Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

6th Lecture / 6. Vorlesung

Dr.-Ing. René Marklein

marklein@uni-kassel.de

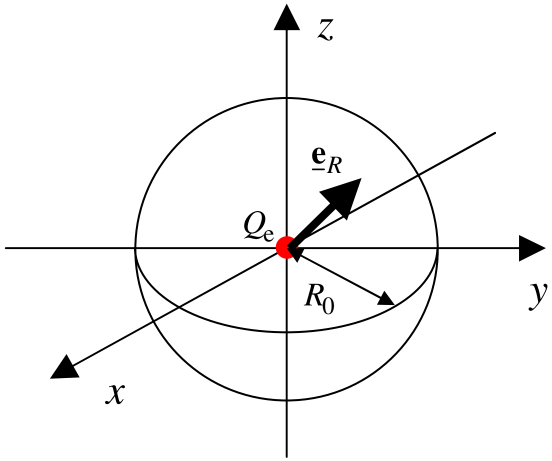
http://www.tet.e-technik.uni-kassel.de

http://www.uni-kassel.de/fb16/tet/marklein/index.html

Universität Kassel
Fachbereich Elektrotechnik / Informatik (FB 16)
Fachgebiet Theoretische Elektrotechnik (FG TET)
Wilhelmshöher Allee 71
Büro: Raum 2113 / 2115
D-34121 Kassel

University of Kassel
Dept. Electrical Engineering / Computer Science (FB 16)
Electromagnetic Field Theory (FG TET)
Wilhelmshöher Allee 71
Office: Room 2113 / 2115
D-34121 Kassel

Electrostatic (ES) Fields – Point Charge Concept /
Elektrostatische (ES) Felder – Konzept der Punktladung

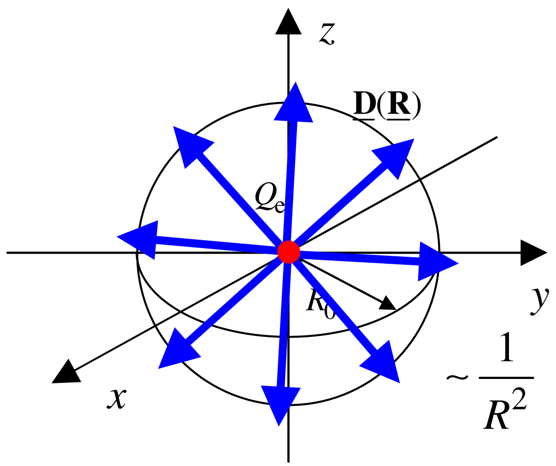


$$\oiint_{\underbrace{S=\partial V}_{=\text{Sphere } R_0}} \underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}\,\mathrm{d}S = \iiint_V \underbrace{\rho_e(\underline{\mathbf{R}})}_{=Q_e\delta(\underline{\mathbf{R}})}\,\mathrm{d}V = Q_e$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}}) = -\nabla\Phi_e(R)$$

$$= -\frac{\partial}{\partial R}\Phi_e(R)\,\underline{\mathbf{e}}_R(\varphi,\vartheta) = \alpha\frac{1}{R^2}\underline{\mathbf{e}}_R(\varphi,\vartheta)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \underline{\mathbf{D}}(R) = \varepsilon_0\alpha\frac{1}{R^2}\underline{\mathbf{e}}_R(\varphi,\vartheta)$$



$$\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}} \varepsilon_0\alpha\frac{1}{R_0^2}\underbrace{\underline{\mathbf{e}}_R(\varphi,\vartheta)\bullet\underline{\mathbf{e}}_R(\varphi,\vartheta)}_{=1}\,\mathrm{d}S = \varepsilon_0\alpha\frac{1}{R_0^2}\underbrace{\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}}\mathrm{d}S}_{4\pi R_0^2}$$

$$= 4\pi\varepsilon_0\alpha$$

$$= Q_e$$

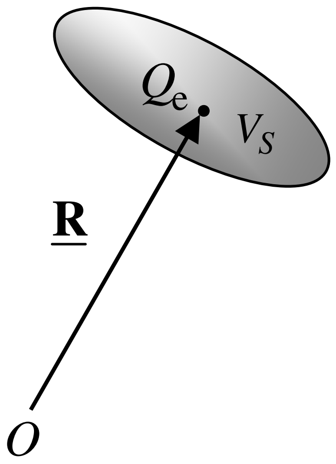
$$\Phi_e(R) = \frac{Q_e}{4\pi\varepsilon_0 R}$$

$$\alpha = \frac{Q_e}{4\pi\varepsilon_0}$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Point Source / Punktquelle

$$Q_\mathrm{e} \quad [\mathrm{As/m^3} = \mathrm{Coulomb}]$$



$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = ?$$

$$= \text{infinite} \quad / \text{unendlich}$$

$$\iiint_{V_S} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = Q_\mathrm{e}$$

Mathematically Nonsense / Mathematischer Unsinn

$$V_S \to 0$$

Integration Theory of Riemann / Riemannsche Integralrechnung:

$$\iiint_{V_S} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = 0$$

To Define Something New / Definiere etwas Neues

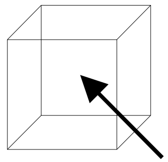
Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Charge /
Elektrostatisches Ladung

$$Q_\mathrm{e} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\, dV$$

Electrostatic Volume Charge Density /
Elektrostatisches Raumladungsdichte

$$\rho_\mathrm{e} = \frac{\Delta Q_\mathrm{e}}{\Delta V}$$



Small Volume /
Kleines Volumen

ΔV

Electrostatic Charge /
Elektrostatisches Ladung

$$\Delta Q_\mathrm{e} = \rho_\mathrm{e} \Delta V$$

In the Limit /
Grenzübergang

$\Delta V \to 0$

Constant / Konstant

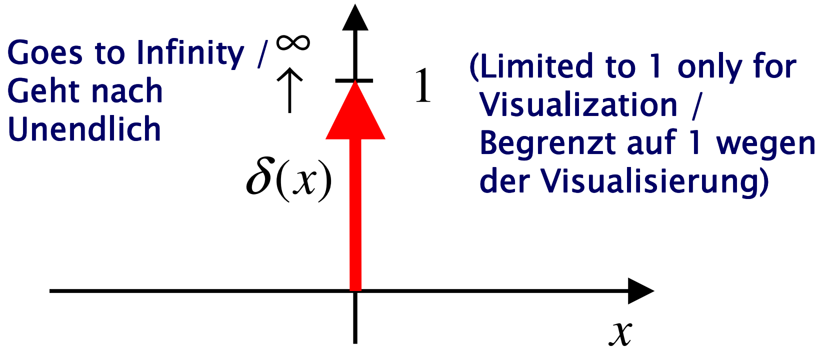
$$\Delta Q_\mathrm{e} = \lim_{\substack{\Delta V \to 0 \\ \rho_\mathrm{e} \to \infty}} \rho_\mathrm{e} \Delta V$$

Point / Punkt

ΔQ_e =Constant if ΔV Goes to Zero, than the Volume Charge Density must go to Infinity. /
ΔQ_e=konstant bleiben soll wenn ΔV nach null geht, dann muss die Raumladungsdichte nach unendlich gehen.

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution



Delta-Function / Delta-Funktion
δ-Distribution / δ-Distribution
δ-Dirac-Pulse / δ-Dirac-Impuls

Distribution ➨ Generalized Function / Verallgemeinerte Funktion

$$\delta(x) = \begin{cases} "\infty" & \text{for/ für} \quad x = 0 \\ 0 & \text{for/ für} \quad x \neq 0 \end{cases}$$

$$x\,[\mathrm{m}],\ \delta(x)\left[\frac{1}{\mathrm{m}}\right]$$

The Unit of the Delta-Distribution is the Inverse Unit of the Argument / Die Einheit der Delta-Distribution ist die inverse Einheit des Argumentes

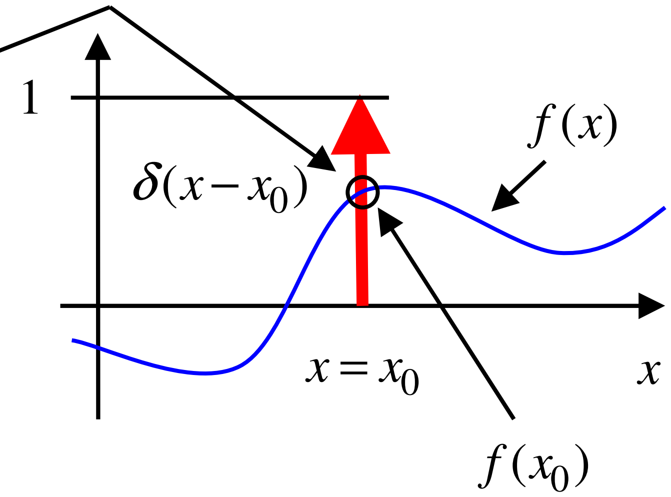
Definition of the δ -Distribution / Definition der δ -Distribution

$$\int_{x=-\infty}^{\infty} \delta(x - x_0)\,\mathrm{d}x = 1$$

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x - x_0)\,\mathrm{d}x = f(x_0)$$

$$f(x)\,\delta(x - x_0) = f(x_0)\,\delta(x - x_0)$$

Sifting Property / Siebeigenschaft



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x-x_0)\,\mathrm{d}x = f(x_0)$$

$$\left\langle f(x), \delta(x-x_0)\right\rangle = f(x_0)$$

Properties: Algebraic and Calculus Properties / Eigenschaften: Algebraische Eigenschaften und Rechenregeln

$$\alpha\,\delta(x-x_0):$$

$$\int_{x=-\infty}^{\infty} \alpha\,\delta(x-x_0)\,f(x)\,\mathrm{d}x = \alpha f(x_0)$$

$$\alpha(x)\,\delta(x-x_0):$$

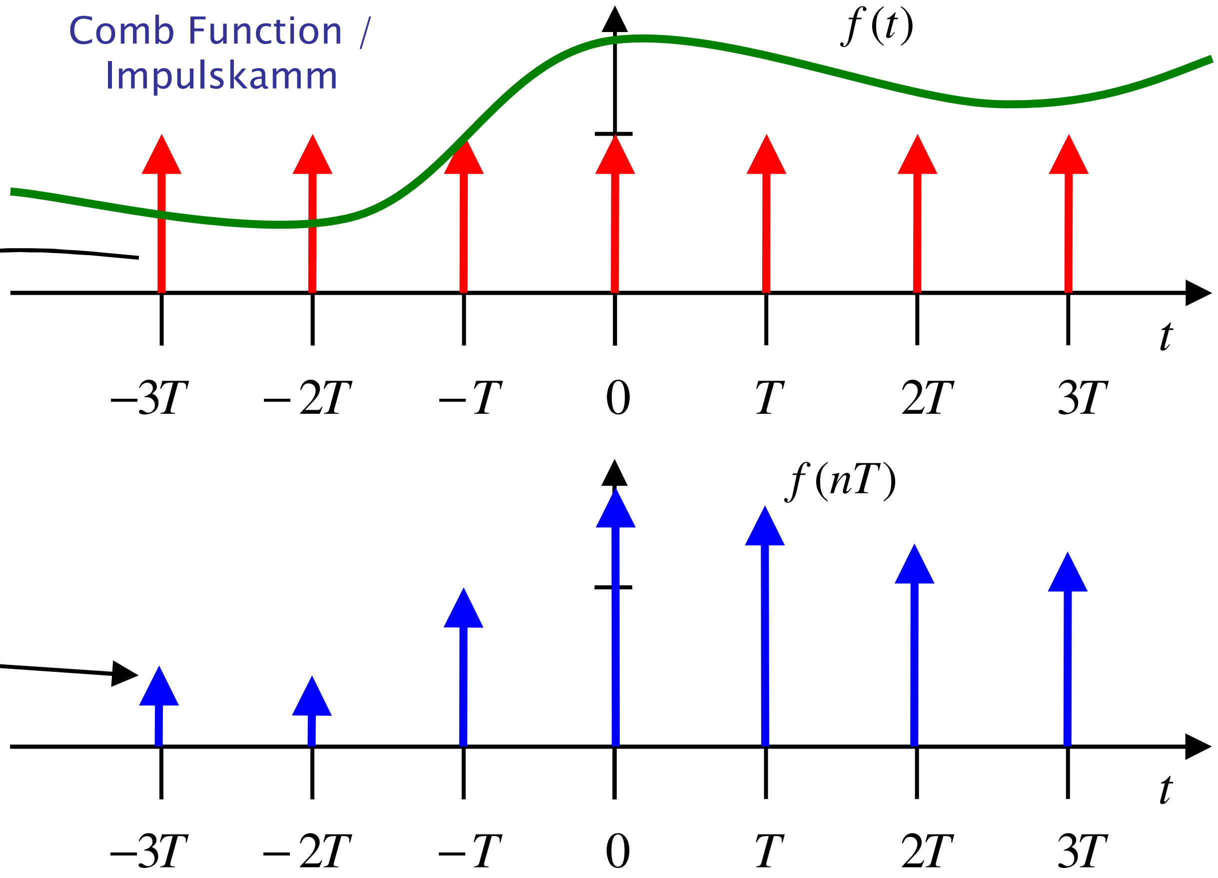
$$\int_{x=-\infty}^{\infty} \alpha(x)\,\delta(x-x_0)\,f(x)\mathrm{d}x = \alpha(x_0)\,f(x_0)$$

$$\alpha(x)\,\delta(x-x_0) = \alpha(x_0)\,\delta(x-x_0)$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution – Signal Processing – Sampling / 1D Delta-Distribution – Signalverarbeitung –Abtastung

$$f(t)\sum_{n=-N}^{N}\delta(t-nT) = \sum_{n=-N}^{N} f(t)\,\delta(t-nT) = \sum_{n=-N}^{N} f(nT)\,\delta(t-nT)$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

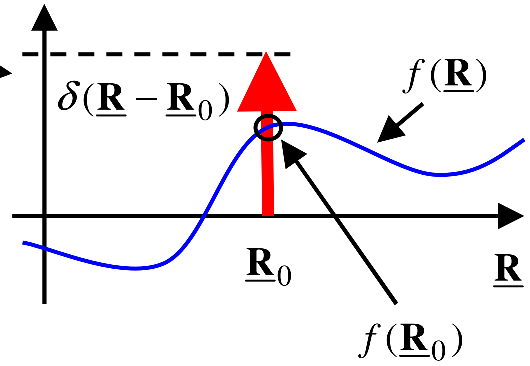
3-D Delta-Distribution / 3D Delta-Distribution

Sifting Property / Siebeigenschaft

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}} = 1$$

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} f(\underline{\mathbf{R}})\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}} = f(\underline{\mathbf{R}}_0)$$

$$f(\underline{\mathbf{R}})\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0) = f(\underline{\mathbf{R}}_0)\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)$$

