Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

6th Lecture / 6. Vorlesung

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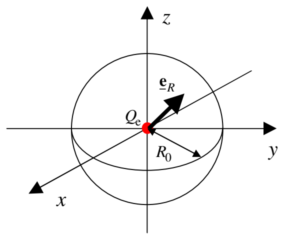
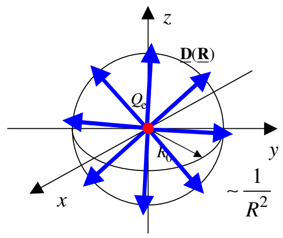
Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung



$$\oiint_{\underbrace{S=\partial V}_{=\text{Sphere } R_0}} \mathbf{D}(\mathbf{R})\cdot\mathbf{n}\,\mathrm{d}S = \iiint_V \underbrace{\rho_e(\mathbf{R})}_{=Q_e\delta(\mathbf{R})}\,\mathrm{d}V = Q_e$$

$$\mathbf{E}(\mathbf{R}) = -\nabla\Phi_e(\mathbf{R}) = -\nabla\Phi_e(R)$$
$$= -\frac{\partial}{\partial R}\Phi_e(R)\,\mathbf{e}_R(\varphi,\vartheta) = \alpha\frac{1}{R^2}\mathbf{e}_R(\varphi,\vartheta)$$

$$\mathbf{D}(\mathbf{R}) = \mathbf{D}(R) = \varepsilon_0\alpha\frac{1}{R^2}\mathbf{e}_R(\varphi,\vartheta)$$



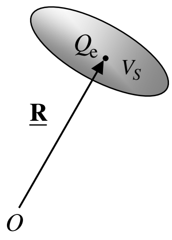
$$\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}} \varepsilon_0\alpha\frac{1}{R_0^2}\underbrace{\mathbf{e}_R(\varphi,\vartheta)\cdot\mathbf{e}_R(\varphi,\vartheta)}_{=1}\,\mathrm{d}S = \varepsilon_0\alpha\frac{1}{R_0^2}\underbrace{\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}}\mathrm{d}S}_{4\pi R_0^2}$$
$$= 4\pi\varepsilon_0\alpha$$
$$= Q_e$$

$$\Phi_e(R) = \frac{Q_e}{4\pi\varepsilon_0 R} \qquad \alpha = \frac{Q_e}{4\pi\varepsilon_0}$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Point Source / Punktquelle

$Q_e \quad [\mathrm{As/m^3} = \mathrm{Coulomb}]$



$\rho_e(\underline{\mathbf{R}}) = ?$

$= \text{infinite} \;\; / \text{ unendlich}$

$$\iiint_{V_S} \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V = Q_e$$

Mathematically Nonsense / Mathematischer Unsinn

$V_S \to 0$

Integration Theory of Riemann / Riemannsche Integralrechnung:

$$\iiint_{V_S} \rho_e(\underline{\mathbf{R}})\, \mathrm{d}V = 0$$

To Define Something New / Definiere etwas Neues

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

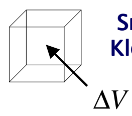
Electrostatic Charge / Elektrostatisches Ladung

$$Q_e = \iiint_V \rho_e(\underline{\mathbf{R}})\, dV$$

Electrostatic Volume Charge Density / Elektrostatisches Raumladungsdichte

$$\rho_e = \frac{\Delta Q_e}{\Delta V}$$

Small Volume / Kleines Volumen



Electrostatic Charge / Elektrostatisches Ladung

$$\Delta Q_e = \rho_e \Delta V$$

In the Limit / Grenzübergang

$\Delta V \to 0$

Constant / Konstant

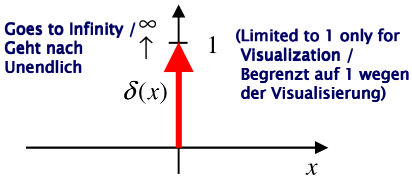
$$\Delta Q_e = \lim_{\substack{\Delta V \to 0 \\ \rho_e \to \infty}} \rho_e \Delta V$$

Point / Punkt

ΔQ_e =Constant if ΔV Goes to Zero, than the Volume Charge Density must go to Infinity. / ΔQ_e=konstant bleiben soll wenn ΔV nach null geht, dann muss die Raumladungsdichte nach unendlich gehen.

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution



Delta-Function / Delta-Funktion
δ-Distribution / δ-Distribution
δ-Dirac-Pulse / δ-Dirac-Impuls

Distribution ➨ Generalized Function / Verallgemeinerte Funktion

$$\delta(x) = \begin{cases} "\infty" & \text{for/für} \quad x = 0 \\ 0 & \text{for/für} \quad x \neq 0 \end{cases} \qquad x\,[\mathrm{m}],\ \delta(x)\left[\frac{1}{\mathrm{m}}\right]$$

The Unit of the Delta-Distribution is the Inverse Unit of the Argument / Die Einheit der Delta-Distribution ist die inverse Einheit des Argumentes

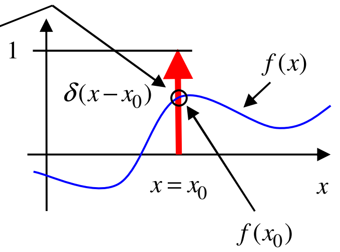
Definition of the δ -Distribution / Definition der δ -Distribution

$$\int_{x=-\infty}^{\infty} \delta(x-x_0)\,\mathrm{d}x = 1$$

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x-x_0)\,\mathrm{d}x = f(x_0)$$

$$f(x)\,\delta(x-x_0) = f(x_0)\,\delta(x-x_0)$$

Sifting Property / Siebeigenschaft



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x-x_0)\,\mathrm{d}x = f(x_0)$$

$$\langle f(x), \delta(x-x_0) \rangle = f(x_0)$$

Properties: Algebraic and Calculus Properties / Eigenschaften: Algebraische Eigenschaften und Rechenregeln

$$\alpha\,\delta(x-x_0):$$

$$\int_{x=-\infty}^{\infty} \alpha\,\delta(x-x_0)\,f(x)\,\mathrm{d}x = \alpha\,f(x_0)$$

$$\alpha(x)\,\delta(x-x_0):$$

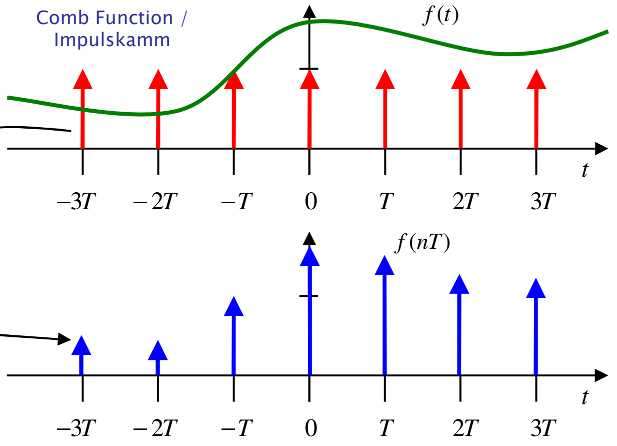
$$\int_{x=-\infty}^{\infty} \alpha(x)\,\delta(x-x_0)\,f(x)\mathrm{d}x = \alpha(x_0)\,f(x_0)$$

$$\alpha(x)\,\delta(x-x_0) = \alpha(x_0)\,\delta(x-x_0)$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution – Signal Processing – Sampling / 1D Delta-Distribution – Signalverarbeitung -Abtastung

$$f(t)\sum_{n=-N}^{N}\delta(t-nT) = \sum_{n=-N}^{N} f(t)\,\delta(t-nT) = \sum_{n=-N}^{N} f(nT)\,\delta(t-nT)$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

