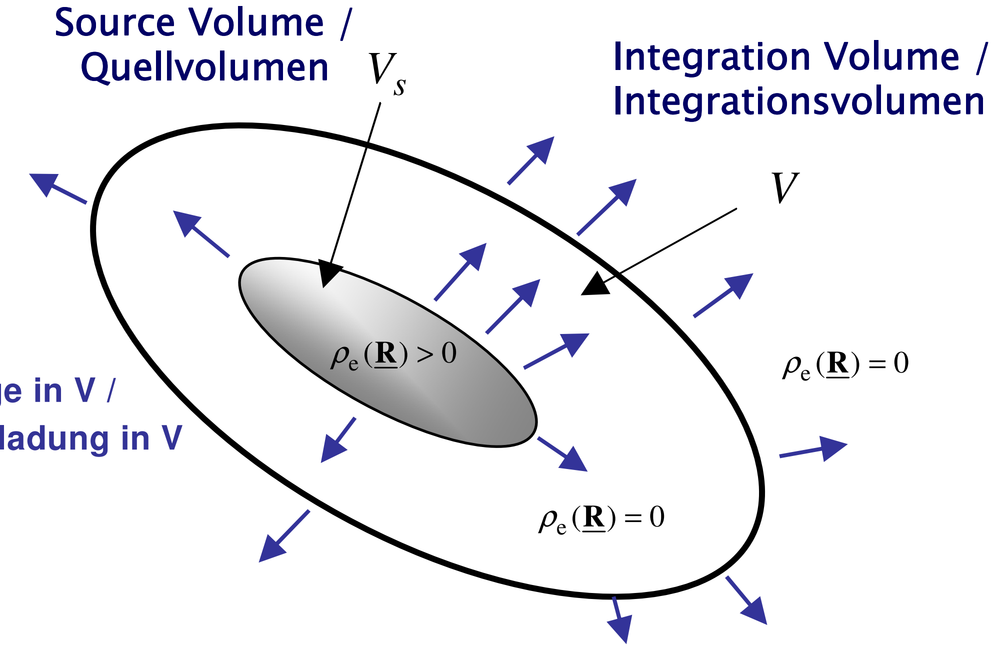
Source Distribution / Quellverteilung

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ = 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$

$$\psi_e = \oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$= Q_e$$

Total Electric Charge in V / Elektrische Gesamtladung in V



$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})}\,\mathrm{d}S$$

$$\underbrace{\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}}}_{\underbrace{\oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})}\,\mathrm{d}S}} = \underbrace{\underbrace{\iiint_V \rho_e(\underline{\mathbf{R}})\,\mathrm{d}V}_{Q_e}}$$

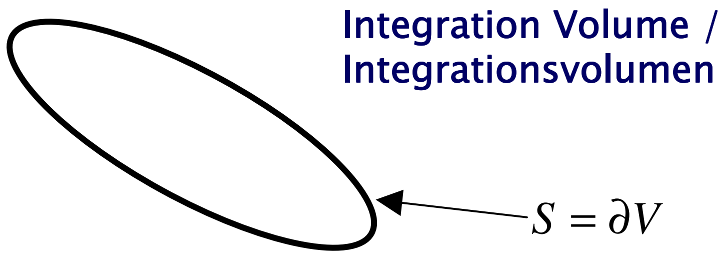
Summation of all $D_n = \underline{\mathbf{n}} \bullet \underline{\mathbf{D}}$ Contributions /
Summation aller $D_n = \underline{\mathbf{n}} \bullet \underline{\mathbf{D}}$-Beiträge

Total electric charge inside the volume V with the closed surface $S = \partial V$ /
Gesamte elektrische Ladung im Volumen V mit der geschlossenen Oberfläche $S = \partial V$