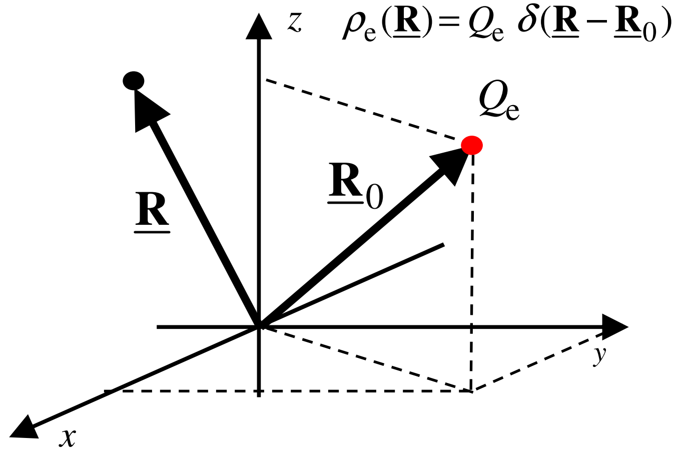


Distribution ➡ Generalized Function / Verallgemeinerte Funktion

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}^3\underline{\mathbf{R}} = \iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} Q_\mathrm{e}\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}$$

$$= Q_\mathrm{e} \underbrace{\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}}_{=1}$$

$$= Q_\mathrm{e}$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)=\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$=\delta(r-r_0)\,\frac{\delta(\varphi-\varphi_0)}{r}\,\delta(z-z_0)=\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$=\delta(R-R_0)\,\frac{\delta(\vartheta-\vartheta_0)}{R}\,\frac{\delta(\varphi-\varphi_0)}{R\sin\vartheta}=\frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}$$

Spherical Coordinate System / Kugelkoordinatensystem

$$=\frac{\delta(\xi_1-\xi_{10})}{h_{\xi_1}}\,\frac{\delta(\xi_2-\xi_{20})}{h_{\xi_2}}\,\frac{\delta(\xi_3-\xi_{30})}{h_{\xi_3}}$$

General Case / Allgemeiner Fall

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\int_{z=-\infty}^{\infty}\int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty}\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$=\underbrace{\left[\int_{z=-\infty}^{\infty}\underbrace{\left[\int_{y=-\infty}^{\infty}\underbrace{\left[\int_{x=-\infty}^{\infty}\delta(x-x_0)\,\mathrm{d}x\right]}_{=1}\delta(y-y_0)\mathrm{d}y\right]}_{=1}\delta(z-z_0)\,\mathrm{d}z\right]}_{=1}=1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}} = \iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}\,\mathrm{d}^3\underline{\mathbf{R}} = \int\limits_{z=-\infty}^{\infty}\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=0}^{\infty} \frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}\,r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$= \underbrace{\left[\int\limits_{z=-\infty}^{\infty} \underbrace{\left[\int\limits_{\varphi=0}^{2\pi} \underbrace{\left[\int\limits_{r=0}^{\infty} \delta(r-r_0)\,\mathrm{d}r\right]}_{=1} \frac{\delta(\varphi-\varphi_0)}{r}\,r\mathrm{d}\varphi\right]}_{=1} \delta(z-z_0)\,\mathrm{d}z\right]}_{=1} = 1$$

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}} = \iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty} \frac{\delta(R-R_0)\,\delta(\vartheta-\vartheta_0)\,\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}\,\mathrm{d}^3\underline{\mathbf{R}} = \int\limits_{\varphi=0}^{2\pi}\int\limits_{\vartheta=0}^{\pi}\int\limits_{R=0}^{\infty} \frac{\delta(R-R_0)\,\delta(\vartheta-\vartheta_0)\,\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}\,R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$= \underbrace{\left[\int\limits_{\varphi=0}^{2\pi} \underbrace{\left[\int\limits_{\vartheta=0}^{\pi} \underbrace{\left[\int\limits_{R=0}^{\infty} \delta(R-R_0)\,\mathrm{d}R\right]}_{=1} \frac{\delta(\vartheta-\vartheta_0)}{R}\,R\,\mathrm{d}\vartheta\right]}_{=1} \frac{\delta(\varphi-\varphi_0)}{R\sin\vartheta}\,R\sin\vartheta\,\mathrm{d}\varphi\right]}_{=1} = 1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Point Charge Density / Elektrostatische Punktladung

$$Q_{\mathrm{e}} = Q_{\mathrm{e}}(x_0, y_0, z_0) \; [\mathrm{As}]$$

Electrostatic Volume Charge Density / Elektrostatische Raumladungsdichte

$$\rho_{\mathrm{e}}(x, y, z) = Q_{\mathrm{e}}\, \delta(x - x_0)\delta(y - y_0)\delta(z - z_0)$$

Q_{e} ● Point / Punkt

Electrostatic Line Charge Density / Elektrostatische Linienladungsdichte

$$\varsigma_{\mathrm{e}}(z) = \varsigma_{\mathrm{e}}(x_0, y_0, z) [\mathrm{As/m}]$$

Electrostatic Line Charge Density / Elektrostatische Linienladungsdichte

$$\rho_{\mathrm{e}}(x, y, z) = \varsigma_{\mathrm{e}}(z)\, \delta(x - x_0)\delta(y - y_0)$$

Line / Linie

$\varsigma_{\mathrm{e}}(z)$

Electrostatic Surface Charge Density / Elektrostatische Flächenladungsdichte

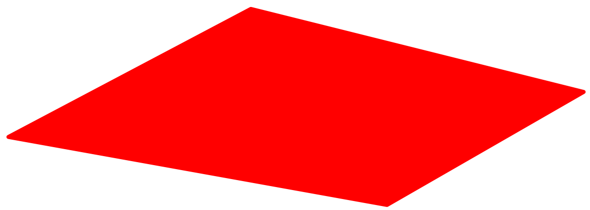
$$\eta_{\mathrm{e}}(x, y) = \eta_{\mathrm{e}}(x, y, z_0) \left[\mathrm{As/m^2} \right]$$

Electrostatic Charge Density / Elektrostatische Ladungsdichte

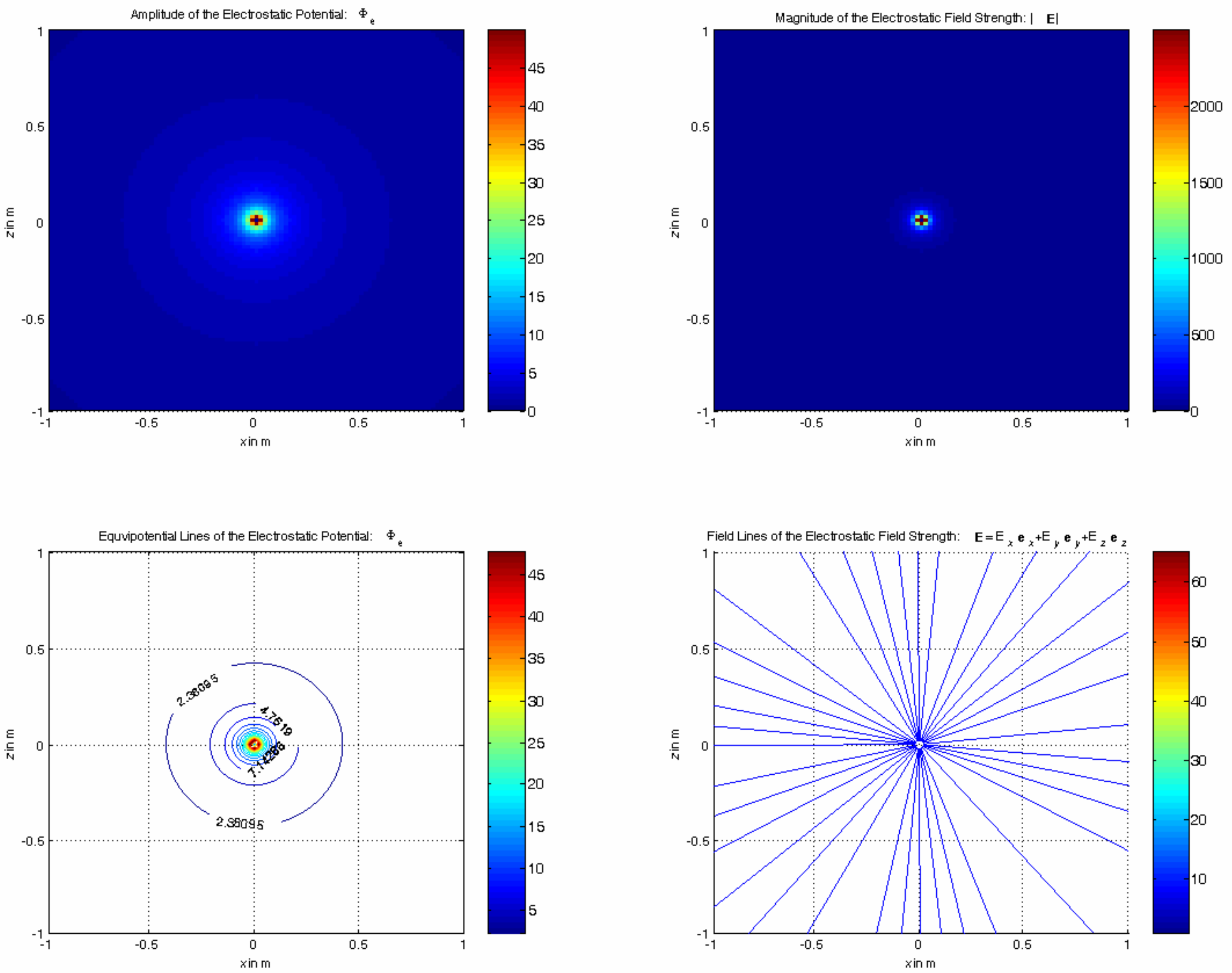
$$\rho_{\mathrm{e}}(x, y, z) = \eta_{\mathrm{e}}(x, y)\, \delta(z - z_0)$$

Surface / Surface

$\eta_{\mathrm{e}}(x, y)$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)



ES Fields – Point Charge Concept / ES Felder – Konzept der Punktladung (...)

Electrostatic Charge Density / Elektrostatische Ladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = Q_\mathrm{e}\,\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0)$$

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q_\mathrm{e}}{\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0\right|}$$

$$\frac{1}{\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0\right|} = \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}$$

Electrostatic Field Strength / Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_\mathrm{e}(\underline{\mathbf{R}})$$

$$= \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_0}{\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0\right|^3}$$

$$\frac{1}{\left|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0\right|^3} = \frac{1}{\left(\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}\right)^3}$$

$$= \frac{1}{\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right]^{3/2}}$$

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral

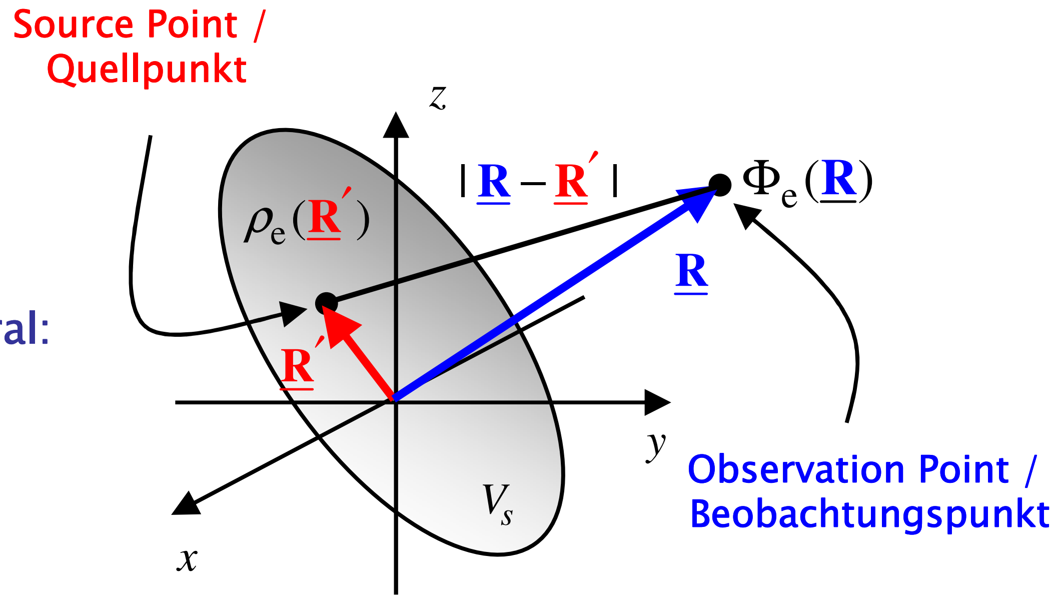
Poisson and Laplace Equation / Poisson- und Laplace-Gleichung

$$\Delta\Phi_e(\mathbf{R}) = \begin{cases} -\dfrac{\rho_e(\mathbf{R})}{\varepsilon_0} & \text{for / für} \quad \rho_e(\mathbf{R}) \neq 0 \quad \text{Poisson Equation / Poisson-Gleichung} \\ 0 & \text{for / für} \quad \rho_e(\mathbf{R}) = 0 \quad \text{Laplace Equation / Laplace-Gleichung} \end{cases}$$

$$\Delta = \nabla^2 = \nabla \bullet \nabla : \text{Laplace Operator / Laplace-Operator}$$

Limited Source Volume / Begrenztes Quellvolumen

$$\rho_e(\mathbf{R}) \begin{cases} \neq 0 & \mathbf{R} \in V_s \\ 0 & \mathbf{R} \notin V_s \end{cases}$$



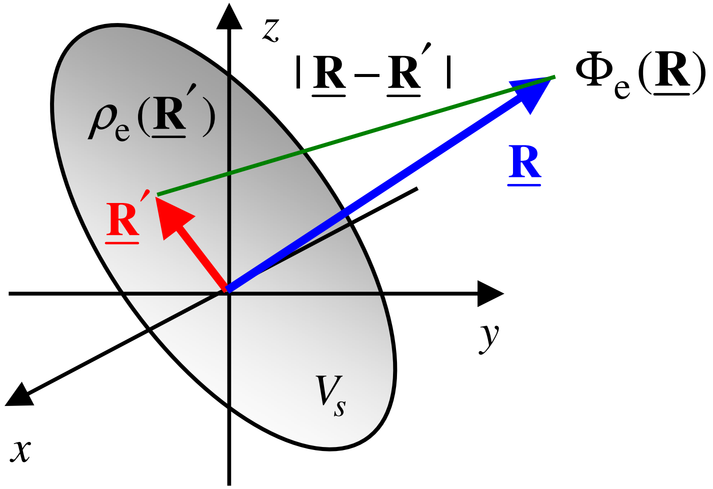
Coulomb Integral / Coulomb-Integral:

$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\rho_e(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|} d^3\mathbf{R}'$$

$\rho_e(\mathbf{R}')$: known / bekannt

$\Phi_e(\mathbf{R})$: unknown / unbekannt

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral (...)