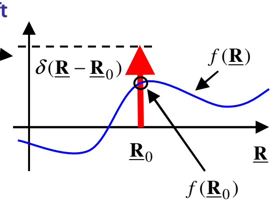
3-D Delta-Distribution / 3D Delta-Distribution

Sifting Property / Siebeigenschaft

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}} = 1$$

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} f(\underline{\mathbf{R}})\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}} = f(\underline{\mathbf{R}}_0)$$

$$f(\underline{\mathbf{R}})\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0) = f(\underline{\mathbf{R}}_0)\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)$$

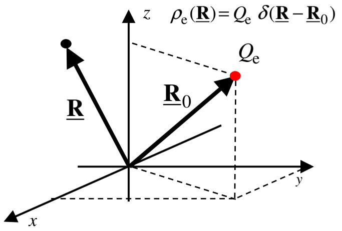


Distribution ➡ Generalized Function / Verallgemeinerte Funktion

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \rho_e(\underline{\mathbf{R}})\,\mathrm{d}^3\underline{\mathbf{R}} = \iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} Q_e\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}$$

$$= Q_e \underbrace{\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}}_{=1}$$

$$= Q_e$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)=\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$=\delta(r-r_0)\,\frac{\delta(\varphi-\varphi_0)}{r}\,\delta(z-z_0)\;=\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$=\delta(R-R_0)\,\frac{\delta(\vartheta-\vartheta_0)}{R}\,\frac{\delta(\varphi-\varphi_0)}{R\sin\vartheta}\;=\frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}$$

Spherical Coordinate System / Kugelkoordinatensystem

$$=\frac{\delta(\xi_1-\xi_{10})}{h_{\xi_1}}\,\frac{\delta(\xi_2-\xi_{20})}{h_{\xi_2}}\,\frac{\delta(\xi_3-\xi_{30})}{h_{\xi_3}}$$

General Case / Allgemeiner Fall

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\int_{z=-\infty}^{\infty}\int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty}\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$=\underbrace{\left[\int_{z=-\infty}^{\infty}\underbrace{\left[\int_{y=-\infty}^{\infty}\underbrace{\left[\int_{x=-\infty}^{\infty}\delta(x-x_0)\,\mathrm{d}x\right]}_{=1}\delta(y-y_0)\mathrm{d}y\right]}_{=1}\delta(z-z_0)\,\mathrm{d}z\right]}_{=1}=1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}\mathrm{d}^3\underline{\mathbf{R}}=\int_{z=-\infty}^{\infty}\int_{\varphi=0}^{2\pi}\int_{r=0}^{\infty}\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}\,r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$=\underbrace{\left[\int_{z=-\infty}^{\infty}\underbrace{\left[\int_{\varphi=0}^{2\pi}\underbrace{\left[\int_{r=0}^{\infty}\delta(r-r_0)\,\mathrm{d}r\right]}_{=1}\frac{\delta(\varphi-\varphi_0)}{r}\,r\mathrm{d}\varphi\right]}_{=1}\delta(z-z_0)\,\mathrm{d}z\right]}_{=1}=1$$

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty}\frac{\delta(R-R_0)\,\delta(\vartheta-\vartheta_0)\,\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}\mathrm{d}^3\underline{\mathbf{R}}=\int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi}\int_{R=0}^{\infty}\frac{\delta(R-R_0)\,\delta(\vartheta-\vartheta_0)\,\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$=\underbrace{\left[\int_{\varphi=0}^{2\pi}\underbrace{\left[\int_{\vartheta=0}^{\pi}\underbrace{\left[\int_{R=0}^{\infty}\delta(R-R_0)\,\mathrm{d}R\right]}_{=1}\frac{\delta(\vartheta-\vartheta_0)}{R}\,R\,\mathrm{d}\vartheta\right]}_{=1}\frac{\delta(\varphi-\varphi_0)}{R\sin\vartheta}\,R\,\sin\vartheta\,\mathrm{d}\varphi\right]}_{=1}=1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Point Charge Density / Elektrostatische Punktladung

$$Q_e = Q_e(x_0, y_0, z_0) \; [\mathrm{As}]$$

Electrostatic Volume Charge Density / Elektrostatische Raumladungsdichte

$$\rho_e(x, y, z) = Q_e \, \delta(x - x_0)\delta(y - y_0)\delta(z - z_0)$$

Q_e Point / Punkt

Electrostatic Line Charge Density / Elektrostatische Linienladungsdichte

$$\varsigma_e(z) = \varsigma_e(x_0, y_0, z)\,[\mathrm{As/m}]$$

Electrostatic Line Charge Density / Elektrostatische Linienladungsdichte

$$\rho_e(x, y, z) = \varsigma_e(z)\,\delta(x - x_0)\delta(y - y_0)$$

Line / Linie

$\varsigma_e(z)$ 

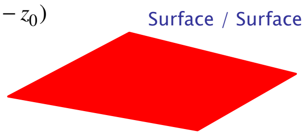
Electrostatic Surface Charge Density / Elektrostatische Flächenladungsdichte

$$\eta_e(x, y) = \eta_e(x, y, z_0)\left[\mathrm{As/m^2}\right]$$

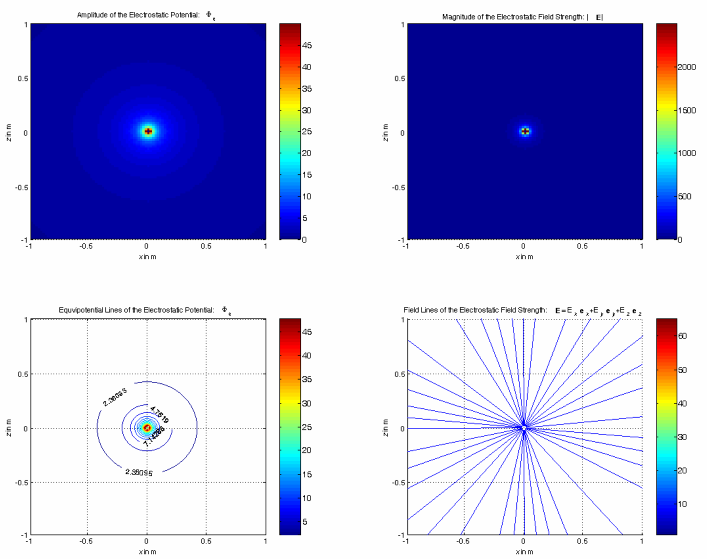
Electrostatic Charge Density / Elektrostatische Ladungsdichte

$$\rho_e(x, y, z) = \eta_e(x, y)\,\delta(z - z_0)$$

Surface / Surface

$\eta_e(x, y)$ 

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)



ES Fields – Point Charge Concept / ES Felder – Konzept der Punktladung (...)