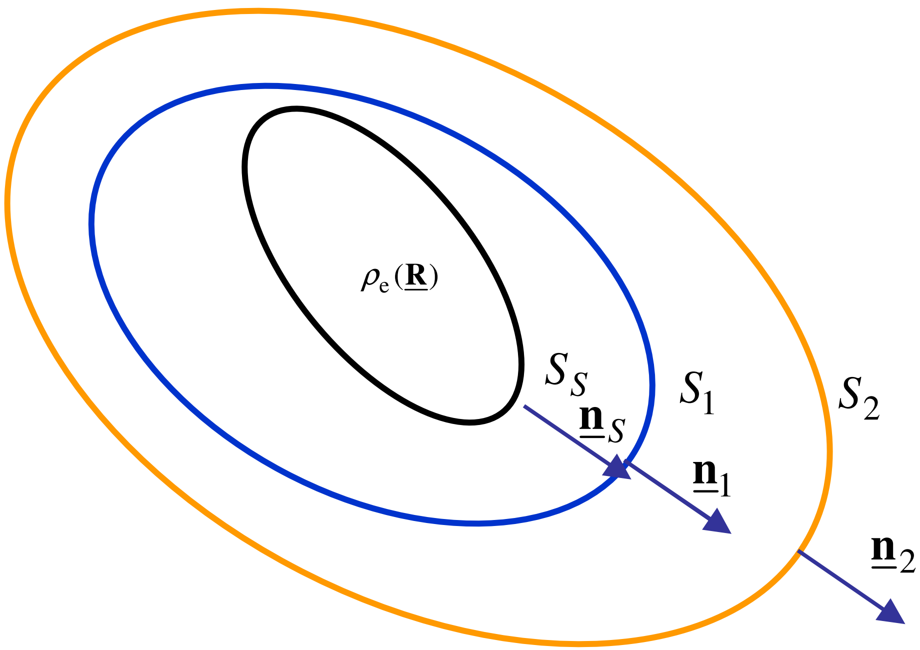
Flux of $\underline{\mathbf{D}}$ through $S = Q_e$ in V /
Fluss von $\underline{\mathbf{D}}$ durch $S = Q_e$ in V

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}})\,\mathrm{d}V$$
$$= Q_{\mathrm{e}}$$

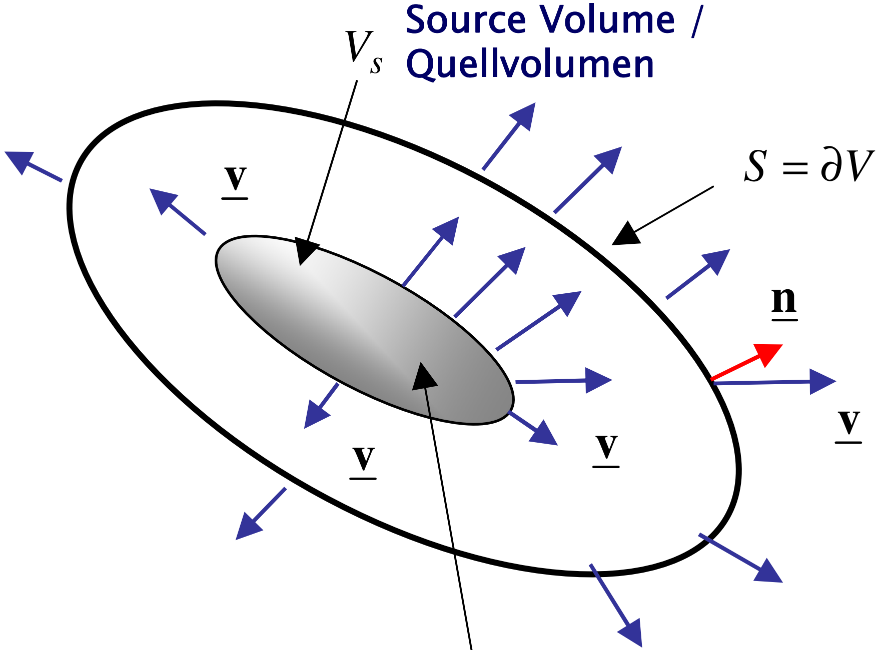


$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} \begin{cases} = 0 & \text{source-free / quellenfrei} \\ > 0 & \text{Source / Quelle} \\ < 0 & \text{Sink / Senke} \end{cases}$$



$$\oiint_{S_S=\partial V_S} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_S \,\mathrm{d}S$$
$$= \oiint_{S_1=\partial V_1} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_1 \,\mathrm{d}S$$
$$= \oiint_{S_2=\partial V_2} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_2 \,\mathrm{d}S$$
$$= Q_{\mathrm{e}}$$