Coulomb Integral / Coulomb-Integral:

$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\rho_e(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|} d^3\mathbf{R}'$$

$\rho_e(\mathbf{R}')$: known / bekannt

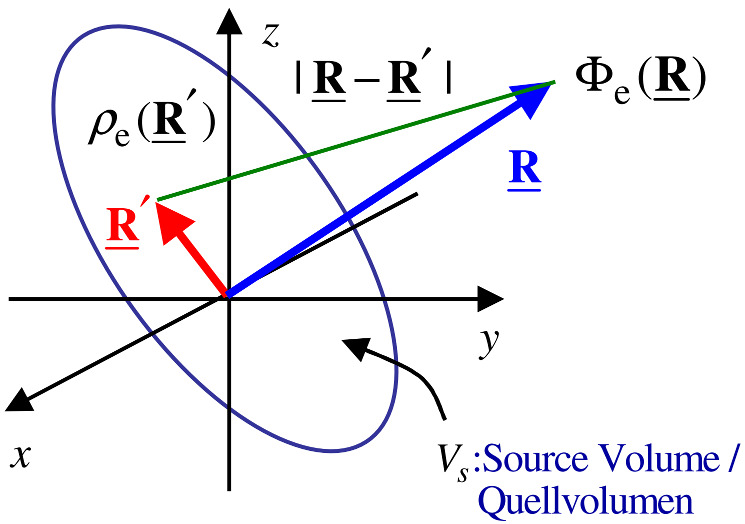
$\Phi_e(\mathbf{R})$: unknown / unbekannt

$$\Delta\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \Delta \iiint_{V_s} \frac{1}{|\mathbf{R}-\mathbf{R}'|} \rho_e(\mathbf{R}') d^3\mathbf{R}'$$

$$= \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \underbrace{\left[\Delta \frac{1}{|\mathbf{R}-\mathbf{R}'|} \right]}_{=-4\pi\delta(\mathbf{R}-\mathbf{R}')} \rho_e(\mathbf{R}') d^3\mathbf{R}' \qquad \text{with} \quad \Delta \frac{1}{4\pi|\mathbf{R}-\mathbf{R}'|} = -\delta\left(\mathbf{R}-\mathbf{R}'\right)$$

$$= -\frac{1}{4\pi\varepsilon_0} \underbrace{\iiint_{V_s} 4\pi\delta(\mathbf{R}-\mathbf{R}')\rho_e(\mathbf{R}') d^3\mathbf{R}'}_{=\rho_e(\mathbf{R})} = -\frac{1}{\varepsilon_0}\rho_e(\mathbf{R})$$

ES Fields – Green's Function / ES Felder – Greensche Funktion



$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_s}\frac{\rho_e(\underline{\mathbf{R}}')}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}d^3\underline{\mathbf{R}}'$$

$$= \frac{1}{\varepsilon_0}\iiint_{V_s}\underbrace{\frac{1}{4\pi}\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}}_{=G_e^{ES}(\underline{\mathbf{R}}-\underline{\mathbf{R}}')}\rho_e(\underline{\mathbf{R}}')d^3\underline{\mathbf{R}}'$$

Electrostatic Green's Function / Elektrostatische Greensche Funktion

$$G_e^{ES}(\underline{\mathbf{R}}-\underline{\mathbf{R}}') = \frac{1}{4\pi}\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|} \qquad \text{for / für } \underline{\mathbf{R}} \neq \underline{\mathbf{R}}'$$

with $\quad \Delta G_e^{ES}(\underline{\mathbf{R}}-\underline{\mathbf{R}}') = -\delta\left(\underline{\mathbf{R}}-\underline{\mathbf{R}}'\right)$

Normalized Potential of a Point Charge / Normiertes Potential einer Punktladung

Electrostatic Potential of an Electrostatic Point Charge / Elektrostatisches Potential einer elektrostatischen Punktladung

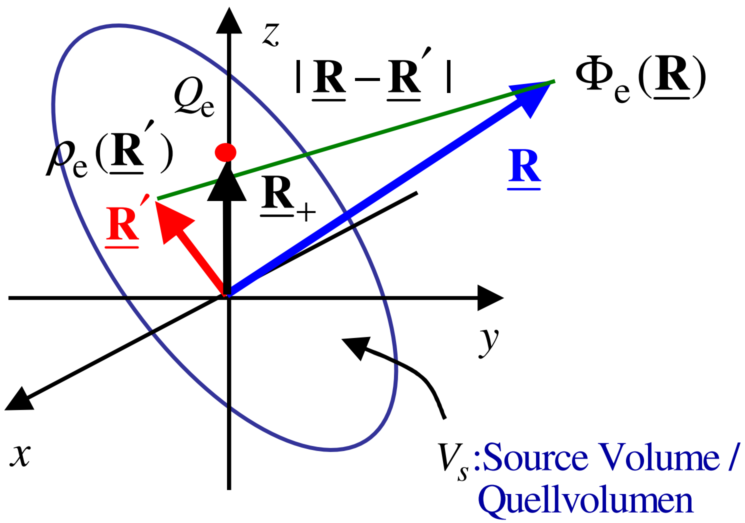
$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi}\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|} \qquad \text{for / für} \quad \rho_e(\underline{\mathbf{R}}) = Q_e\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_+)$$

ES Fields – Potential of a Point Charge / ES Felder – Potential einer Punktladung

Electrostatic Volume Charge Density /
Elektrostatisches Raumladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = Q_e \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_+)$$

with / mit $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$ $\quad \underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$



$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{Q_e \delta(\underline{\mathbf{R}}' - \underline{\mathbf{R}}_+)}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} d^3\underline{\mathbf{R}}'$$

$$= \frac{Q_e}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\delta(\underline{\mathbf{R}}' - \underline{\mathbf{R}}_+)}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} d^3\underline{\mathbf{R}}'$$

$$= \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

Field of an Electrostatic Point Charge / Feld einer elektrostatischen Punktladung

Electrostatic Potential of a Point Charge /
Elektrostatisches Potential einer Punktladung

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

with $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$\underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|} = \frac{1}{\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}} = \frac{1}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2\right]^{1/2}}$$

Electrostatic Field Strength of a Point Charge /
Elektrostatische Feldstärke einer Punktladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3}$$

with $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

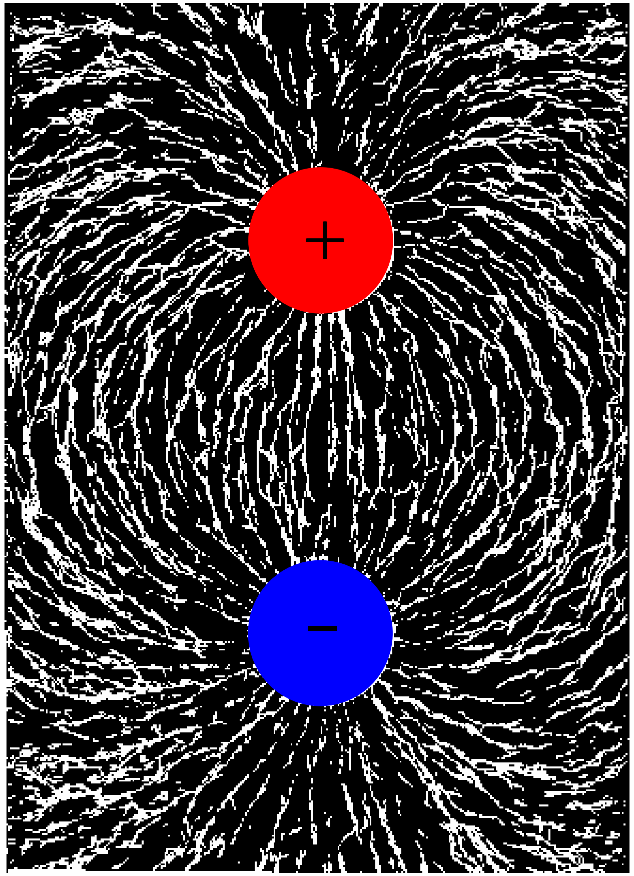
$\underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$

$$\frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3} = \frac{(x-x_+)\underline{\mathbf{e}}_x + (y-y_+)\underline{\mathbf{e}}_y + (z-z_+)\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}\right]^3} = \frac{(x-x_+)\underline{\mathbf{e}}_x + (y-y_+)\underline{\mathbf{e}}_y + (z-z_+)\underline{\mathbf{e}}_z}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2\right]^{3/2}}$$

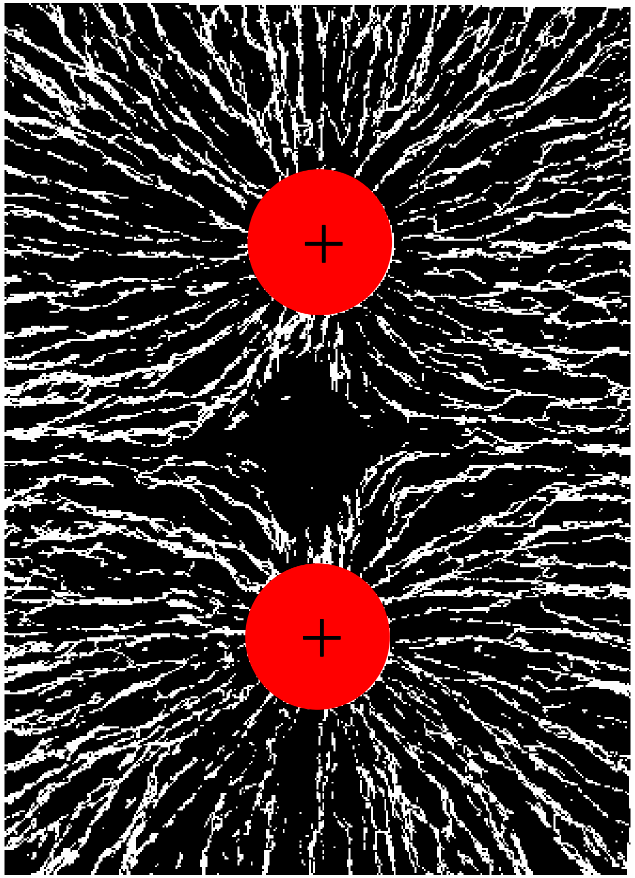
Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischen Dipol

Field Lines of the Electric Field Strength of Two Spheres Carrying Charges of Opposite Sign / Feldlinien der elektrischen Feldstärke zweier ungleich geladener Kugeln



Electric Field Lines of Two Spheres Carrying Charges of the Same Sign / Feldlinien der elektrischen Feldstärke zweier gleich geladener Kugeln



Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischen Dipol

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_{\mathrm{e}+}}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|} + \frac{Q_{\mathrm{e}-}}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_-|}\right)$$

with/mit

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{R}}_\pm = x_\pm\underline{\mathbf{e}}_x + y_\pm\underline{\mathbf{e}}_y + z_\pm\underline{\mathbf{e}}_z$$

$$\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm|} = \frac{1}{\sqrt{(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2}} = \frac{1}{\left[(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2\right]^{1/2}}$$

Electrostatic Field Strength / Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\left(Q_{\mathrm{e}+}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_+}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|^3} + Q_{\mathrm{e}-}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_-}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_-|^3}\right)$$

with/mit

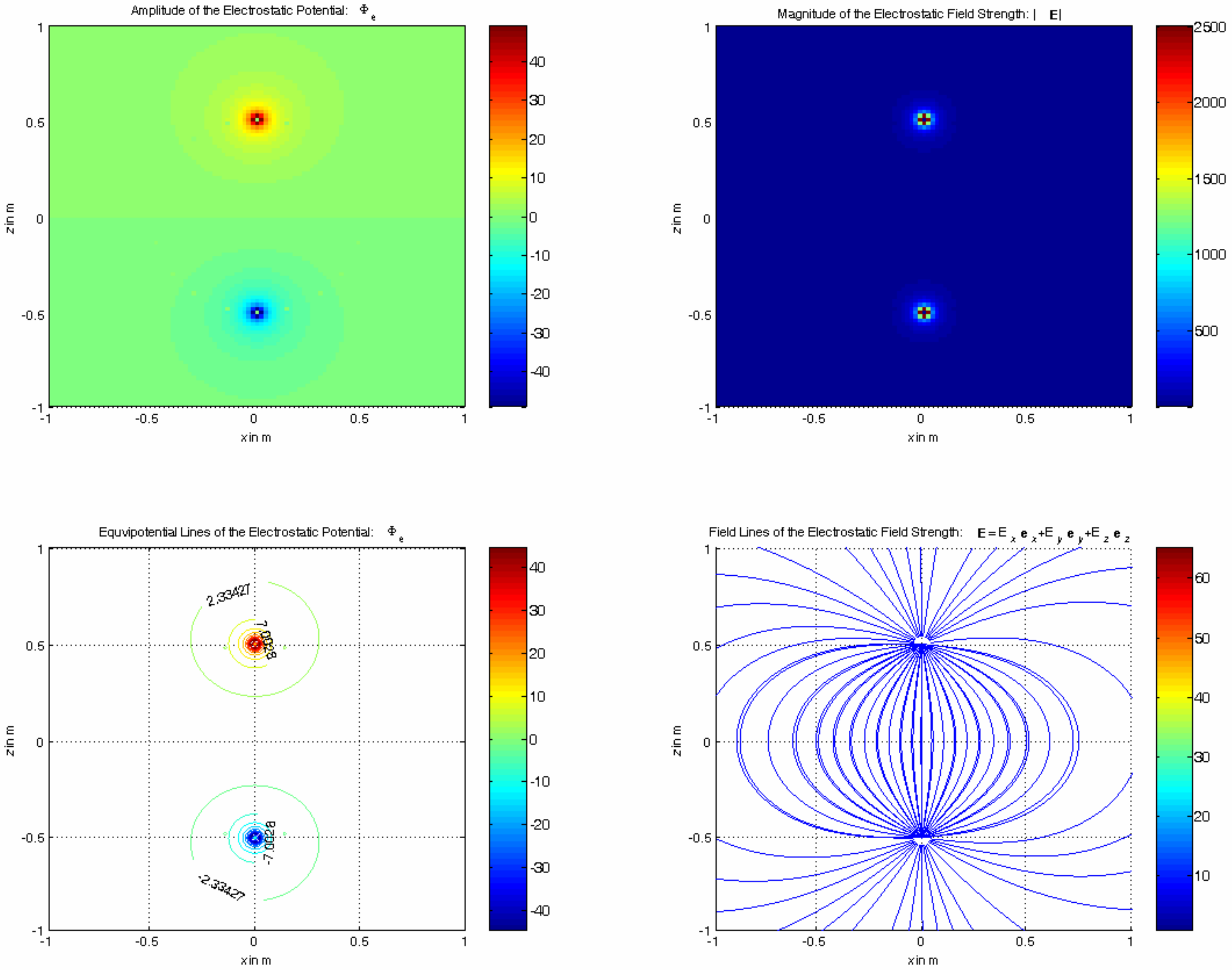
$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{R}}_\pm = x_\pm\underline{\mathbf{e}}_x + y_\pm\underline{\mathbf{e}}_y + z_\pm\underline{\mathbf{e}}_z$$

$$\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm|^3} = \frac{(x-x_\pm)\underline{\mathbf{e}}_x+(y-y_\pm)\underline{\mathbf{e}}_y+(z-z_\pm)\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2}\right]^3} = \frac{(x-x_\pm)\underline{\mathbf{e}}_x+(y-y_\pm)\underline{\mathbf{e}}_y+(z-z_\pm)\underline{\mathbf{e}}_z}{\left[(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2\right]^{3/2}}$$