Electrostatic Charge Density / Elektrostatische Ladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = Q_e\,\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0)$$

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q_e}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|}$$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|} = \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}$$

Electrostatic Field Strength / Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0}\frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_0}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|^3}$$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|^3} = \frac{1}{\left(\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}\right)^3} = \frac{1}{\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right]^{3/2}}$$

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral

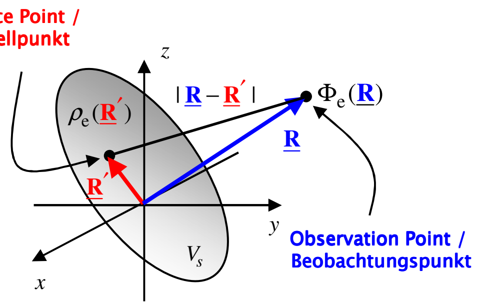
Poisson and Laplace Equation / Poisson- und Laplace-Gleichung

$$\Delta\Phi_e(\underline{\mathbf{R}}) = \begin{cases} -\dfrac{\rho_e(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für } \rho_e(\underline{\mathbf{R}}) \neq 0 \quad \text{Poisson Equation / Poisson-Gleichung} \\ 0 & \text{for / für } \rho_e(\underline{\mathbf{R}}) = 0 \quad \text{Laplace Equation / Laplace-Gleichung} \end{cases}$$

$\Delta = \nabla^2 = \nabla \bullet \nabla$: Laplace Operator / Laplace-Operator

Limited Source Volume / Begrenztes Quellvolumen

$$\rho_e(\underline{\mathbf{R}}) \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ 0 & \underline{\mathbf{R}} \not\in V_s \end{cases}$$



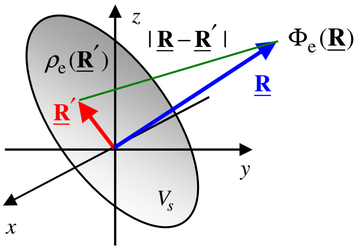
Coulomb Integral / Coulomb-Integral:

$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_s}\frac{\rho_e(\underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}\,d^3\underline{\mathbf{R}}'$$

$\rho_e(\underline{\mathbf{R}}')$: known / bekannt

$\Phi_e(\underline{\mathbf{R}})$: unknown / unbekannt

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral (...)



Coulomb Integral / Coulomb-Integral:

$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_s} \frac{\rho_e(\mathbf{R}')}{\mid \mathbf{R}-\mathbf{R}' \mid} d^3\mathbf{R}'$$

$\rho_e(\mathbf{R}')$: known / bekannt

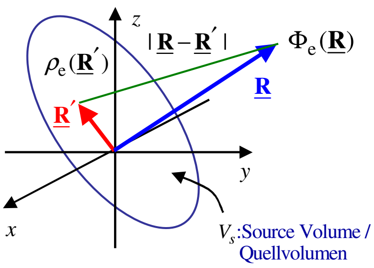
$\Phi_e(\mathbf{R})$: unknown / unbekannt

$$\Delta\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0}\Delta\iiint_{V_s} \frac{1}{\mid \mathbf{R}-\mathbf{R}' \mid}\rho_e(\mathbf{R}')d^3\mathbf{R}'$$

$$= \frac{1}{4\pi\varepsilon_0}\iiint_{V_s} \underbrace{\left[\Delta\frac{1}{\mid \mathbf{R}-\mathbf{R}' \mid}\right]}_{=-4\pi\delta(\mathbf{R}-\mathbf{R}')}\rho_e(\mathbf{R}')d^3\mathbf{R}' \quad \text{with} \quad \Delta\frac{1}{4\pi\left|\mathbf{R}-\mathbf{R}'\right|} = -\delta\left(\mathbf{R}-\mathbf{R}'\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}\underbrace{\iiint_{V_s} 4\pi\delta(\mathbf{R}-\mathbf{R}')\rho_e(\mathbf{R}')d^3\mathbf{R}'}_{=\rho_e(\mathbf{R})} = -\frac{1}{\varepsilon_0}\rho_e(\mathbf{R})$$

ES Fields – Green's Function / ES Felder – Greensche Funktion



$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_s} \frac{\rho_e(\mathbf{R}')}{\mid \mathbf{R}-\mathbf{R}' \mid} d^3\mathbf{R}'$$

$$= \frac{1}{\varepsilon_0}\iiint_{V_s} \underbrace{\frac{1}{4\pi}\frac{1}{\mid \mathbf{R}-\mathbf{R}' \mid}}_{=G_e^{ES}(\mathbf{R}-\mathbf{R}')}\rho_e(\mathbf{R}')d^3\mathbf{R}'$$

Electrostatic Green's Function / Elektrostatische Greensche Funktion

$$G_e^{ES}(\mathbf{R}-\mathbf{R}') = \frac{1}{4\pi}\frac{1}{\mid \mathbf{R}-\mathbf{R}' \mid} \qquad \text{for / für } \mathbf{R} \neq \mathbf{R}'$$

with $\Delta G_e^{ES}(\mathbf{R}-\mathbf{R}') = -\delta\left(\mathbf{R}-\mathbf{R}'\right)$

Normalized Potential of a Point Charge / Normiertes Potential einer Punktladung

Electrostatic Potential of an Electrostatic Point Charge / Elektrostatisches Potential einer elektrostatischen Punktladung

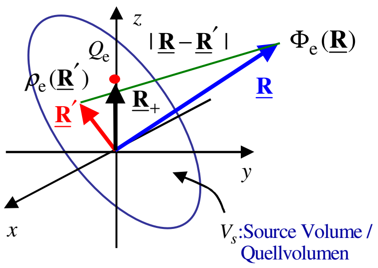
$$\Phi_e(\mathbf{R}) = \frac{Q_e}{4\pi}\frac{1}{\mid \mathbf{R}-\mathbf{R}_+ \mid} \qquad \text{for / für} \quad \rho_e(\mathbf{R}) = Q_e\delta(\mathbf{R}-\mathbf{R}_+)$$

ES Fields – Potential of a Point Charge / ES Felder – Potential einer Punktladung

Electrostatic Volume Charge Density / Elektrostatisches Raumladungsdichte

$$\rho_e(\underline{\mathbf{R}}) = Q_e \delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_+)$$

with / mit $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$ $\quad \underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$



$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{Q_e \delta(\underline{\mathbf{R}}' - \underline{\mathbf{R}}_+)}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} d^3\underline{\mathbf{R}}'$$

$$= \frac{Q_e}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\delta(\underline{\mathbf{R}}' - \underline{\mathbf{R}}_+)}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|} d^3\underline{\mathbf{R}}'$$

$$= \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of an Electrostatic Point Charge / Feld einer elektrostatischen Punktladung

Electrostatic Potential of a Point Charge / Elektrostatisches Potential einer Punktladung

$$\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|}$$

with $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$\underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|} = \frac{1}{\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}} = \frac{1}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2\right]^{1/2}}$$

Electrostatic Field Strength of a Point Charge / Elektrostatische Feldstärke einer Punktladung

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3}$$

with $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

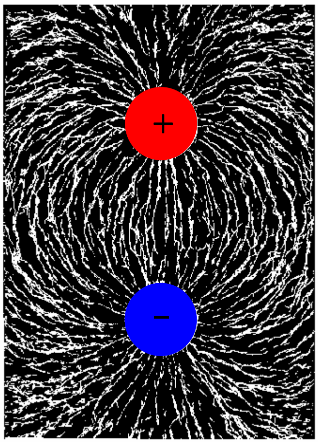
$\underline{\mathbf{R}}_+ = x_+\underline{\mathbf{e}}_x + y_+\underline{\mathbf{e}}_y + z_+\underline{\mathbf{e}}_z$

$$\frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_+}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_+|^3} = \frac{(x-x_+)\underline{\mathbf{e}}_x + (y-y_+)\underline{\mathbf{e}}_y + (z-z_+)\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}\right]^3} = \frac{(x-x_+)\underline{\mathbf{e}}_x + (y-y_+)\underline{\mathbf{e}}_y + (z-z_+)\underline{\mathbf{e}}_z}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2\right]^{3/2}}$$

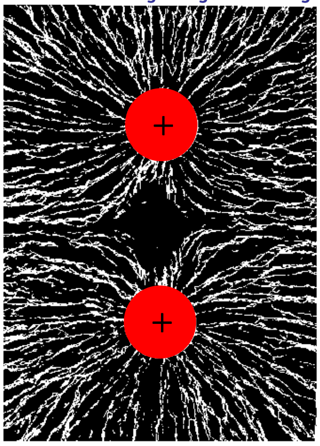
Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischen Dipol