Example: Fluid Mechanics – Spring of Water / Beispiel: Strömungsmechanik – Wasserquelle



Total Flux through the Closed Surface / Gesamtfluss durch die geschlossene Oberfläche

$$\oiint_{S=\partial V} \underline{\mathbf{v}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{v}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=\mathrm{v}_n(\underline{\mathbf{R}})} \, \mathrm{d}S$$

$$= \oiint_{S=\partial V} \mathrm{v}_n(\underline{\mathbf{R}}) \, \mathrm{d}S$$

$$= \psi_{\mathrm{v}}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen (ES) Fall

$$\oint\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \oint\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \, \mathrm{d}S = \underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V}_{=Q_\mathrm{e}}$$

Prescribed: Electric Charge Density / Vorgegeben: Elektrische Raumladungsdichte

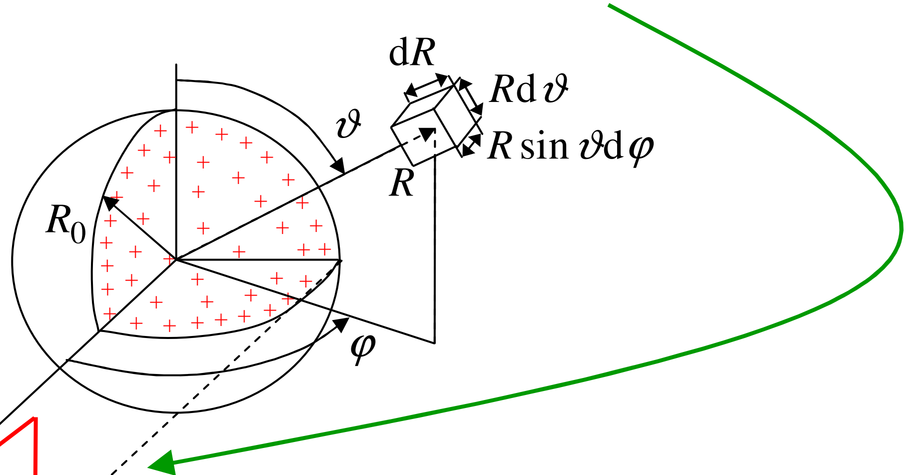
Radial Symmetry / Radialsymmetrie !

$$\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{e}}_R}_{=D_R(\underline{\mathbf{R}})}$$

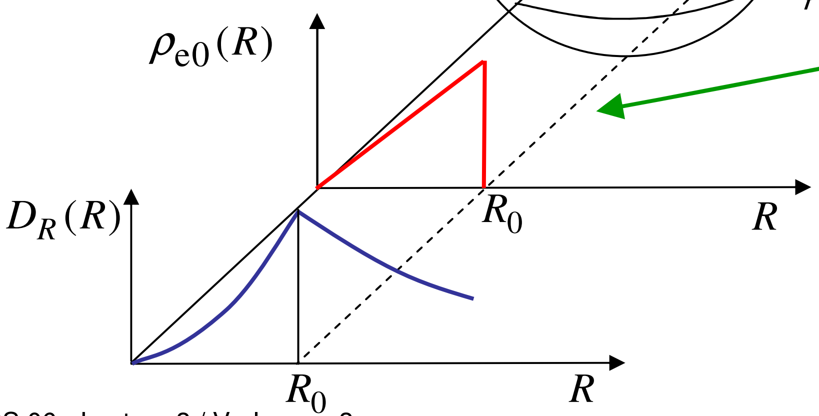
$$D_n(\underline{\mathbf{R}}) = D_R(\underline{\mathbf{R}})$$