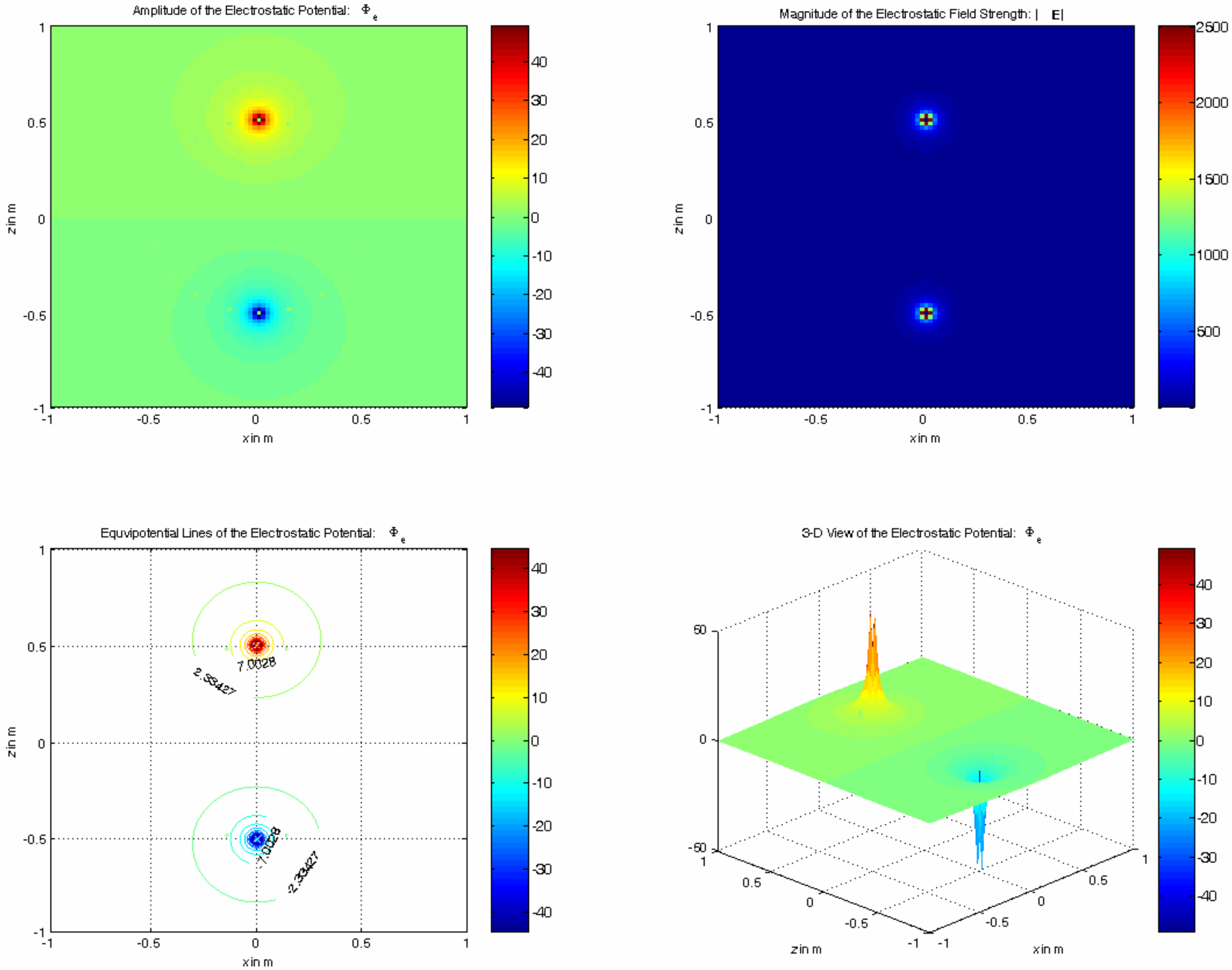
Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischer Dipol

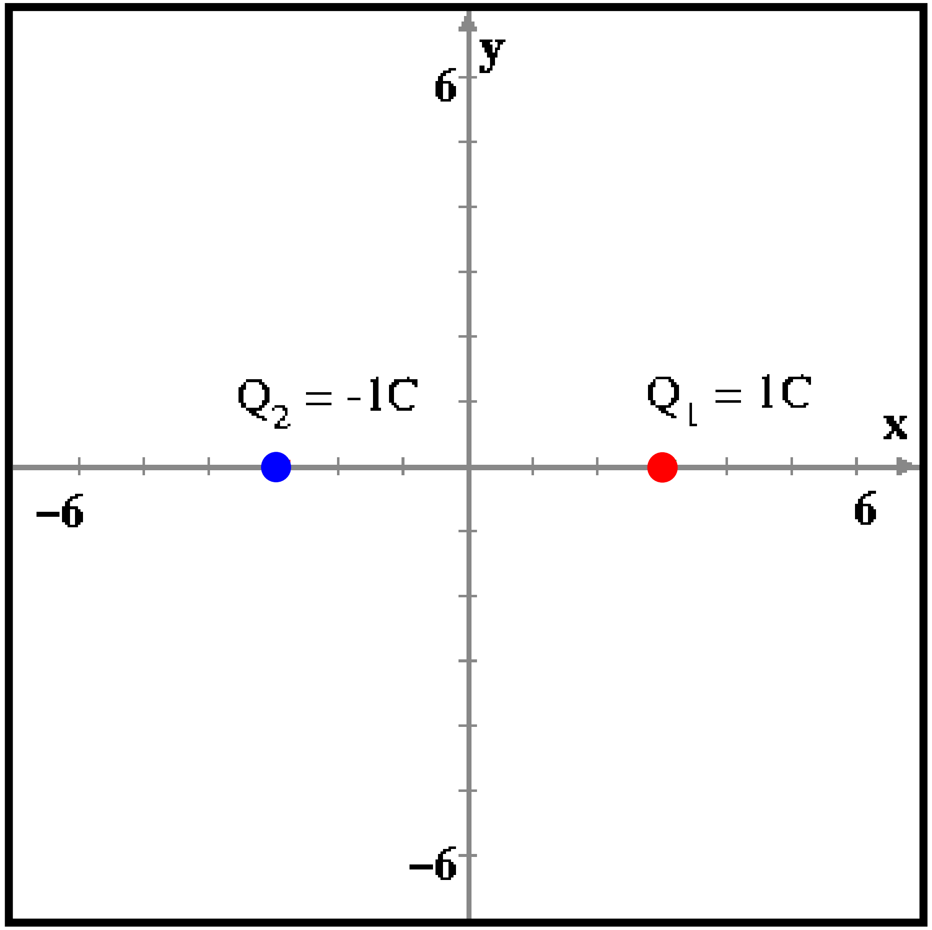


Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischer Dipol



Electrostatic Field Due To Two Point Charges / Elektrostatische Feld von zwei Punktladungen



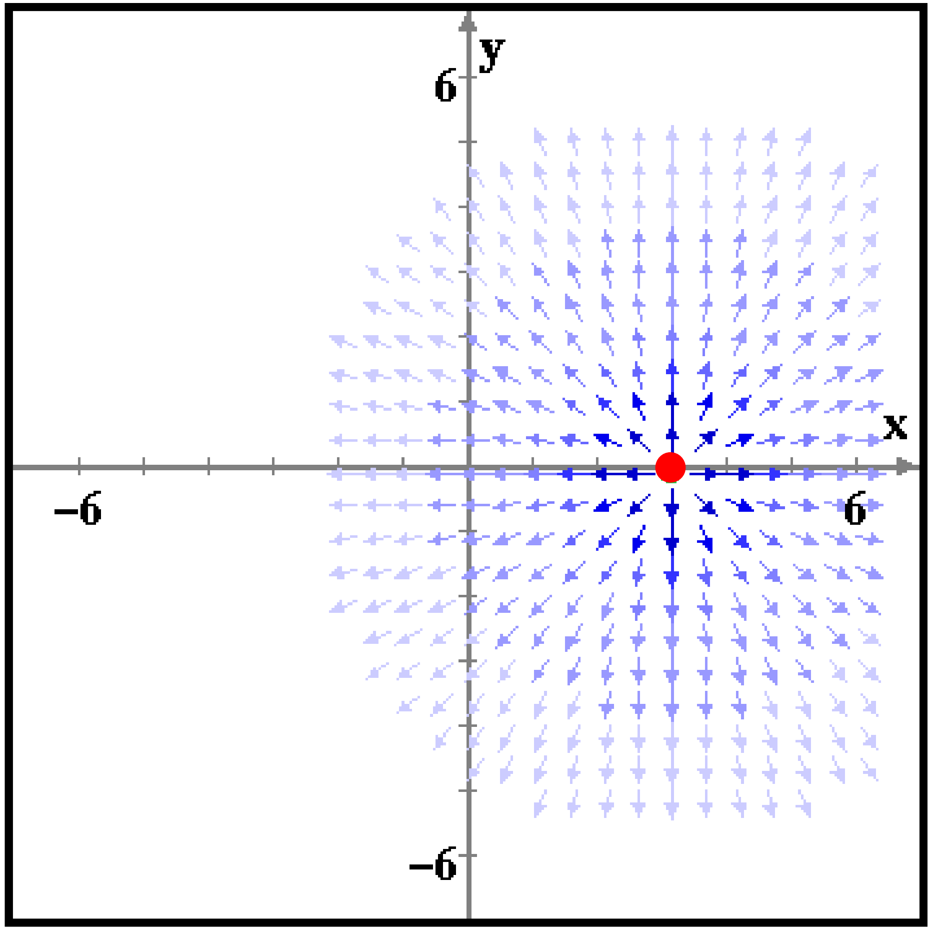
$Q_1 = 1$ As located at $P(x,y,z) = (3,0,0)$
and
$Q_2 = -1$ As located at $P(x,y,z) = (-3,0,0)$

In this demo, arrows are used to sketch the electric field pattern in the *xy* plane due to:

Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone
Press	Q_1 and Q_2 / Q_1 and Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

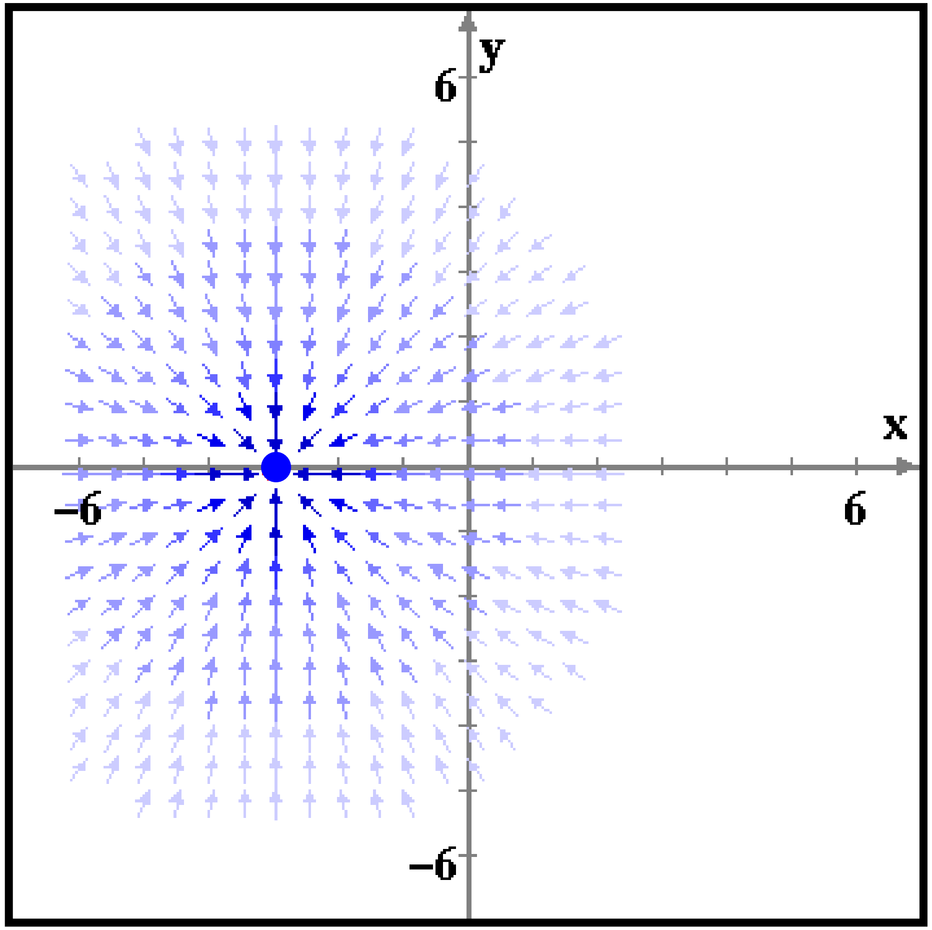
Electrostatic Field... / Elektrostatische Feld...
Q_1 alone / Q_1 alleine



$Q_1 = \ 1$ As located at $P(x,y,z) = (3,0,0)$
and
$Q_2 = -1$ As located at $P(x,y,z) = (-3,0,0)$

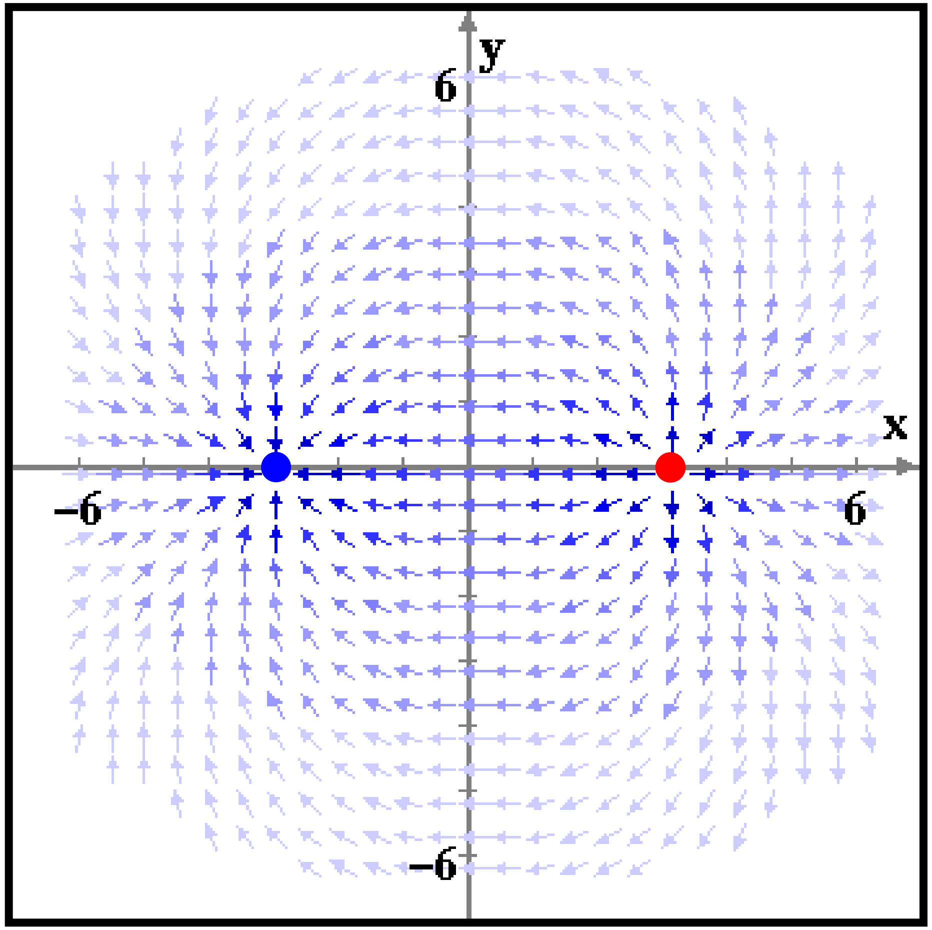
	In this demo, arrows are used to sketch the electric field pattern in the *xy* plane due to:
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
	Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Electrostatic Field… / Elektrostatische Feld…
Q_2 alone / Q_2 alleine



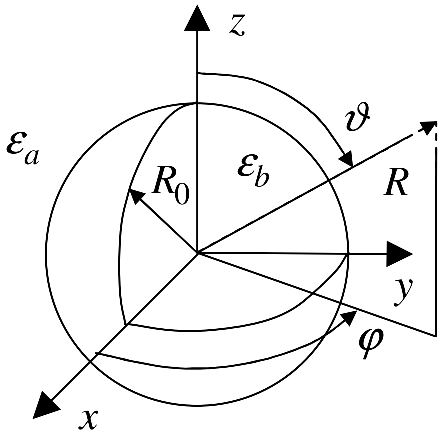
$Q_1 = \ 1$ As located at $P(x,y,z) = (3,0,0)$ and $Q_2 = -1$ As located at $P(x,y,z) = (-3,0,0)$	
	In this demo, arrows are used to sketch the electric field pattern in the *xy* plane due to:
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
	Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Electrostatic Field... / Elektrostatische Feld...
Q_1 and Q_2 / Q_1 und Q_2

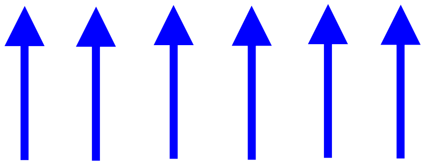


$Q_1 = \ \ 1$ As located at $P(x,y,z) = (3,0,0)$ and $Q_2 = -1$ As located at $P(x,y,z) = (-3,0,0)$	
	In this demo, arrows are used to sketch the electric field pattern in the xy plane due to:
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
	Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Transition Conditions = ? / Übergangsbedingungen = ?