Field Lines of the Electric Field Strength of Two Spheres Carrying Charges of Opposite Sign / Feldlinien der elektrischen Feldstärke zweier ungleich geladener Kugeln



Electric Field Lines of Two Spheres Carrying Charges of the Same Sign / Feldlinien der elektrischen Feldstärke zweier gleich geladener Kugeln



Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischen Dipol

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_{e+}}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|} + \frac{Q_{e-}}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_-|}\right)$$

with/mit $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$\underline{\mathbf{R}}_\pm = x_\pm\underline{\mathbf{e}}_x + y_\pm\underline{\mathbf{e}}_y + z_\pm\underline{\mathbf{e}}_z$

$$\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm|} = \frac{1}{\sqrt{(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2}} = \frac{1}{\left[(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2\right]^{1/2}}$$

Electrostatic Field Strength / Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_e(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\left(Q_{e+}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_+}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|^3} + Q_{e-}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_-}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_-|^3}\right)$$

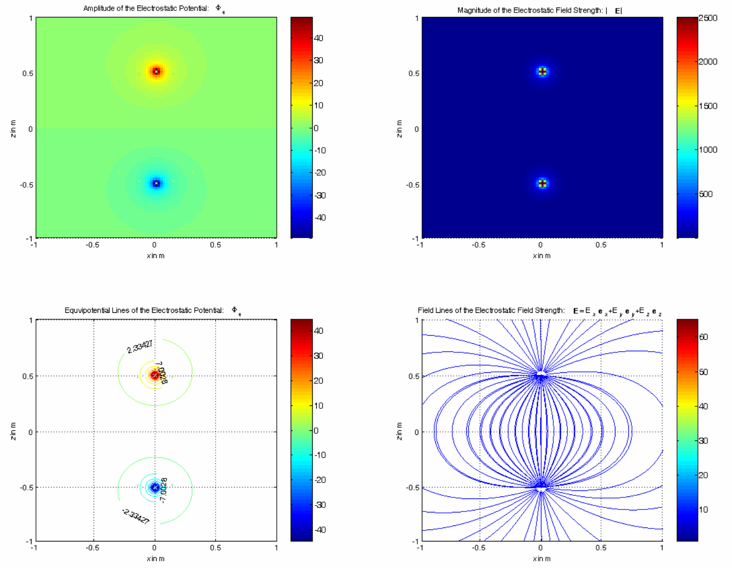
with/mit $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$\underline{\mathbf{R}}_\pm = x_\pm\underline{\mathbf{e}}_x + y_\pm\underline{\mathbf{e}}_y + z_\pm\underline{\mathbf{e}}_z$

$$\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm|^3} = \frac{(x-x_\pm)\underline{\mathbf{e}}_x+(y-y_\pm)\underline{\mathbf{e}}_y+(z-z_\pm)\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2}\right]^3} = \frac{(x-x_\pm)\underline{\mathbf{e}}_x+(y-y_\pm)\underline{\mathbf{e}}_y+(z-z_\pm)\underline{\mathbf{e}}_z}{\left[(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2\right]^{3/2}}$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

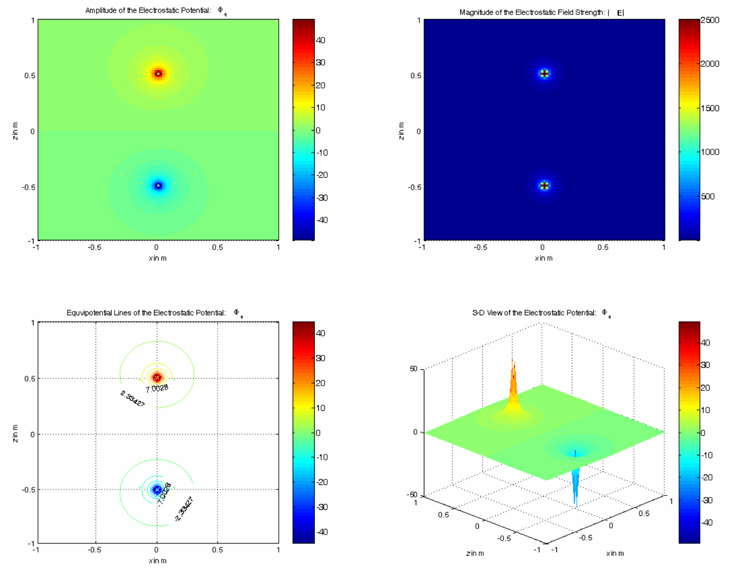
Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischer Dipol



Dr.-Ing

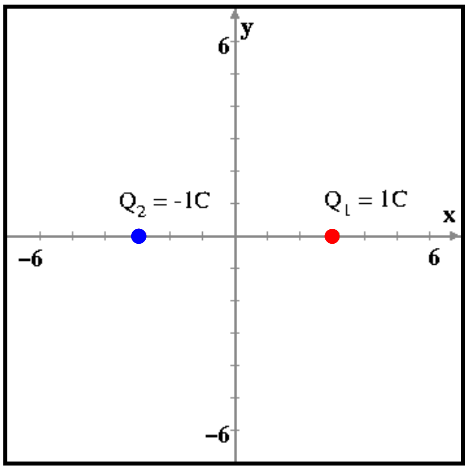
Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Field of Two Electrostatic Point Charges – Electrostatic Dipole / Feld von zwei elektrostatischen Punktladungen – Elektrostatischer Dipol



Dr.-Ing

Electrostatic Field Due To Two Point Charges / Elektrostatische Feld von zwei Punktladungen



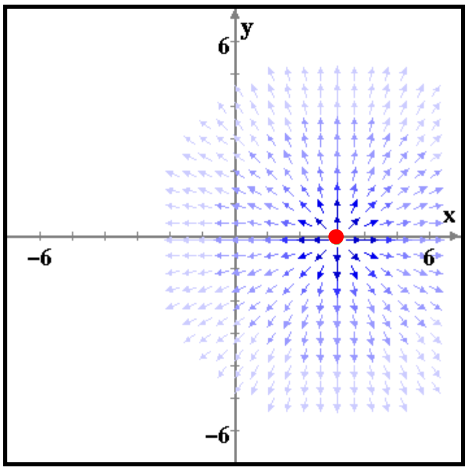
Q_1 = 1 As located at $P(x,y,z)$ = (3,0,0)
and
Q_2 = −1 As located at $P(x,y,z)$ = (−3,0,0)

In this demo, arrows are used to sketch the electric field pattern in the *xy* plane due to:

Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone
Press	Q_1 and Q_2 / Q_1 and Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Electrostatic Field... / Elektrostatische Feld... Q_1 alone / Q_1 alleine



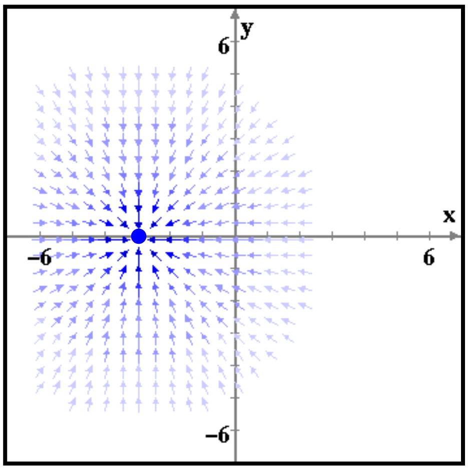
Q_1 = 1 As located at $P(x,y,z)$ = (3,0,0)
and
Q_2 = −1 As located at $P(x,y,z)$ = (−3,0,0)