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(R) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Charged Sphere with Radius R_0 / Geladene Kugel mit dem Radius R_0

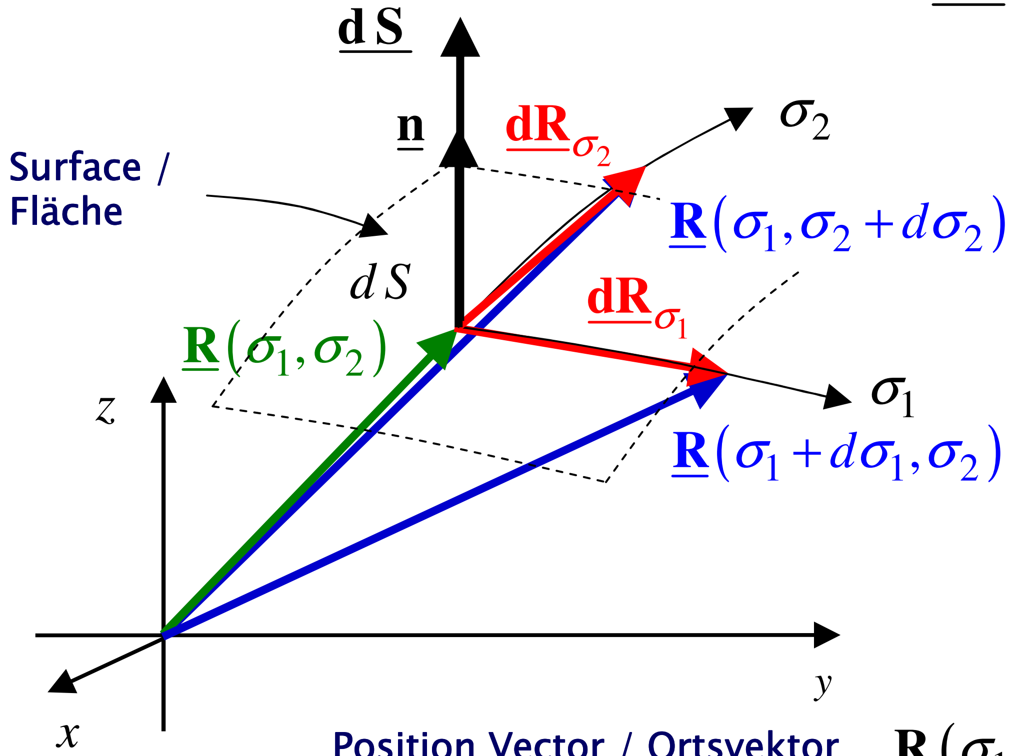


Solution for D(R) / Lösung für D(R)



Vector Differential Surface Element / Vektorielles differentielles Flächenelement (1)

Definition: $\underline{\mathbf{d\,S}} = \underline{\mathbf{n}}\,\mathrm{d}S$



σ_1,σ_2	Surface Parameters / Flächenparameter
$\underline{\mathbf{R}}(\sigma_1,\sigma_2)$	Position Vector / Ortsvektor
$\underline{\mathbf{R}}(\sigma_1+\mathrm{d}\sigma_1,\sigma_2)$	Position Vector / Ortsvektor
$\underline{\mathbf{R}}(\sigma_1,\sigma_2+\mathrm{d}\sigma_2)$	Position Vector / Ortsvektor
$\underline{\mathbf{dR}}_{\sigma_1}$, $\underline{\mathbf{dR}}_{\sigma_2}$	Vector Differential Line Elements / Vektorielle differentielle Linienelemente

Position Vector / Ortsvektor $\underline{\mathbf{R}}(\sigma_1,\sigma_2)$

Tangential Vectors / Tangentialvektoren

$$\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2) = \frac{\partial}{\partial\sigma_1}\underline{\mathbf{R}}(\sigma_1,\sigma_2)$$

$$\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2) = \frac{\partial}{\partial\sigma_2}\underline{\mathbf{R}}(\sigma_1,\sigma_2)$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement (2)