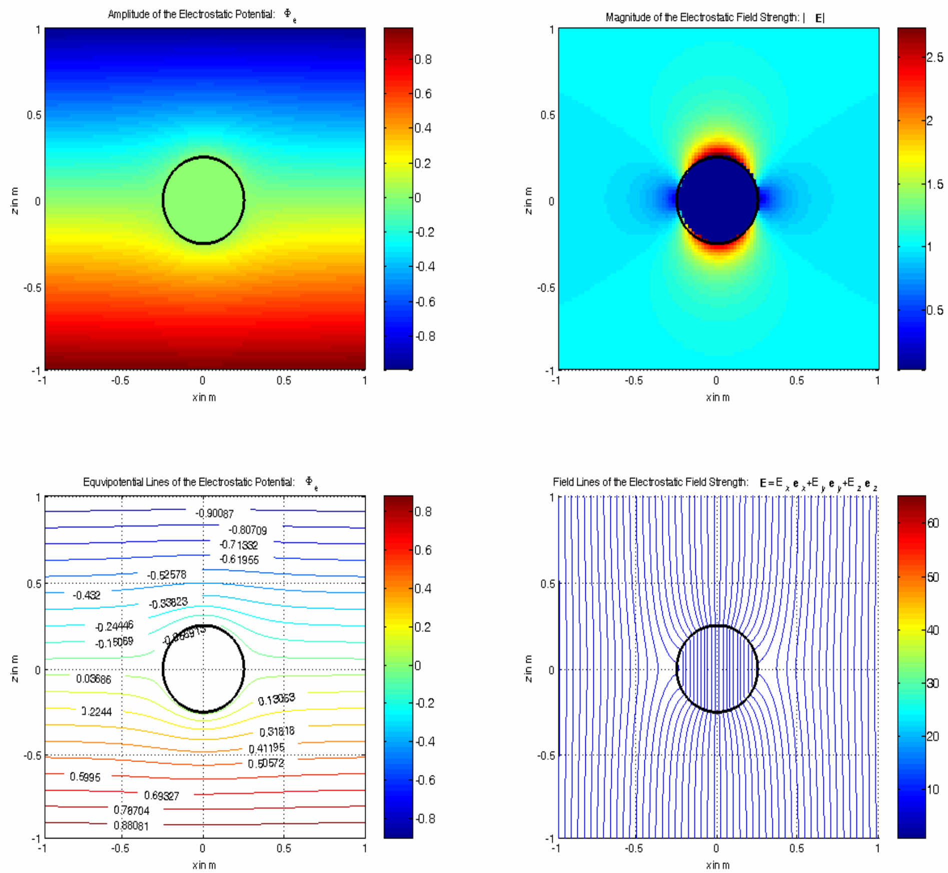
$$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$$



$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \leq R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

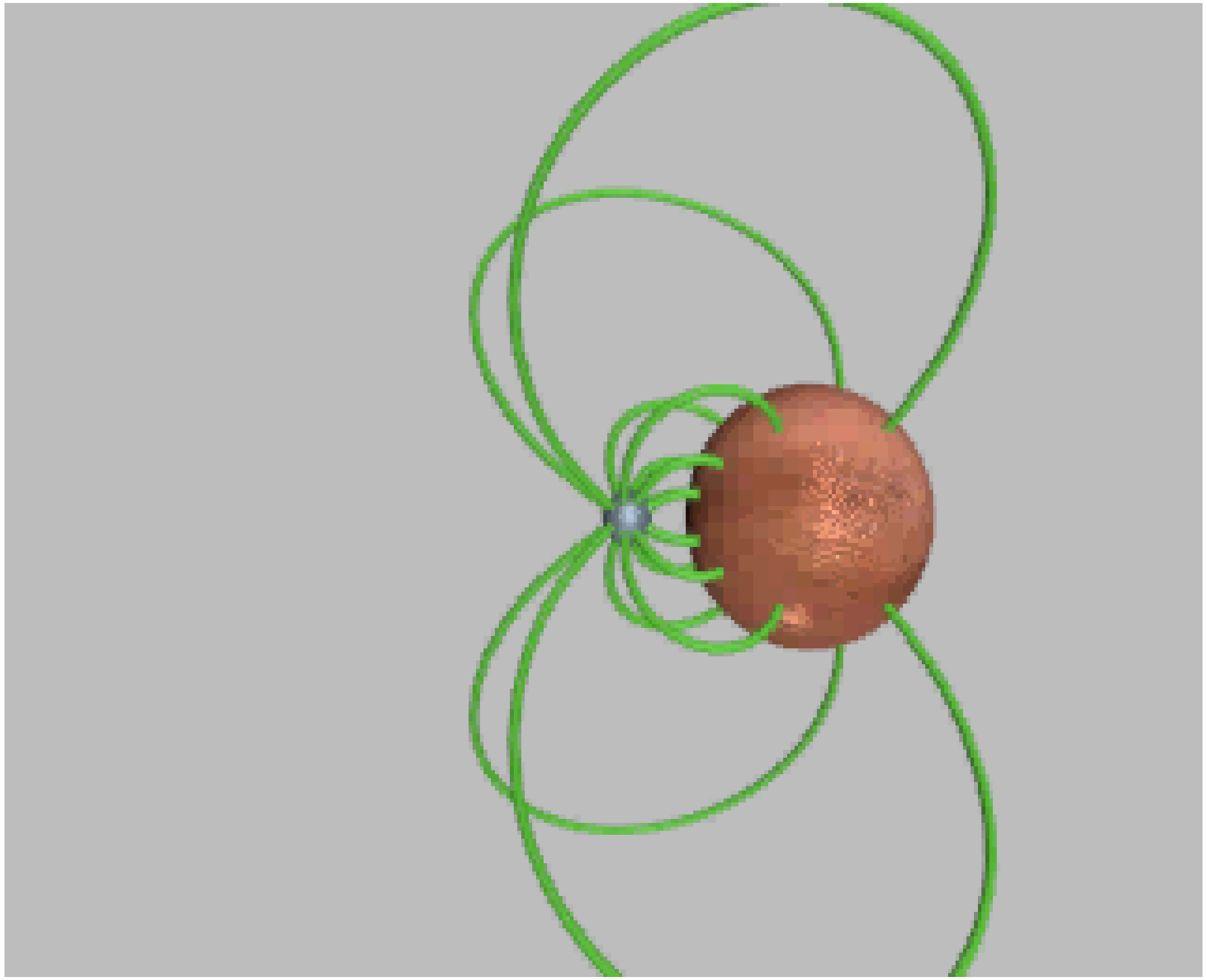
$$\varepsilon_a = \varepsilon_0$$

$$\varepsilon_b = 100\varepsilon_0$$



Boundary Conditions = ? / Randbedingungen = ?

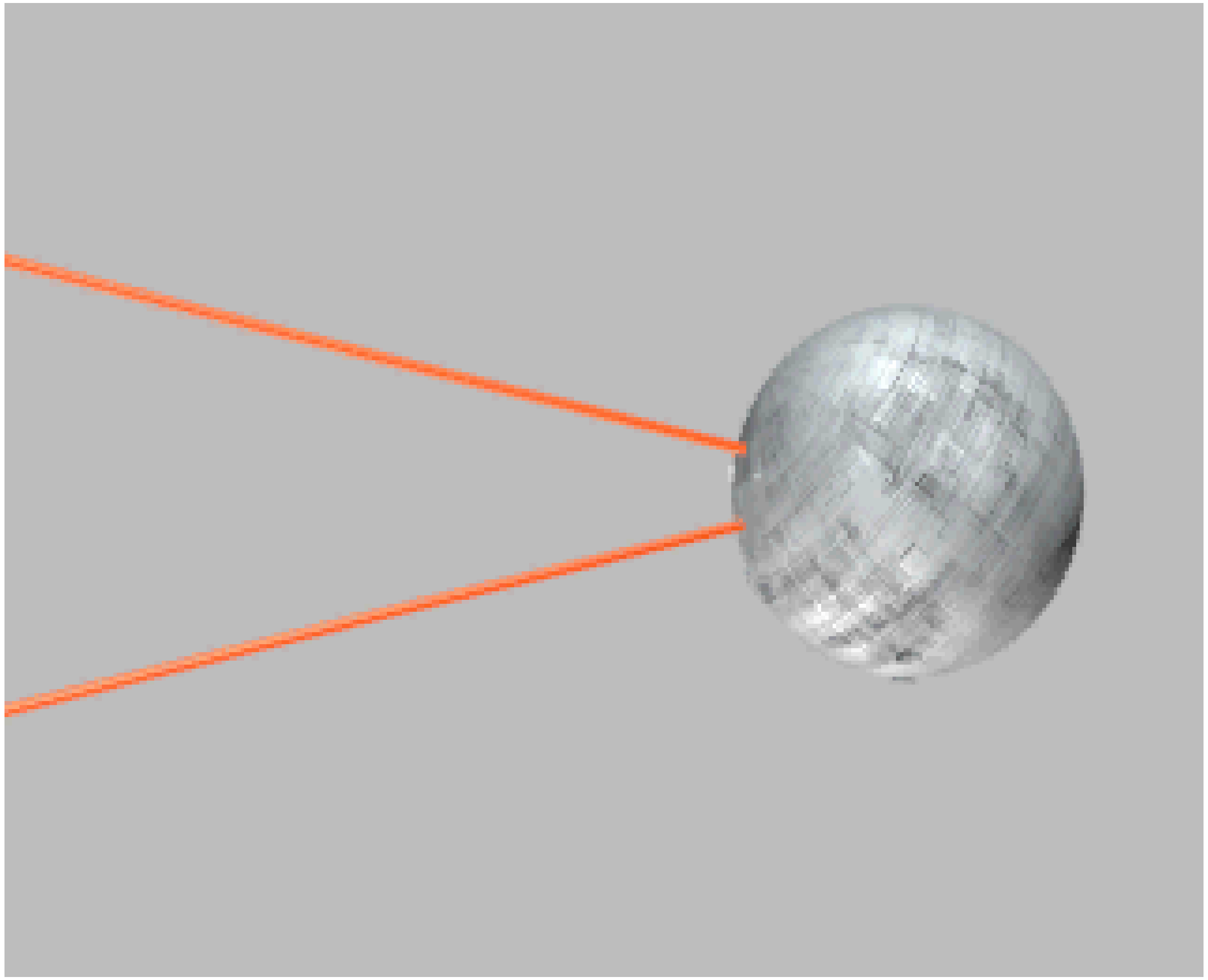
Point Charge Attracted to a Electrically Charged Sphere / Punktladung angezogen von einer elektrisch geladenen Kugel



http://web.mit.edu/jbelcher/www/att.html

Boundary Conditions = ? / Randbedingungen = ?

Point Charge Repulsed By A Charged Sphere / Punktladung abgestoßen von einer elektrisch geladenen Kugel



http://web.mit.edu/jbelcher/www/att.html

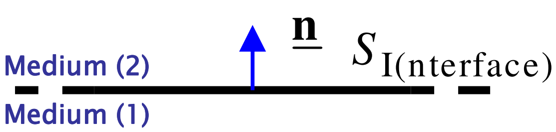
ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Transition Conditions / Übergangsbedingungen



For / Für

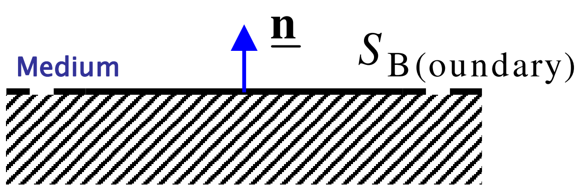
$\underline{\mathbf{R}} \in S_\mathrm{I}$

$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t) \right] = \underline{\mathbf{0}}$$

$$\underline{\mathbf{n}} \bullet \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t) \right] = \begin{cases} \eta_\mathrm{e}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

ws: with sources; sf = source-free / mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen



For / Für

$\underline{\mathbf{R}} \in S_\mathrm{B}$

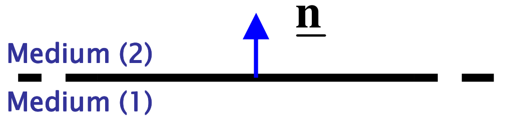
$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}} \qquad \text{pec / iel}$$

$$\underline{\mathbf{n}} \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \eta_\mathrm{e}(\underline{\mathbf{R}},t) \qquad \text{pec / iel}$$

pec = perfectly electric conducting; pmc = perfectly magnetic conducting / iel = ideal elektrisch leitend; iml = ideal magnetisch leitend

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$$\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$$

$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}}) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}}) \right] = \underline{\mathbf{0}}$$

$$\underline{\mathbf{n}} \bullet \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}}) - \underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}}) \right] = \eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{E}}_{tan}(\underline{\mathbf{R}}) = E_{tan}(\underline{\mathbf{R}})\underline{\mathbf{e}}_{tan}$$

$\underline{\mathbf{E}}_{tan}(\underline{\mathbf{R}})$: Vector Tangential Component of / Vektorielle Tangentialkomponente von $\underline{\mathbf{E}}(\underline{\mathbf{R}})$

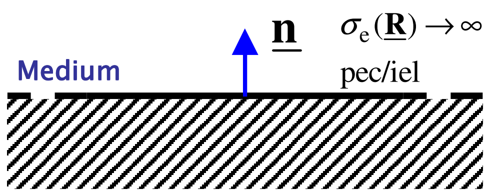
$E_{tan}(\underline{\mathbf{R}})$: Scalar Tangential Component of / Skalare Tangentialkomponente von $\underline{\mathbf{E}}(\underline{\mathbf{R}})$

$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

Boundary Conditions / Randbedingungen

$$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$$



$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}} \qquad \text{pec / iel}$$

$$\underline{\mathbf{n}} \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

pec = perfectly electric conducting /
iel = ideal elektrisch leitend

$$\underline{\mathbf{n}} \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = D_n(\underline{\mathbf{R}})$$

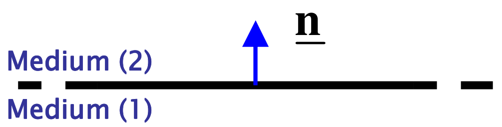
$D_n(\underline{\mathbf{R}})$: Scalar Normal Component of / Skalare Normalkomponente von $\underline{\mathbf{D}}(\underline{\mathbf{R}})$

$$E_{tan}(\underline{\mathbf{R}}) = 0 \qquad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$$\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$$