In this demo, arrows are used to sketch the electric field pattern in the *xy* plane due to:

Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

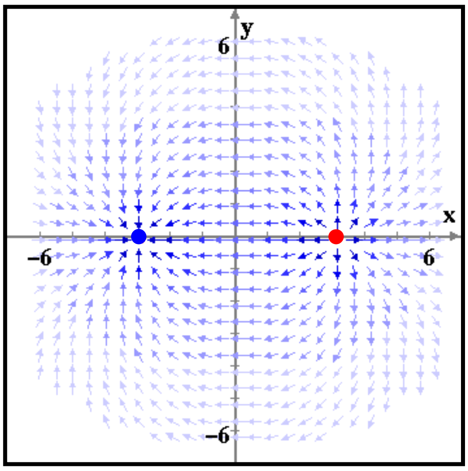
Electrostatic Field… / Elektrostatische Feld…
Q_2 alone / Q_2 alleine



Q_1 = 1 As located at $P(x,y,z)$ = (3,0,0)
and
Q_2 = –1 As located at $P(x,y,z)$ = (–3,0,0)

	In this demo, arrows are used to sketch the electric field pattern in the xy plane due to:
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
	Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

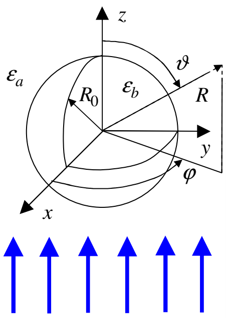
Electrostatic Field… / Elektrostatische Feld…
Q_1 and Q_2 / Q_1 und Q_2



Q_1 = 1 As located at $P(x,y,z)$ = (3,0,0)
and
Q_2 = –1 As located at $P(x,y,z)$ = (–3,0,0)

	In this demo, arrows are used to sketch the electric field pattern in the xy plane due to:
Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2
	Note: Color Intensity is Proportional to the Magnitude of the Electric Field Strength. / Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

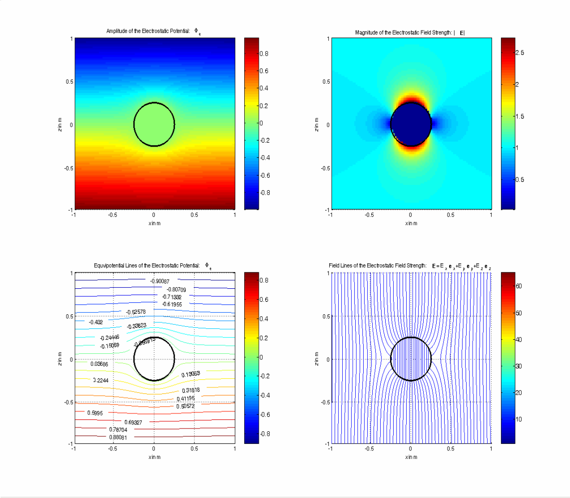
Transition Conditions = ? / Übergangsbedingungen = ?



$$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$$

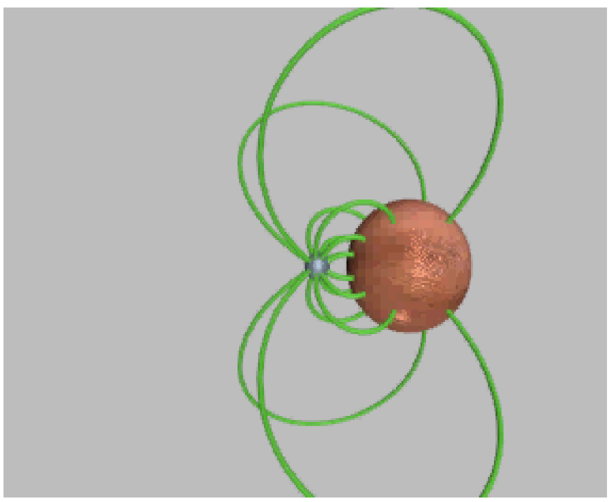
$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \le R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

$$\varepsilon_a = \varepsilon_0$$
$$\varepsilon_b = 100\varepsilon_0$$



Boundary Conditions = ? / Randbedingungen = ?

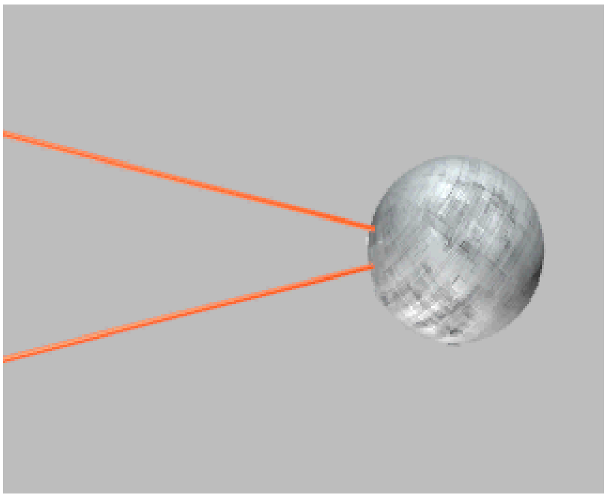
Point Charge Attracted to a Electrically Charged Sphere / Punktladung angezogen von einer elektrisch geladenen Kugel



http://web.mit.edu/jbelcher/www/att.html

Boundary Conditions = ? / Randbedingungen = ?

Point Charge Repulsed By A Charged Sphere / Punktladung abgestoßen von einer elektrisch geladenen Kugel



http://web.mit.edu/jbelcher/www/att.html

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Transition Conditions / Übergangsbedingungen

Medium (2) / Medium (1), $\underline{\mathbf{n}}$, $S_{\mathrm{I(nterface)}}$

For / Für $\underline{\mathbf{R}} \in S_{\mathrm{I}}$

$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t) \right] = \underline{\mathbf{0}}$$

$$\underline{\mathbf{n}} \bullet \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t) \right] = \begin{cases} \eta_e(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

ws: with sources; sf = source-free / mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen

Medium, $\underline{\mathbf{n}}$, $S_{\mathrm{B(oundary)}}$

For / Für $\underline{\mathbf{R}} \in S_{\mathrm{B}}$

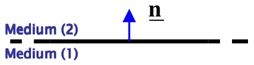
$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}} \qquad \text{pec / iel}$$

$$\underline{\mathbf{n}} \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \eta_e(\underline{\mathbf{R}},t) \qquad \text{pec / iel}$$

pec = perfectly electric conducting; pmc = perfectly magnetic conducting / iel = ideal elektrisch leitend; iml = ideal magnetisch leitend

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$

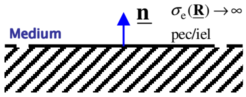
$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}}) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}}) \right] = \underline{\mathbf{0}}$$

$$\underline{\mathbf{n}} \cdot \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}}) - \underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}}) \right] = \eta_e(\underline{\mathbf{R}})$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen

$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$



$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}} \qquad \text{pec / iel}$$

$$\underline{\mathbf{n}} \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

pec = perfectly electric conducting /
iel = ideal elektrisch leitend

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{E}}_{tan}(\underline{\mathbf{R}}) = E_{tan}(\underline{\mathbf{R}})\underline{\mathbf{e}}_{tan}$$

$$\underline{\mathbf{n}} \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = D_n(\underline{\mathbf{R}})$$

$\underline{\mathbf{E}}_{tan}(\underline{\mathbf{R}})$: Vector Tangential Component of / Vektorielle Tangentialkomponente von $\underline{\mathbf{E}}(\underline{\mathbf{R}})$