Vector Differential Line Elements / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}}_{\sigma_1} = \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1$$

$$\underline{\mathbf{dR}}_{\sigma_2} = \underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_2$$

Scalar Differential Surface Elements / Skalares differentielles Flächenelement

$$\begin{aligned}\mathrm{d}S &= \left|\underline{\mathbf{dR}}_{\sigma_1} \times \underline{\mathbf{dR}}_{\sigma_2}\right| \\ &= \left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2\end{aligned}$$

Normal Unit-Vector / Normaleneinheitsvektor

$$\underline{\mathbf{n}} = \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement

$$\begin{aligned}\underline{\mathbf{dS}} &= \underline{\mathbf{n}}\,\mathrm{d}S \\ &= \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2 \\ &= \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1\mathrm{d}\sigma_2\end{aligned}$$

Gauss’ Electric Law / Gaußsches elektrisches Gesetz

$$\underbrace{\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathrm{d}\underline{\mathbf{S}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \mathrm{d}S = \underbrace{\iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}})\,\mathrm{d}V}_{\text{Volume Integral / Volumenintegral}}$$

Closed Surface Integral / Geschlossenes Flächenintegral

Summation of all Normal Componentes of $\underline{\mathbf{D}}$ at the Closed Surface $S=\partial V$ of the Volume V / Summation aller Normalkomponenten von $\underline{\mathbf{D}}$ auf der geschlossenen Oberfläche $S=\partial V$ des Volumens V