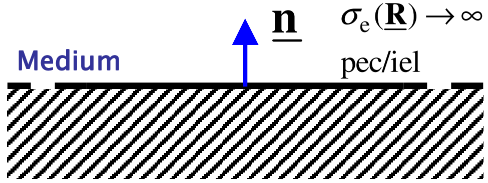
$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen

$$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$$



$$\sigma_{\mathrm{e}}(\underline{\mathbf{R}}) \to \infty$$

$$E_{tan}(\underline{\mathbf{R}}) = 0 \qquad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

pec = perfectly electric conducting /
iel = ideal elektrisch leitend

$$\underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}}) = -\nabla\Phi^{(i)}(\underline{\mathbf{R}}) \qquad \underline{\mathbf{D}}^{(i)}(\underline{\mathbf{R}}) = \varepsilon_0 \varepsilon_{\mathrm{r}}^{(i)} \underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}}) \qquad i = 1, 2$$

$$= -\varepsilon_0 \varepsilon_{\mathrm{r}}^{(i)} \nabla\Phi_{\mathrm{e}}^{(i)}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{n}} \times \nabla\left[\Phi_{\mathrm{e}}^{(2)}(\underline{\mathbf{R}}) - \Phi_{\mathrm{e}}^{(1)}(\underline{\mathbf{R}})\right] = \underline{\mathbf{0}}$$

$$\Phi_{\mathrm{e}}^{(2)}(\underline{\mathbf{R}}) - \Phi_{\mathrm{e}}^{(1)}(\underline{\mathbf{R}}) = \Phi_{\mathrm{e}}$$

$$\underline{\mathbf{n}} \times \nabla\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = 0$$

$$\underbrace{\underline{\mathbf{n}} \bullet \nabla}_{=\frac{\partial}{\partial n}}\left[\varepsilon_{\mathrm{r}}^{(2)} \Phi_{\mathrm{e}}^{(2)}(\underline{\mathbf{R}}) - \varepsilon_{\mathrm{r}}^{(1)} \Phi_{\mathrm{e}}^{(1)}(\underline{\mathbf{R}})\right] = -\frac{\eta_{\mathrm{e}}(\underline{\mathbf{R}})}{\varepsilon_0}$$

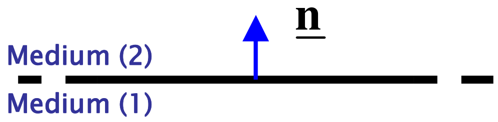
$$\varepsilon_0 \varepsilon_{\mathrm{r}} \underbrace{\underline{\mathbf{n}} \bullet \nabla}_{=\frac{\partial}{\partial n}} \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = -\eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}^{(2)}(\underline{\mathbf{R}}) - \frac{\varepsilon_{\mathrm{r}}^{(1)}}{\varepsilon_{\mathrm{r}}^{(2)}} \frac{\partial}{\partial n}\Phi_{\mathrm{e}}^{(1)}(\underline{\mathbf{R}}) = -\frac{\eta_{\mathrm{e}}(\underline{\mathbf{R}})}{\varepsilon_0 \varepsilon_{\mathrm{r}}^{(2)}}$$

$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = -\frac{\eta_{\mathrm{e}}(\underline{\mathbf{R}})}{\varepsilon_0 \varepsilon_{\mathrm{r}}}$$

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$

$$E_{tan}^{(2)}\left(\underline{\mathbf{R}}\right) - E_{tan}^{(1)}\left(\underline{\mathbf{R}}\right) = 0$$

$$D_n^{(2)}\left(\underline{\mathbf{R}}\right) - D_n^{(1)}\left(\underline{\mathbf{R}}\right) = \eta_{\mathrm{e}}\left(\underline{\mathbf{R}}\right)$$

$$E_{tan}^{(2)}\left(\underline{\mathbf{R}}\right) - E_{tan}^{(1)}\left(\underline{\mathbf{R}}\right) = 0$$

$$\Downarrow$$

$$\Phi_{\mathrm{e}}^{(2)}\left(\underline{\mathbf{R}}\right) - \Phi_{\mathrm{e}}^{(1)}\left(\underline{\mathbf{R}}\right) = \Phi_{\mathrm{e0}} = \mathrm{const.}$$

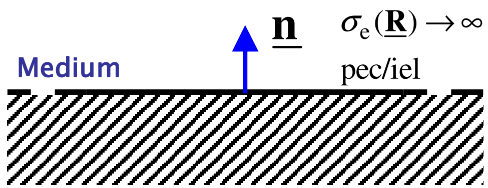
$$D_n^{(2)}\left(\underline{\mathbf{R}}\right) - D_n^{(1)}\left(\underline{\mathbf{R}}\right) = \eta_{\mathrm{e}}\left(\underline{\mathbf{R}}\right)$$

$$\Downarrow$$

$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}^{(2)}\left(\underline{\mathbf{R}}\right) - \frac{\varepsilon_{\mathrm{r}}^{(1)}}{\varepsilon_{\mathrm{r}}^{(2)}}\frac{\partial}{\partial n}\Phi_{\mathrm{e}}^{(1)}\left(\underline{\mathbf{R}}\right) = -\frac{1}{\varepsilon_0\varepsilon_{\mathrm{r}}^{(2)}}\eta_{\mathrm{e}}\left(\underline{\mathbf{R}}\right)$$

Boundary Conditions / Randbedingungen

$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$



$$E_{tan}\left(\underline{\mathbf{R}}\right) = 0 \qquad \text{pec / iel}$$

$$D_n\left(\underline{\mathbf{R}}\right) = \eta_{\mathrm{e}}\left(\underline{\mathbf{R}}\right) \qquad \text{pec / iel}$$

$$E_{tan}\left(\underline{\mathbf{R}}\right) = 0$$

$$\Downarrow$$

$$\Phi_{\mathrm{e}}\left(\underline{\mathbf{R}}\right) = \Phi_{\mathrm{e0}} = \mathrm{const.}$$

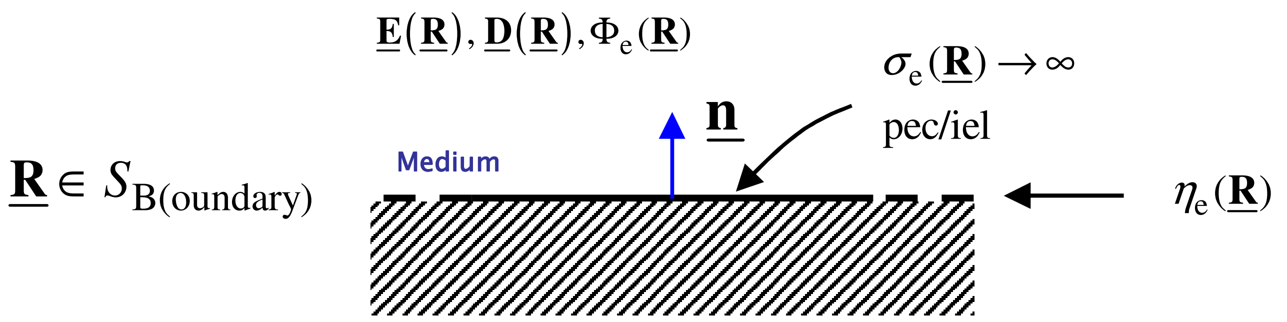
$$D_n\left(\underline{\mathbf{R}}\right) = \eta_{\mathrm{e}}\left(\underline{\mathbf{R}}\right)$$

$$\Downarrow$$

$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}\left(\underline{\mathbf{R}}\right) = -\frac{1}{\varepsilon_0\varepsilon_{\mathrm{r}}}\eta_{\mathrm{e}}\left(\underline{\mathbf{R}}\right)$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Boundary Conditions / Randbedingungen



$$\Phi_e(\underline{\mathbf{R}}) = \Phi_{e0} = \text{const.} \quad (\Phi_{e0} = 0 \text{ V})$$

$$\frac{\partial}{\partial n}\Phi_e(\underline{\mathbf{R}}) = -\frac{1}{\varepsilon_0 \varepsilon_r}\eta_e(\underline{\mathbf{R}})$$

Neumann Boundary Conditions for Φ_e /
Neumann-Randbedingung für Φ_e

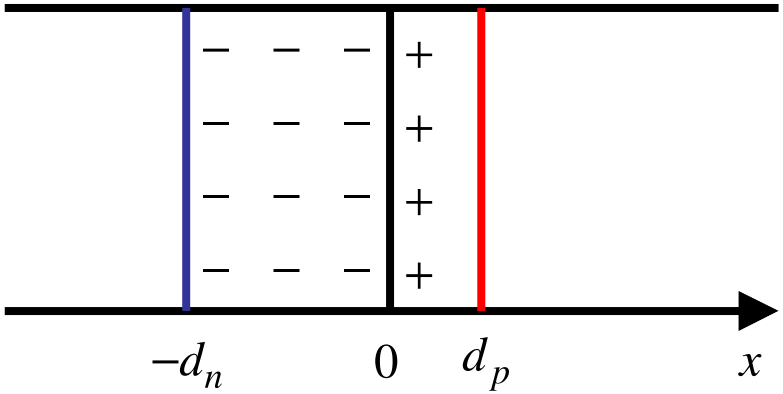
Dirichlet Boundary Conditions for Φ_e /
Dirichlet-Randbedingung für Φ_e

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (3)

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

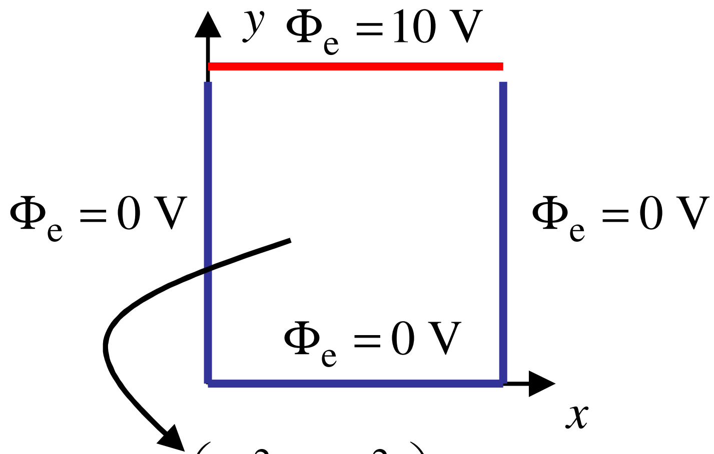
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_\mathrm{e}(x,y,z)=\begin{cases}-\dfrac{\rho_\mathrm{e}(x,y,z)}{\varepsilon_0} & \text{for / für} \quad \rho_\mathrm{e}(x,y,z)\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für} \quad \rho_\mathrm{e}(x,y,z)=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Example: pn Junction – pn Diode / Beispiel: pn-Übergang – pn Diode



$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Phi_\mathrm{e}(x)=\frac{e}{\varepsilon}\begin{cases}-n_e & \text{for / für} \quad -d_n\le x\le 0\\ n_e & \text{for / für} \quad 0\le x\le d_p\end{cases}$$

Example: / Beispiel:



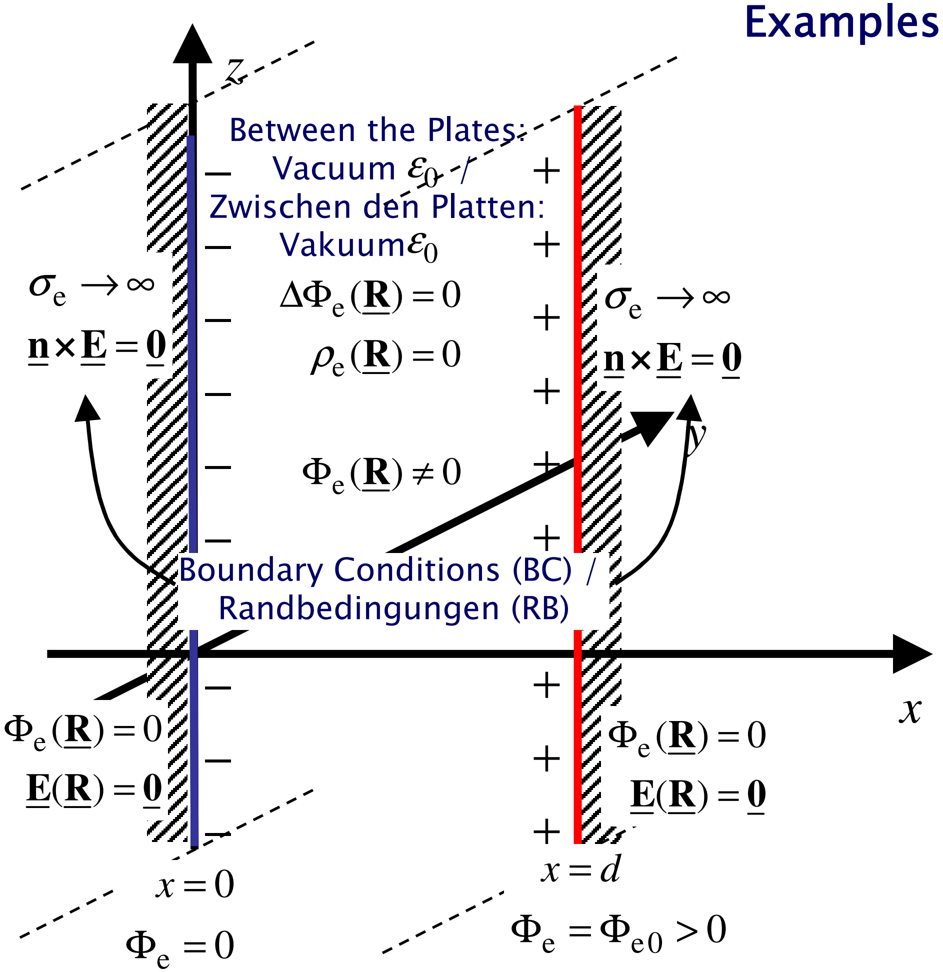
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_\mathrm{e}(x,y)=0$$

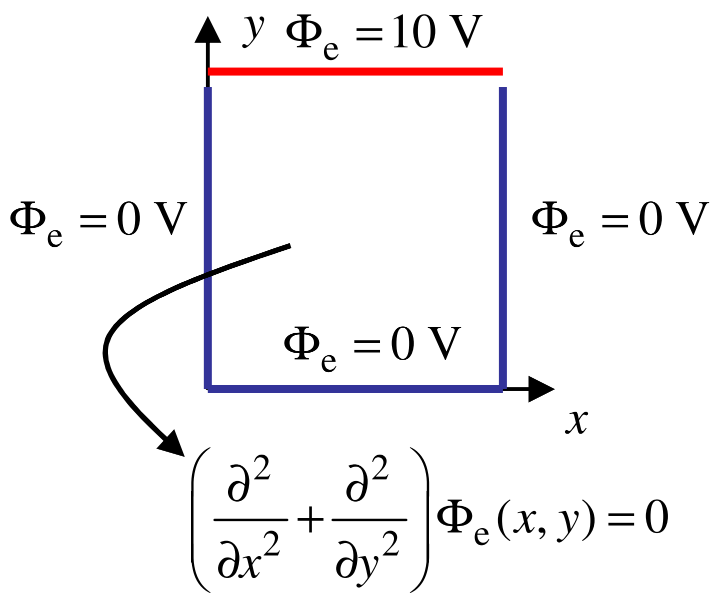
Separation of Variables / Separation der Variablen !