$E_{tan}(\underline{\mathbf{R}})$: Scalar Tangential Component of / Skalare Tangentialkomponente von $\underline{\mathbf{E}}(\underline{\mathbf{R}})$

$D_n(\underline{\mathbf{R}})$: Scalar Normal Component of / Skalare Normalkomponente von $\underline{\mathbf{D}}(\underline{\mathbf{R}})$

$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

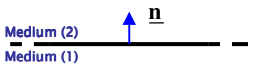
$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}})$$

$$E_{tan}(\underline{\mathbf{R}}) = 0 \qquad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$

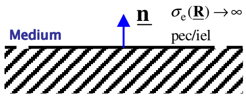
$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}})$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen

$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$



$$E_{tan}(\underline{\mathbf{R}}) = 0 \qquad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_e(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

pec = perfectly electric conducting /
iel = ideal elektrisch leitend

$$\underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}}) = -\nabla\Phi^{(i)}(\underline{\mathbf{R}}) \qquad \underline{\mathbf{D}}^{(i)}(\underline{\mathbf{R}}) = \varepsilon_0 \varepsilon_r^{(i)} \underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}}) \qquad i = 1,2$$

$$= -\varepsilon_0 \varepsilon_r^{(i)} \nabla \Phi_e^{(i)}(\underline{\mathbf{R}})$$

$$\underline{\mathbf{n}} \times \nabla \left[\Phi_e^{(2)}(\underline{\mathbf{R}}) - \Phi_e^{(1)}(\underline{\mathbf{R}}) \right] = \underline{\mathbf{0}}$$

$$\Phi_e^{(2)}(\underline{\mathbf{R}}) - \Phi_e^{(1)}(\underline{\mathbf{R}}) = \Phi_e$$

$$\underline{\mathbf{n}} \times \nabla \Phi_e(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\Phi_e(\underline{\mathbf{R}}) = 0$$

$$\underbrace{\underline{\mathbf{n}} \cdot \nabla}_{=\frac{\partial}{\partial n}} \left[\varepsilon_r^{(2)} \Phi_e^{(2)}(\underline{\mathbf{R}}) - \varepsilon_r^{(1)} \Phi_e^{(1)}(\underline{\mathbf{R}}) \right] = -\frac{\eta_e(\underline{\mathbf{R}})}{\varepsilon_0}$$

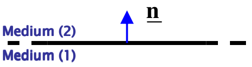
$$\varepsilon_0 \varepsilon_r \underbrace{\underline{\mathbf{n}} \cdot \nabla}_{=\frac{\partial}{\partial n}} \Phi_e(\underline{\mathbf{R}}) = -\eta_e(\underline{\mathbf{R}})$$

$$\frac{\partial}{\partial n} \Phi_e^{(2)}(\underline{\mathbf{R}}) - \frac{\varepsilon_r^{(1)}}{\varepsilon_r^{(2)}} \frac{\partial}{\partial n} \Phi_e^{(1)}(\underline{\mathbf{R}}) = -\frac{\eta_e(\underline{\mathbf{R}})}{\varepsilon_0 \varepsilon_r^{(2)}}$$

$$\frac{\partial}{\partial n} \Phi_e(\underline{\mathbf{R}}) = -\frac{\eta_e(\underline{\mathbf{R}})}{\varepsilon_0 \varepsilon_r}$$

ES Fields: Transition and Boundary Conditions / ES-Felder: Übergangs- und Randbedingungen

Transition Conditions / Übergangsbedingungen



$\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$

$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$E_{tan}^{(2)}(\underline{\mathbf{R}}) - E_{tan}^{(1)}(\underline{\mathbf{R}}) = 0$$

$$\Downarrow$$

$$\Phi_{\mathrm{e}}^{(2)}(\underline{\mathbf{R}}) - \Phi_{\mathrm{e}}^{(1)}(\underline{\mathbf{R}}) = \Phi_{\mathrm{e}0} = \mathrm{const.}$$

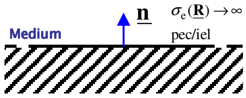
$$D_n^{(2)}(\underline{\mathbf{R}}) - D_n^{(1)}(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\Downarrow$$

$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}^{(2)}(\underline{\mathbf{R}}) - \frac{\varepsilon_{\mathrm{r}}^{(1)}}{\varepsilon_{\mathrm{r}}^{(2)}}\frac{\partial}{\partial n}\Phi_{\mathrm{e}}^{(1)}(\underline{\mathbf{R}}) = -\frac{1}{\varepsilon_0\varepsilon_{\mathrm{r}}^{(2)}}\eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

Boundary Conditions / Randbedingungen

$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$



$$E_{tan}(\underline{\mathbf{R}}) = 0 \qquad \text{pec / iel}$$

$$D_n(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}}) \qquad \text{pec / iel}$$

$$E_{tan}(\underline{\mathbf{R}}) = 0$$

$$\Downarrow$$

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \Phi_{\mathrm{e}0} = \mathrm{const.}$$

$$D_n(\underline{\mathbf{R}}) = \eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\Downarrow$$

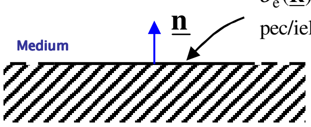
$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = -\frac{1}{\varepsilon_0\varepsilon_{\mathrm{r}}}\eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Boundary Conditions / Randbedingungen

$\underline{\mathbf{E}}(\underline{\mathbf{R}}), \underline{\mathbf{D}}(\underline{\mathbf{R}}), \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$

$\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$



$\sigma_{\mathrm{e}}(\underline{\mathbf{R}}) \to \infty$ pec/iel

$\eta_{\mathrm{e}}(\underline{\mathbf{R}})$

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \Phi_{\mathrm{e}0} = \mathrm{const.} \quad (\Phi_{\mathrm{e}0} = 0\ \mathrm{V})$$

$$\frac{\partial}{\partial n}\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = -\frac{1}{\varepsilon_0\varepsilon_{\mathrm{r}}}\eta_{\mathrm{e}}(\underline{\mathbf{R}})$$

Neumann Boundary Conditions for Φ_{e} / Neumann-Randbedingung für Φ_{e}

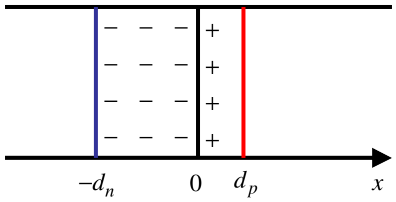
Dirichlet Boundary Conditions for Φ_{e} / Dirichlet-Randbedingung für Φ_{e}

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (3)

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

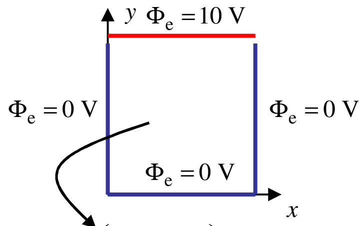
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_e(x,y,z)=\begin{cases}-\dfrac{\rho_e(x,y,z)}{\varepsilon_0} & \text{for / für } \rho_e(x,y,z)\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für } \rho_e(x,y,z)=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Example: pn Junction – pn Diode / Beispiel: pn-Übergang – pn Diode



$$\frac{d^2}{dx^2}\Phi_e(x)=\frac{e}{\varepsilon}\begin{cases}-n_e & \text{for / für } -d_n\le x\le 0\\ n_e & \text{for / für } 0\le x\le d_p\end{cases}$$

Example: / Beispiel:



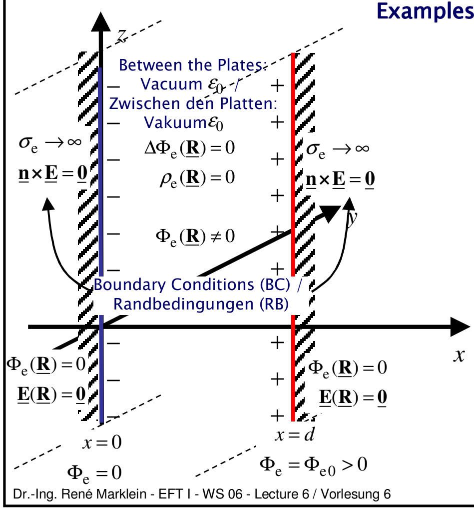
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_e(x,y)=0$$

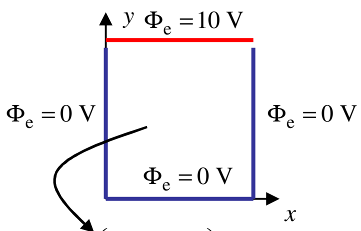
Separation of Variables / Separation der Variablen !