$=\psi_{\mathrm{e}}$

Flux Through the Colsed Surface / Fluss durch die geschlossene Oberfläche

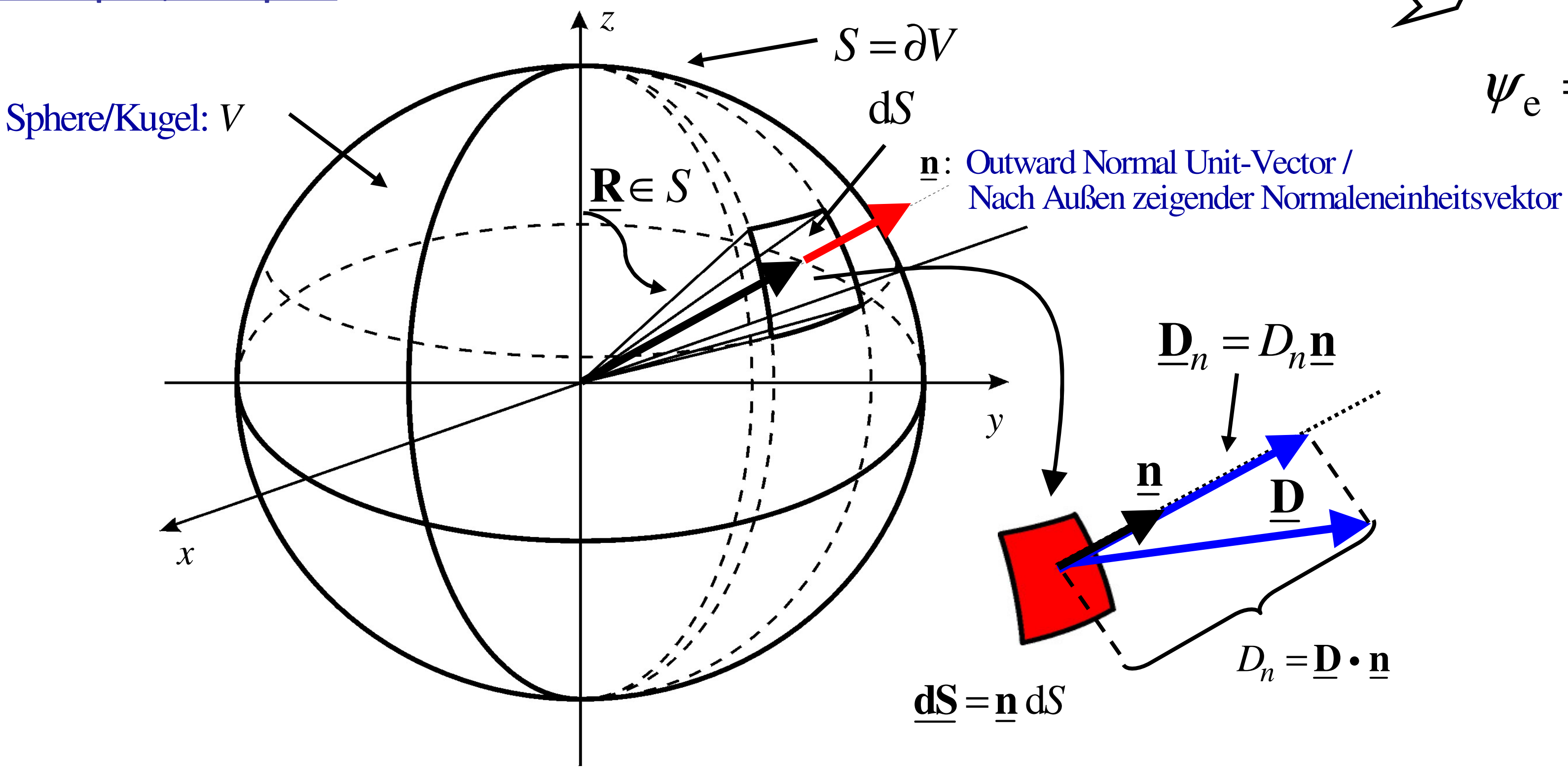
Volume Integral / Volumenintegral

Summation of all charges inside the Volume V / Summation aller Ladungen in dem Volumen V

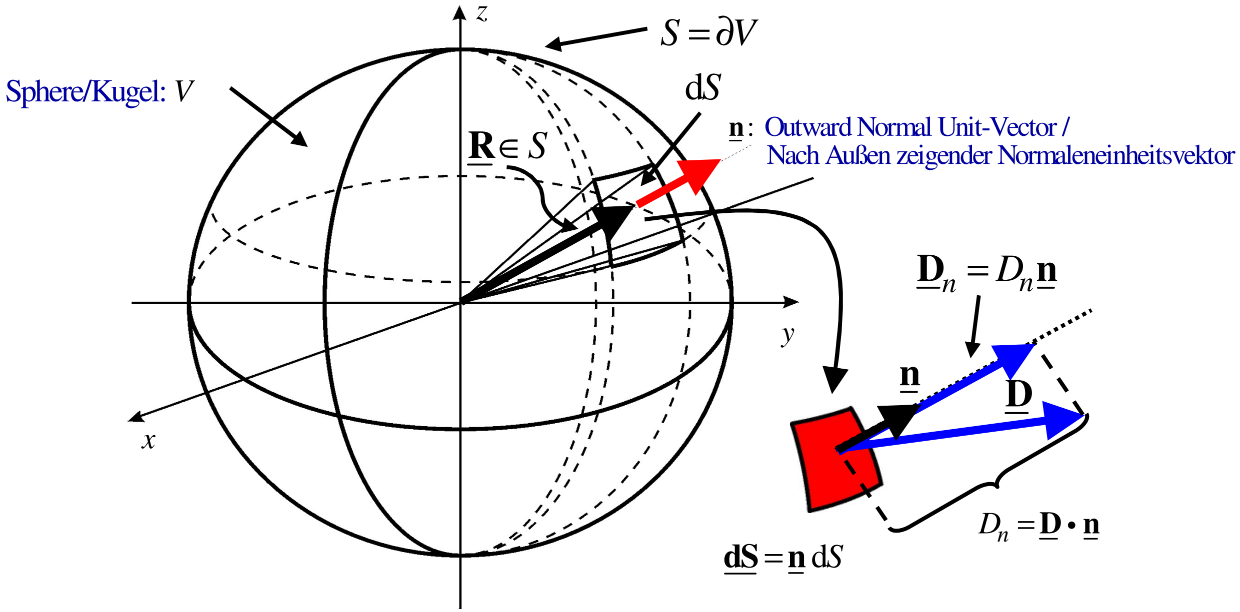
$=Q_{\mathrm{e}}$

$$\psi_{\mathrm{e}} = Q_{\mathrm{e}}$$

Example / Beispiel:



Example: Sphere with Radius a / Beispiel: Kugel mit Radius a (1)



$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \, \mathrm{d}S$$

$$= \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V$$

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}} \, \mathrm{d}S \quad (= \underline{\mathbf{n}}_{\vartheta\varphi} \, h_\vartheta \, h_\varphi \, \mathrm{d}\vartheta \, \mathrm{d}\varphi)$$

$$= \underbrace{\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\underline{\mathbf{n}}} \underbrace{R^2 \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi}_{\mathrm{d}S} \Bigg|_{R=a} = \underbrace{\underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\underline{\mathbf{n}}} \underbrace{a^2 \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi}_{\mathrm{d}S}$$

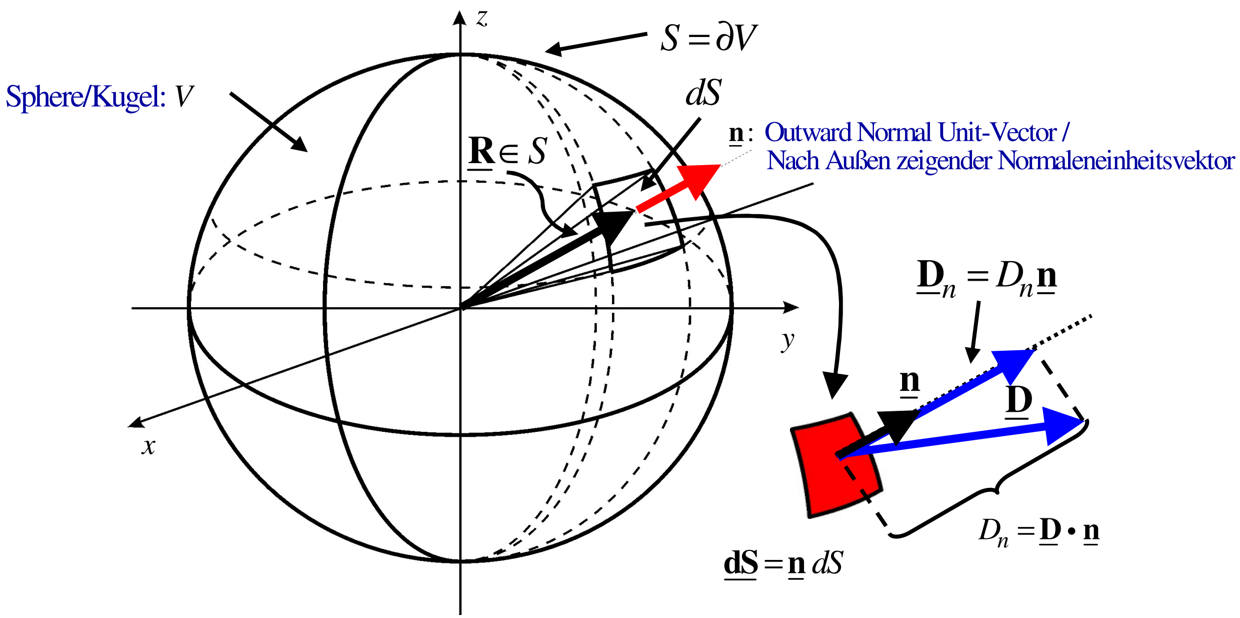
$$0 \le \vartheta \le \pi$$

$$0 \le \varphi < 2\pi$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \, \mathrm{d}S = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \underbrace{\underline{\mathbf{D}}[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)] \bullet \underline{\mathbf{e}}_R(\vartheta,\varphi)}_{\substack{=D_R[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)] \\ =D_n[\underline{\mathbf{R}}(R=a,\vartheta,\varphi)]}} a^2 \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$$