Electrostatic (ES) Fields – Boundary Value Problem (BVP) / Elektrostatische (ES) Felder – Randwertproblem (RWP)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_e(x,y,z)=\begin{cases}-\dfrac{\rho_e(x,y,z)}{\varepsilon_0} & \text{for / für } \rho_e(x,y,z)\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für } \rho_e(x,y,z)=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Examples: / Beispiele:

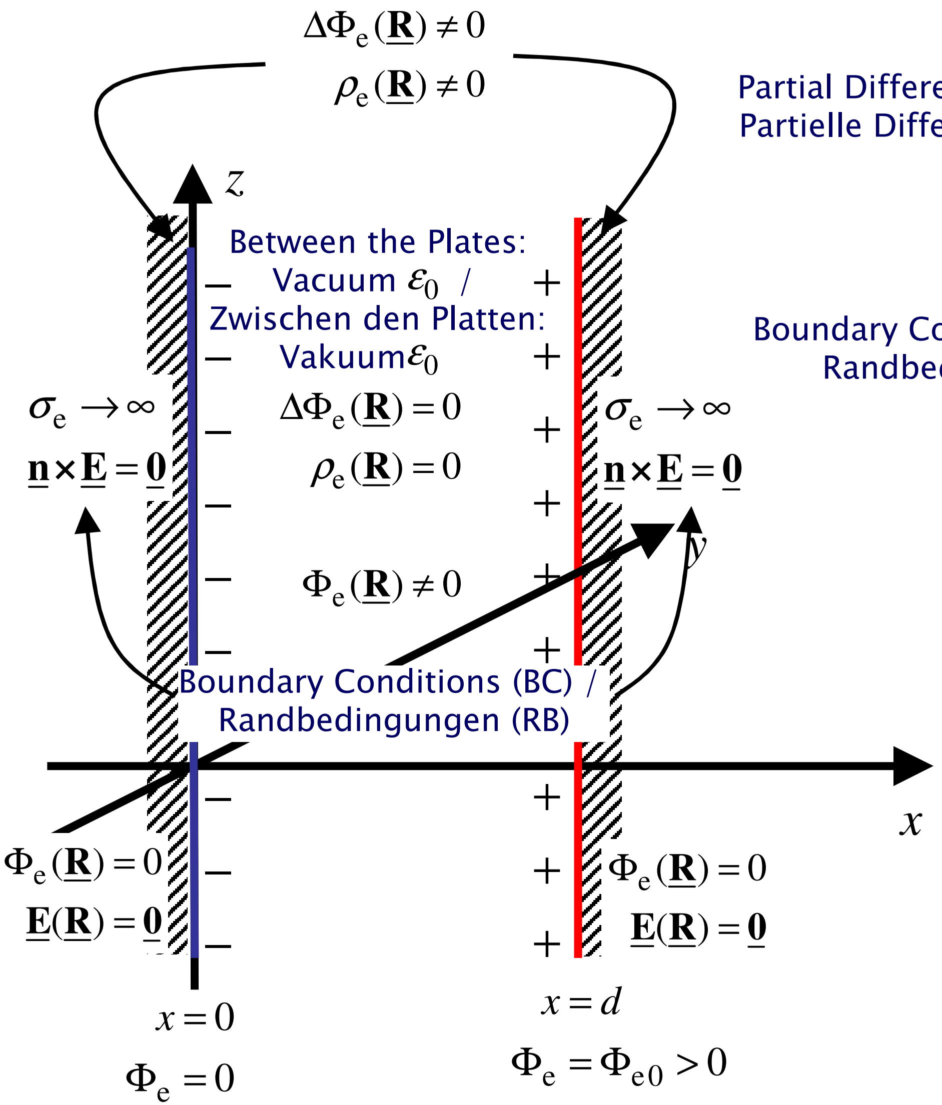




⇒ Separation of Variables / Separation der Variablen !

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Partial Differential Equation / Partielle Differentialgleichung

$$\Delta\Phi_e(\underline{\mathbf{R}}) \begin{cases} = 0 & \text{for / für} \quad 0 < x < d \\ \neq \text{const.} & \text{for / für} \quad x = 0, \; x = d \end{cases}$$

Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \quad \Phi_e = 0$$

$$x = d: \quad \Phi_e = \Phi_{e0} > 0$$

Between the Plates Laplace Equation: Zwischen den Platten: Laplace-Gleichung

$$\Delta\Phi_e(\underline{\mathbf{R}}) = 0$$

... Cartesian Coordinates / ... Kartesische Koordinaten

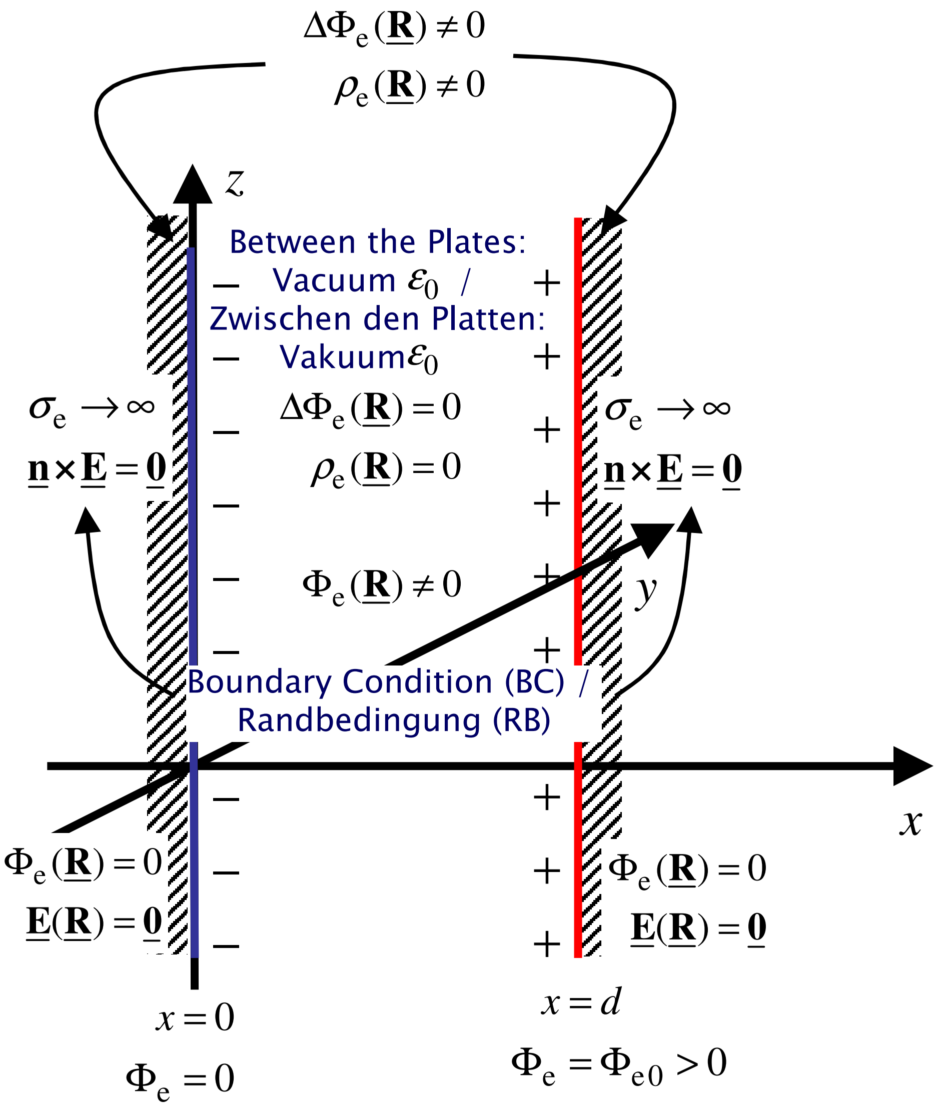
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi_e(x, y, z) = 0$$

... Because of the Symmetry / ... wegen der Symmetrie

$$\frac{d^2}{dx^2}\Phi_e(x) = 0$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatische Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\frac{d^2}{dx^2}\Phi_e(x) = 0 \qquad 0 < x < d$$

Integrating once / Integriere einmal

$$\int \frac{d^2}{dx^2}\Phi_e(x)\,dx = \left[\frac{d}{dx}\Phi_e(x)\right] = \text{const} = a$$

$$\left[\frac{d}{dx}\Phi_e(x)\right] = \text{const} = a$$

Integrating twice / Zweifache Integration ergibt

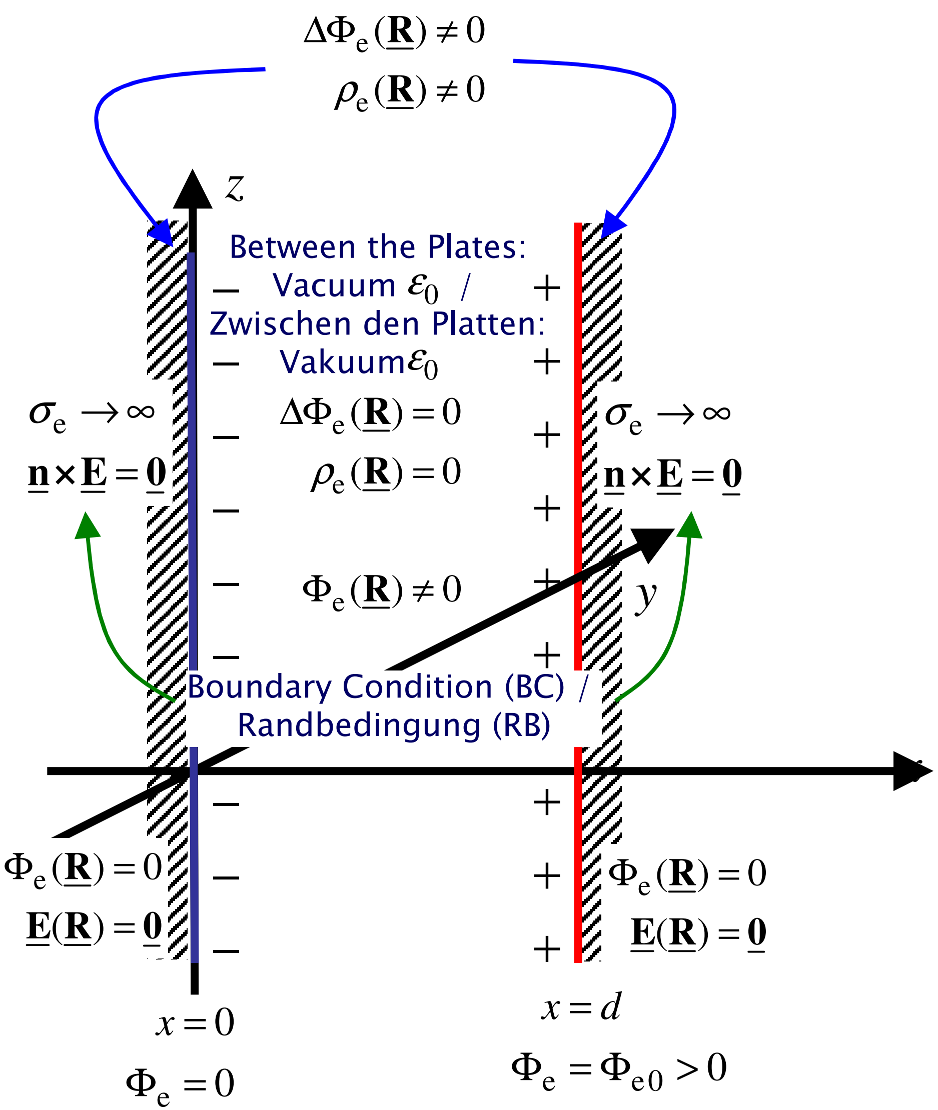
$$\int \left\{ \left[\frac{d}{dx}\Phi_e(x)\right] = \text{const} = a \right\} dx = \Phi_e(x) = ax + b$$

$$\Phi_e(x) = ax + b$$

$$\Rightarrow \quad \Phi_e(x) = ax + b \qquad 0 < x < d$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\Rightarrow \quad \Phi_e(x) = ax + b$$

Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \qquad \Phi_e = 0$$

$$x = d: \qquad \Phi_e = \Phi_{e0} > 0$$

$$\Phi_e(x=0) = a(x=0) + b = 0$$

$$b = 0$$

$$\Phi_e(x) = ax$$

$$\Phi_e(x=b) = a(x=b) = \Phi_{e0}$$

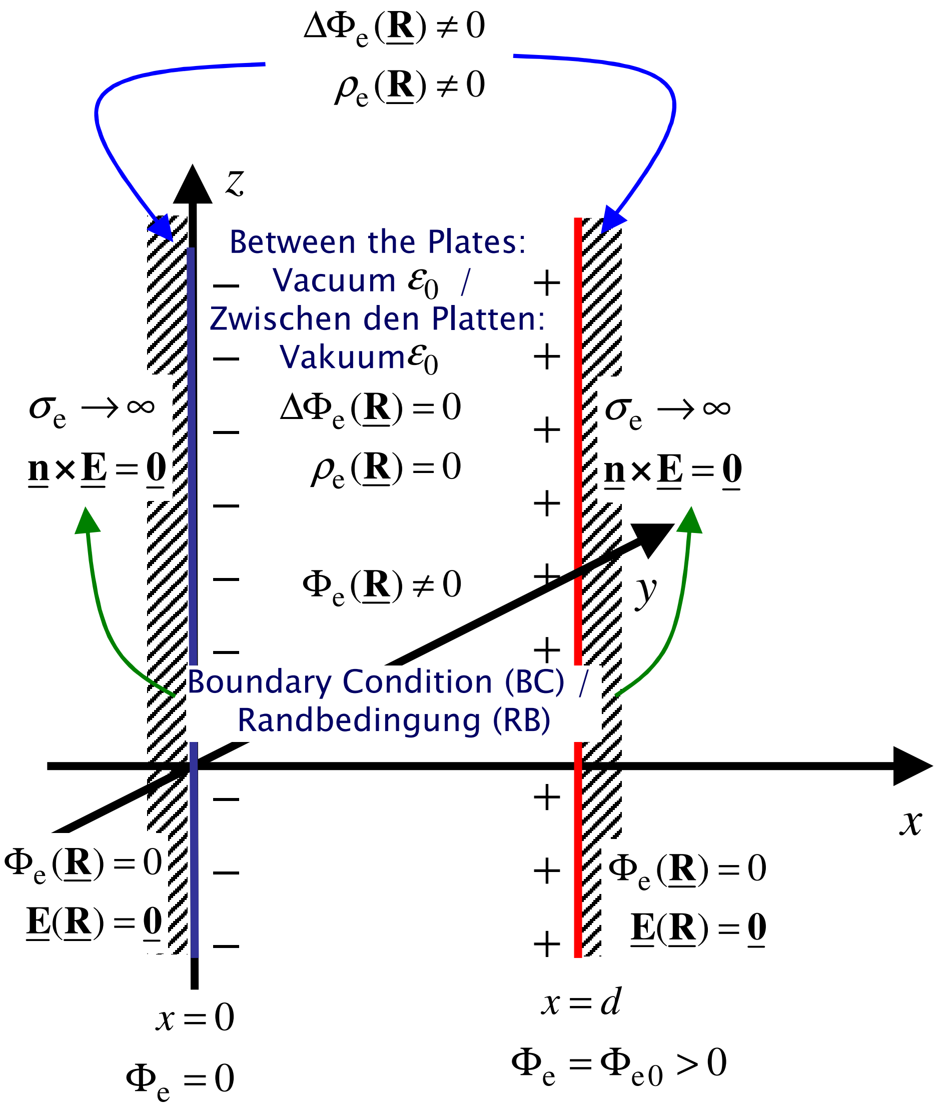
$$a = \frac{\Phi_{e0}}{d}$$

Solution for the Electrostatic Potential / Lösung für das elektrostatische Potential

$$\Rightarrow \quad \Phi_e(x) = \frac{\Phi_{e0}}{d}x \qquad 0 \le x \le d$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Partial Differential Equation (PDE) / Partielle Differentialgleichung (DGL)

$$\frac{d^2}{dx^2}\Phi_e(x) = 0 \qquad 0 < x < d$$

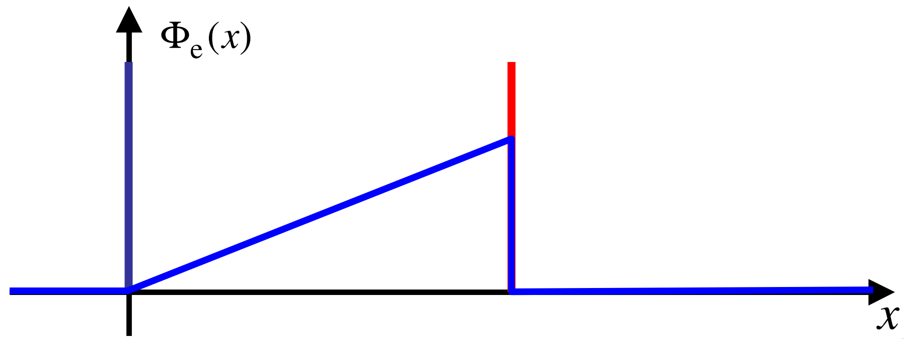
Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \qquad \Phi_e = 0$$