Electrostatic (ES) Fields – Boundary Value Problem (BVP) / Elektrostatische (ES) Felder – Randwertproblem (RWP)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_e(x,y,z)=\begin{cases}-\dfrac{\rho_e(x,y,z)}{\varepsilon_0} & \text{for / für } \rho_e(x,y,z)\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für } \rho_e(x,y,z)=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Examples: / Beispiele:



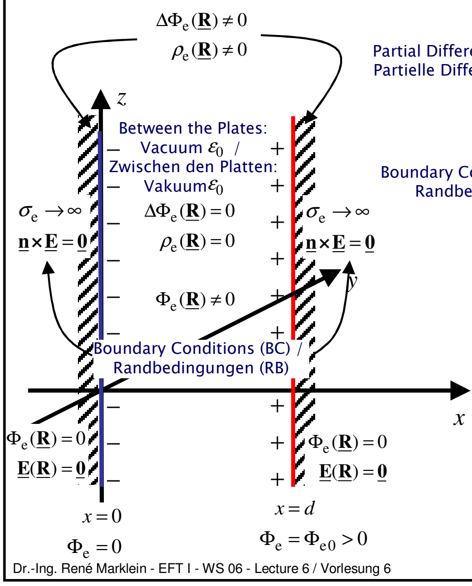


$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_e(x,y)=0$$

Separation of Variables / Separation der Variablen !

ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Partial Differential Equation / Partielle Differentialgleichung

$$\Delta\Phi_e(\underline{\mathbf{R}}) \begin{cases} = 0 & \text{for / für} \quad 0 < x < d \\ \neq \text{const.} & \text{for / für} \quad x = 0, \; x = d \end{cases}$$

Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \quad \Phi_e = 0$$
$$x = d: \quad \Phi_e = \Phi_{e0} > 0$$

Between the Plates Laplace Equation: Zwischen den Platten: Laplace-Gleichung

$$\Delta\Phi_e(\underline{\mathbf{R}}) = 0$$

... Cartesian Coordinates / ... Kartesische Koordinaten

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi_e(x, y, z) = 0$$

... Because of the Symmetry / ... wegen der Symmetrie

$$\frac{d^2}{dx^2}\Phi_e(x) = 0$$

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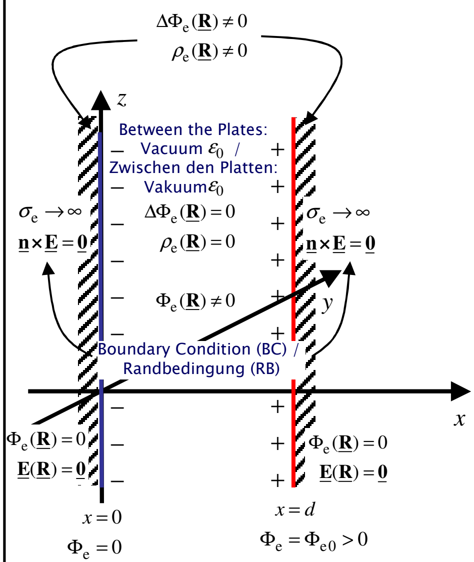
Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\frac{d^2}{dx^2}\Phi_e(x) = 0 \qquad 0 < x < d$$

Integrating once / Integriere einmal

$$\int \frac{d^2}{dx^2}\Phi_e(x)\,dx = \left[\frac{d}{dx}\Phi_e(x)\right] = \text{const} = a$$

$$\left[\frac{d}{dx}\Phi_e(x)\right] = \text{const} = a$$

Integrating twice / Zweifache Integration ergibt

$$\int \left\{\left[\frac{d}{dx}\Phi_e(x)\right] = \text{const} = a\right\} dx = \Phi_e(x) = ax + b$$

$$\Phi_e(x) = ax + b$$

$$\Rightarrow \quad \Phi_e(x) = ax + b \qquad 0 < x < d$$

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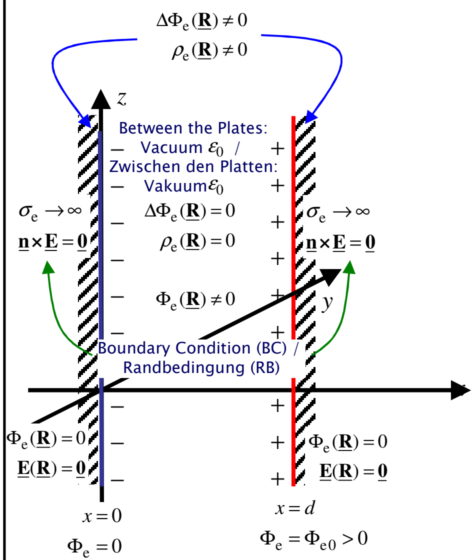
Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$\Rightarrow \quad \Phi_e(x) = ax + b$

Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \quad \Phi_e = 0$$
$$x = d: \quad \Phi_e = \Phi_{e0} > 0$$

$$\Phi_e(x=0) = a(x=0) + b = 0$$
$$b = 0$$
$$\Phi_e(x) = ax$$
$$\Phi_e(x=b) = a(x=b) = \Phi_{e0}$$
$$a = \frac{\Phi_{e0}}{d}$$

Solution for the Electrostatic Potential / Lösung für das elektrostatische Potential

$$\Rightarrow \quad \Phi_e(x) = \frac{\Phi_{e0}}{d} x \qquad 0 \le x \le d$$

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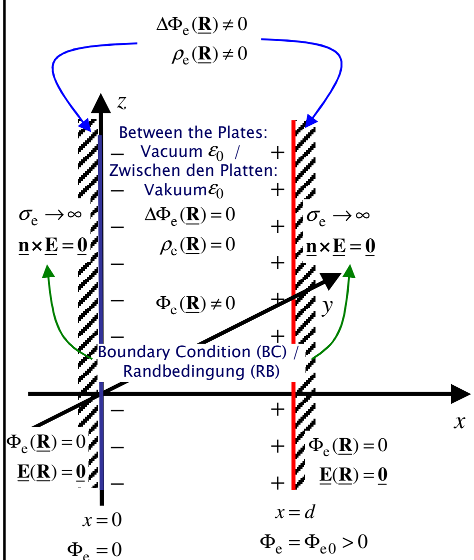
ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Partial Differential Equation (PDE) / Partielle Differentialgleichung (DGL)

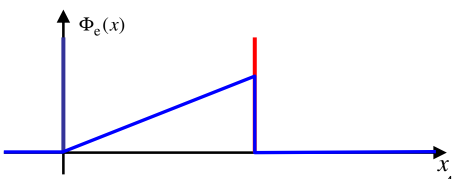
$$\frac{d^2}{dx^2}\Phi_e(x) = 0 \qquad 0 < x < d$$