$$= \psi_\mathrm{e}$$

Example: Sphere with Radius a / Beispiel: Kugel mit Radius a (2)



$$\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})} \, \mathrm{d}S$$

$$= \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\, \mathrm{d}V$$

$$\mathrm{d}V = R^2 \sin\vartheta \, \mathrm{d}R \, \mathrm{d}\vartheta \, \mathrm{d}\varphi \;\left(= h_R h_\vartheta h_\varphi \; \mathrm{d}R \, \mathrm{d}\vartheta \, \mathrm{d}\varphi \right)$$

$$\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\, \mathrm{d}V = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \int_{R=0}^{a} \rho_\mathrm{e}[\underline{\mathbf{R}}(R,\vartheta,\varphi)]\, R^2 \sin\vartheta \, \mathrm{d}R \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$$

$$= Q_\mathrm{e}$$

$$0 \le R \le a$$
$$0 \le \vartheta \le \pi$$
$$0 \le \varphi < 2\pi$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Consider the Electrostatic (ES) Case /
Betrachte den elektrostatischen Fall

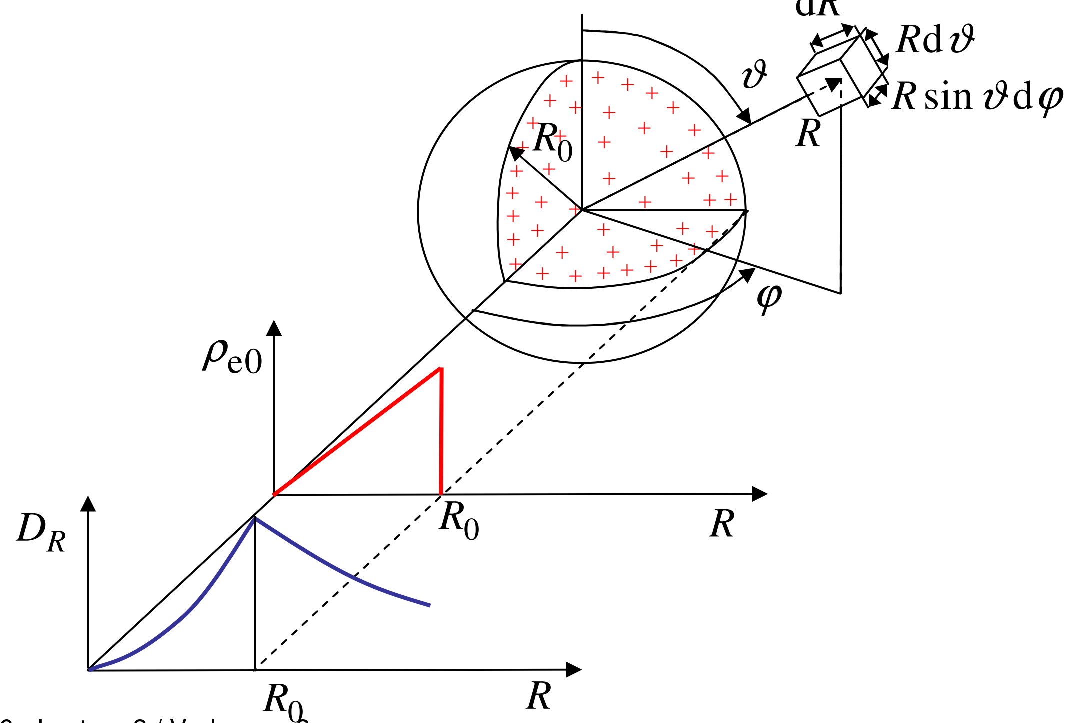
$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})} \, \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}}) \, \mathrm{d}V$$

Electric Charge Density /
Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \rho_{\mathrm{e}0} \dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry /
Radialsymmetrisch

!



End of Lecture 3 / Ende der 3. Vorlesung