$$x = d: \qquad \Phi_e = \Phi_{e0} > 0$$

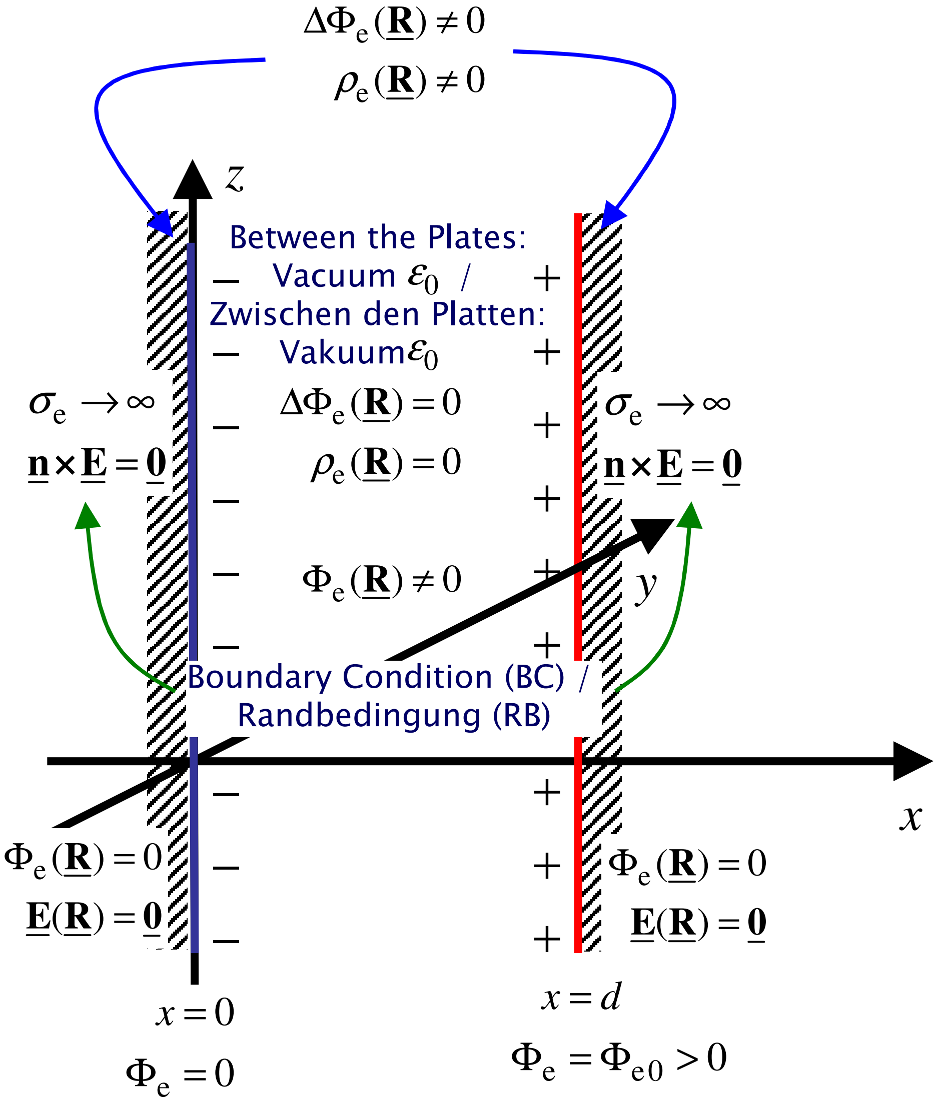
Solution for the Electrostatic Potential / Lösung für das elektrostatische Potential

$$\Phi_e(x) = \begin{cases} \dfrac{\Phi_{e0}}{d}x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$



ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Electrostatic Potential / Elektrostatisches Potential

$$\Phi_e(x) = \begin{cases} \dfrac{\Phi_{e0}}{d} x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}})$$

$$\underline{\mathbf{E}}(x) = -\frac{\mathrm{d}}{\mathrm{d}x}\Phi_e(x)\underline{\mathbf{e}}_x \qquad -\infty < x < \infty$$

$$= \begin{cases} -\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\Phi_{e0}}{d} x\right)\underline{\mathbf{e}}_x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

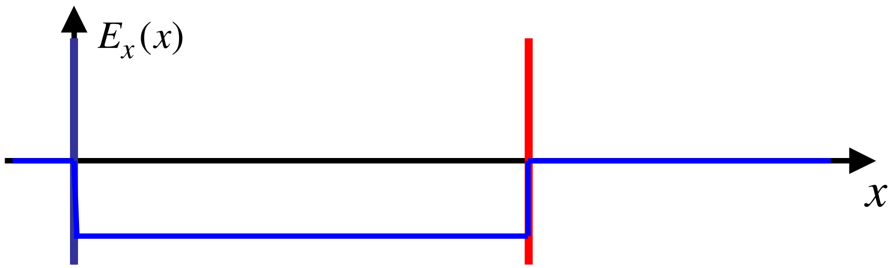
$$= \begin{cases} -\dfrac{\Phi_{e0}}{d}\underline{\mathbf{e}}_x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

⇨ The Electrostatic Potential and Electrostatic Field Strenth are Discontinuous at the Plates / Das elektrostatische Potential und die elektrostatische Feldstärke sind unstetig an den Platten

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

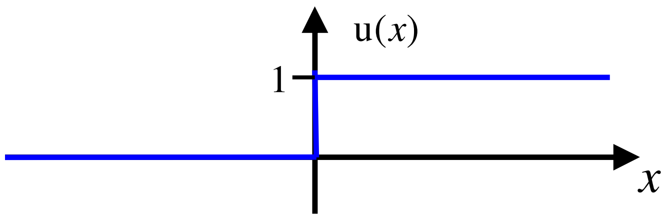
Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung

$$\underline{\mathbf{E}}(x) = \begin{cases} -\dfrac{\Phi_{e0}}{d}\underline{\mathbf{e}}_x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

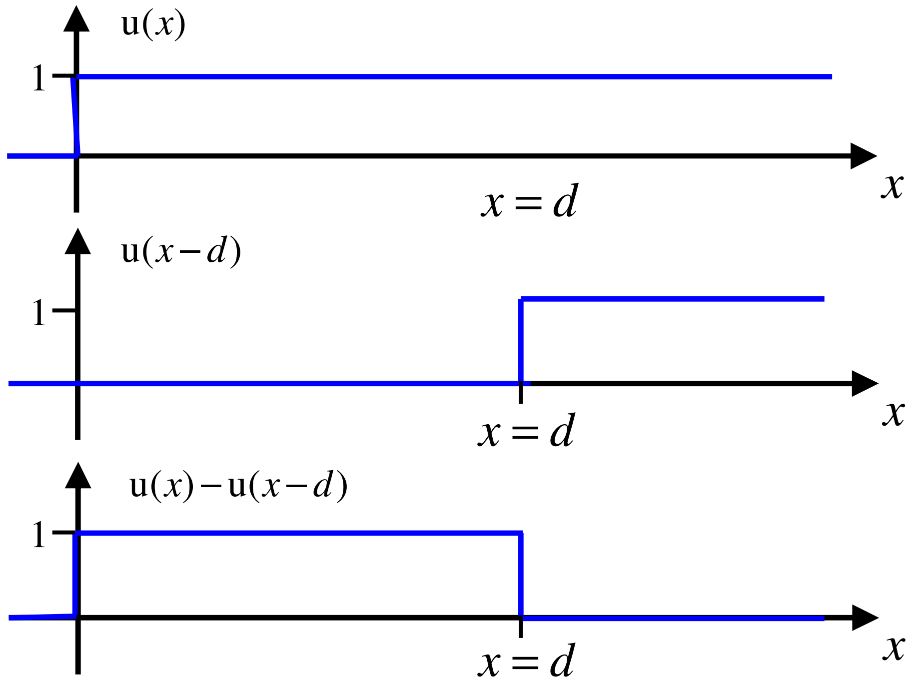


Step Functions / Einheitssprungfunktionen

$$\mathrm{u}(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$



Representation of the Electrostatic Field Strenth using the Unit Step Functions: / Darstellung der elektrostatischen Feldstärke durch Einheitssprungfunktionen:



$$\underline{\mathbf{E}}(x) = -\frac{\Phi_{e0}}{d}\left[\mathrm{u}(x) - \mathrm{u}(x-d)\right]\underline{\mathbf{e}}_x \qquad -\infty < x < \infty$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

$$\frac{d}{dx} D_x(x) = \rho_e(x)$$

$$\varepsilon_0 \frac{d}{dx} E_x(x) = \rho_e(x)$$

$$\frac{d}{dx} E_x(x) = \frac{\rho_e(x)}{\varepsilon_0}$$

$$\frac{d}{dx} E_x(x) = -\frac{\Phi_{e0}}{d} \frac{d}{dx}\left[u(x) - u(x-d)\right]$$

$$= -\frac{\Phi_{e0}}{d}\left[\underbrace{\frac{d}{dx} u(x)}_{\delta(x)} - \underbrace{\frac{d}{dx} u(x-d)}_{\delta(x-d)}\right]$$

$$= -\frac{\Phi_{e0}}{d}\left[\delta(x) - \delta(x-d)\right]$$

$$= \frac{\rho_e(x)}{\varepsilon_0}$$

$$\underbrace{\frac{d}{dx} u(x)}_{=u'(x)} = \delta(x)$$

$$\int_{-\infty}^{\infty} u'(x) f(x)\,dx = u(x) f(x)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} u(x) f'(x)\,dx$$

$$= \left[\underbrace{\underbrace{u(\infty)}_{=1} f(\infty)}_{=f(\infty)} - \underbrace{\underbrace{u(-\infty)}_{0} f(-\infty)}_{=0}\right] - \int_{0}^{\infty} f'(x)\,dx$$

$$= f(\infty) - f(x)\Big|_{0}^{\infty}$$

$$= f(\infty) - \left[f(\infty) - f(0)\right]$$

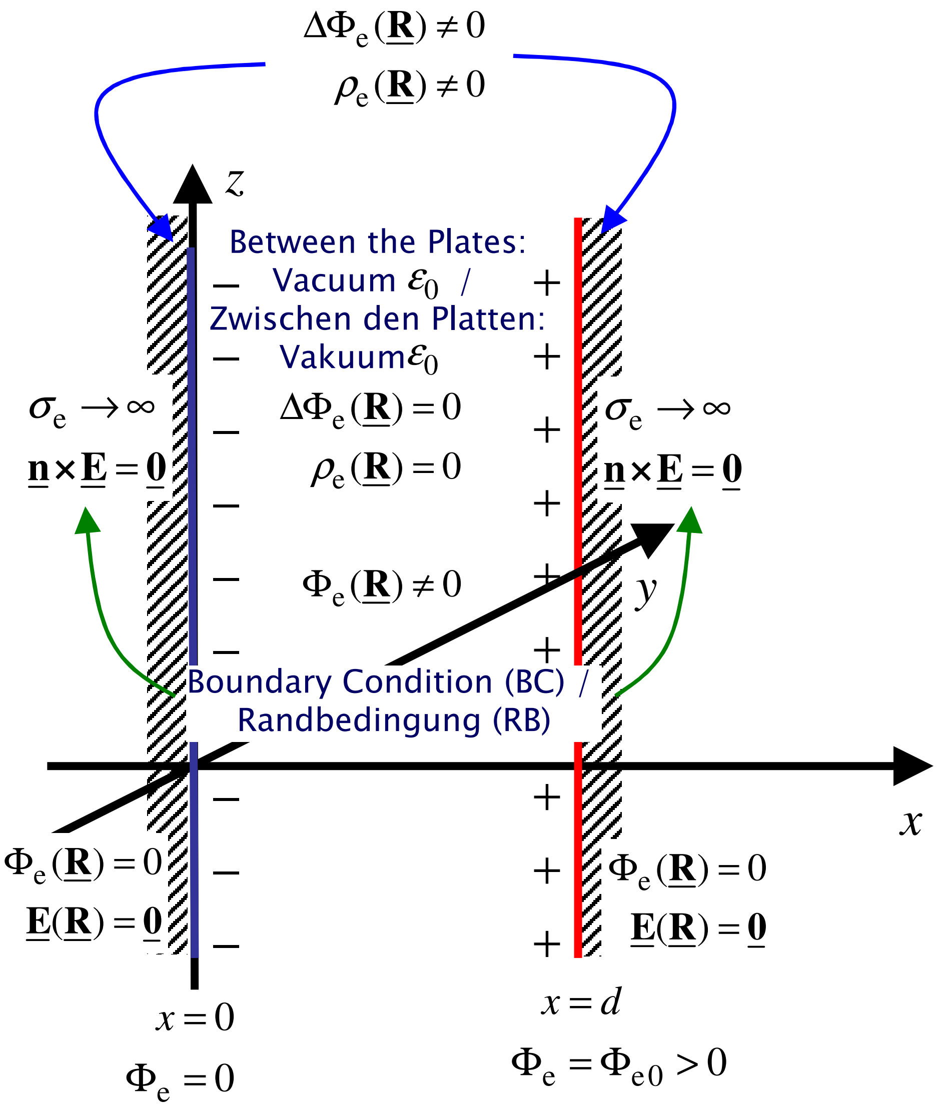
$$= f(0)$$

$$\int_{-\infty}^{\infty} \underbrace{u'(x)}_{=\delta(x)} f(x)\,dx = f(0)$$

$$u'(x) = \delta(x)$$

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

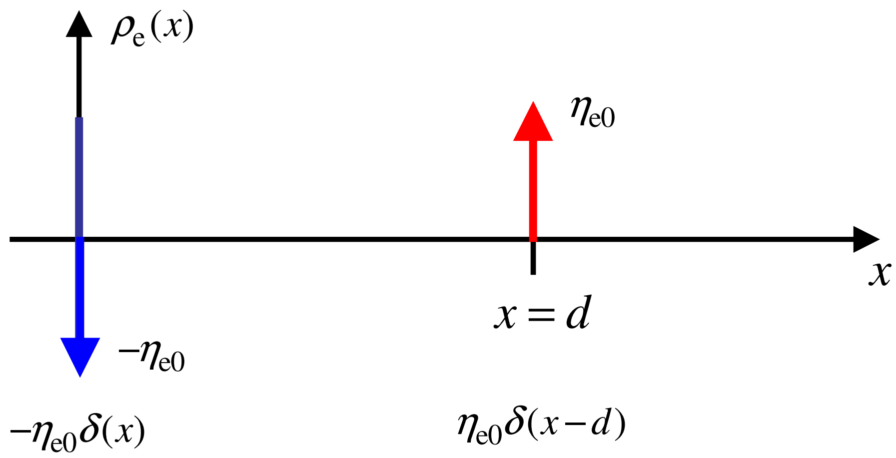
Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\rho_e(x) = \underbrace{\varepsilon_0 \frac{\Phi_{e0}}{d}}_{=\eta_{e0}} \left[-\delta(x) + \delta(x-d)\right]$$

Electric Surface Charge Density / Elektrische Flächenladungsdichte

$$= -\eta_{e0}\delta(x) + \eta_{e0}\delta(x-d)$$



End of Lecture 6 / Ende der 6. Vorlesung