Boundary Conditions (BC) / Randbedingungen (RB)

$$x = 0: \quad \Phi_e = 0$$
$$x = d: \quad \Phi_e = \Phi_{e0} > 0$$

Solution for the Electrostatic Potential / Lösung für das elektrostatische Potential

$$\Phi_e(x) = \begin{cases} \frac{\Phi_{e0}}{d} x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$



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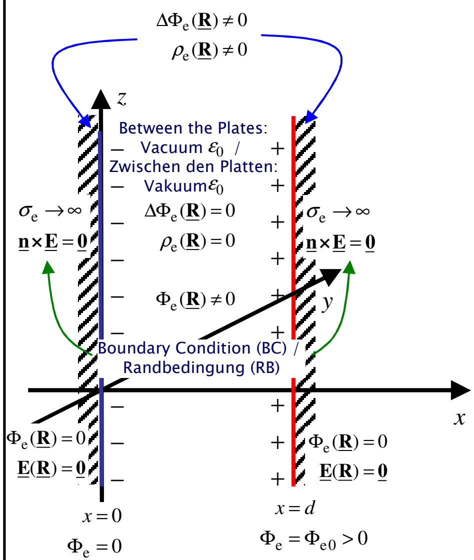
ES Fields – Electrostatic Field Between Two Parallel PEC Plates / ES Felder – Elektrostatisches Feld zwischen zwei parallelen IEL Platten

Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



Electrostatic Potential / Elektrostatisches Potential

$$\Phi_e(x) = \begin{cases} \dfrac{\Phi_{e0}}{d} x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_e(\underline{\mathbf{R}})$$

$$\underline{\mathbf{E}}(x) = -\frac{\mathrm{d}}{\mathrm{d}x} \Phi_e(x) \underline{\mathbf{e}}_x \qquad -\infty < x < \infty$$

$$= \begin{cases} -\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\Phi_{e0}}{d} x\right) \underline{\mathbf{e}}_x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

$$= \begin{cases} -\dfrac{\Phi_{e0}}{d} \underline{\mathbf{e}}_x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

⇨ The Electrostatic Potential and Electrostatic Field Strenth are Discontinuous at the Plates / Das elektrostatisches Potential und die elektrostatische Feldstärke sind unstetig an den Platten

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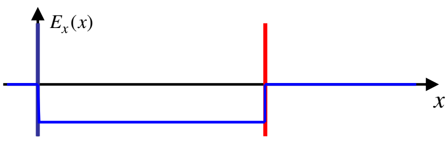
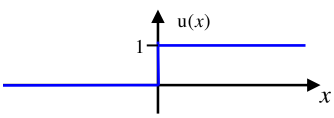
Boundary Value Problem (BVP) – Electrostatic Poisson Equation / Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung

$$\underline{\mathbf{E}}(x) = \begin{cases} -\dfrac{\Phi_{e0}}{d} \underline{\mathbf{e}}_x & 0 \le x \le d \\ 0 & \text{else / sonst} \end{cases}$$

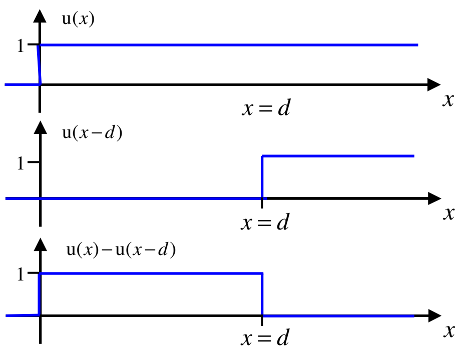


Step Functions / Einheitssprungfunktionen

$$\mathrm{u}(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$



Representation of the Electrostatic Field Strenth using the Unit Step Functions: / Darstellung der elektrostatischen Feldstärke durch Einheitssprungfunktionen:



$$\underline{\mathbf{E}}(x) = -\frac{\Phi_{e0}}{d}\left[\mathrm{u}(x) - \mathrm{u}(x-d)\right]\underline{\mathbf{e}}_x \qquad -\infty < x < \infty$$

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Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung

$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_e(\underline{\mathbf{R}})$$

$$\frac{\mathrm{d}}{\mathrm{d}x} D_x(x) = \rho_e(x)$$

$$\varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}x} E_x(x) = \rho_e(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} E_x(x) = \frac{\rho_e(x)}{\varepsilon_0}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} E_x(x) = -\frac{\Phi_{e0}}{d}\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{u}(x) - \mathrm{u}(x-d)\right]$$

$$= -\frac{\Phi_{e0}}{d}\left[\underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{u}(x)}_{\delta(x)} - \underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{u}(x-d)}_{\delta(x-d)}\right]$$

$$= -\frac{\Phi_{e0}}{d}\left[\delta(x) - \delta(x-d)\right]$$

$$= \frac{\rho_e(x)}{\varepsilon_0}$$

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{u}(x)}_{=\mathrm{u}'(x)} = \delta(x)$$

$$\int_{-\infty}^{\infty} \mathrm{u}'(x) f(x)\,\mathrm{d}x = \mathrm{u}(x) f(x)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \mathrm{u}(x) f'(x)\,\mathrm{d}x$$

$$= \left[\underbrace{\underbrace{\mathrm{u}(\infty)}_{=1} f(\infty)}_{=f(\infty)} - \underbrace{\underbrace{\mathrm{u}(-\infty)}_{0} f(-\infty)}_{=0}\right] - \int_0^{\infty} f'(x)\,\mathrm{d}x$$

$$= f(\infty) - f(x)\Big|_0^{\infty}$$

$$= f(\infty) - \left[f(\infty) - f(0)\right]$$

$$= f(0)$$

$$\int_{-\infty}^{\infty} \underbrace{\mathrm{u}'(x)}_{=\delta(x)} f(x)\,\mathrm{d}x = f(0)$$

$$\mathrm{u}'(x) = \delta(x)$$

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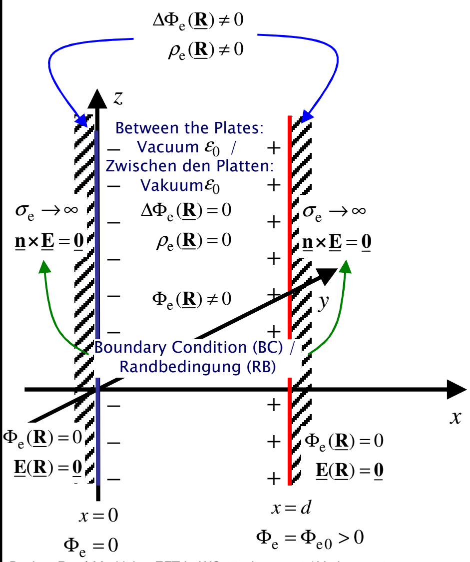
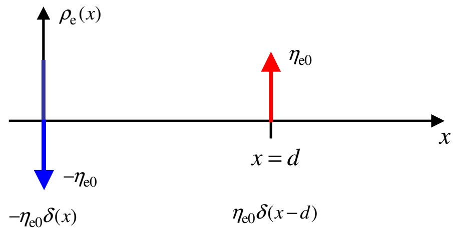
Boundary Value Problem (BVP) – Electrostatic Poisson Equation /
Randwertproblem (RWP) – Elektrostatische Poisson-Gleichung



$$\rho_e(x) = \underbrace{\varepsilon_0 \frac{\Phi_{e0}}{d}}_{=\eta_{e0}}\left[-\delta(x) + \delta(x-d)\right]$$

Electric Surface Charge Density /
Elektrische Flächenladungsdichte

$$= -\eta_{e0}\delta(x) + \eta_{e0}\delta(x-d)$$



End of Lecture 6 /
Ende der 6. Vorlesung