Numerical Methods of Electromagnetic Field Theory I (NFT I) Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

5th Lecture / 5. Vorlesung

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3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

Maxwell's equations / Maxwellsche Gleichungen

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum / Konstituierende Gleichungen (Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

Continuity equations / Kontinuitätsgleichungen

$$\nabla\bullet\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

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$$\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \tag{1}$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \tag{2}$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \tag{3}$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \tag{4}$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\left[\frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\right] - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \tag{5}$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\left[-\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\right] - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \tag{6}$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \tag{7}$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \tag{8}$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)+\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=+\frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)-\frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (1)$$

$$\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)+\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=-\frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)-\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (2)$$

$$c_0=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$-\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (3)$$

$$-\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (4)$$

Vector identity / Vektoridentität

$$\nabla\times\nabla\times=\nabla\nabla\bullet-\underbrace{\nabla\bullet\nabla}_{=\nabla^2=\Delta}=\nabla\nabla\bullet-\Delta$$

Short-hand notation / Abkürzende Schreibweise

$$\nabla\bullet\nabla=\nabla^2=\Delta$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \quad (5)$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \quad (6)$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

3rd and 4th Maxwell's equations / 3. und 4. Maxwellsche Gleichung

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Constitutive equations / Materialgleichungen

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$\Rightarrow$

$$\nabla\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\frac{1}{\mu_0}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\frac{1}{\varepsilon_0}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\nabla\underbrace{\nabla\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)}_{=\frac{1}{\mu_0}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)}-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\nabla\underbrace{\nabla\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)}_{=\frac{1}{\varepsilon_0}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)}-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\nabla\left[\frac{1}{\mu_0}\rho_\mathrm{m}(\underline{\mathbf{R}},t)\right]-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\nabla\left[\frac{1}{\varepsilon_0}\rho_\mathrm{e}(\underline{\mathbf{R}},t)\right]-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

Laplace operator in Cartesian coordinates / Laplace-Operator in Kartesischen Koordinaten

$$\Delta=\nabla\bullet\nabla$$

$$=\left(\underline{\mathrm{e}}_x\frac{\partial}{\partial x}+\underline{\mathrm{e}}_y\frac{\partial}{\partial y}+\underline{\mathrm{e}}_z\frac{\partial}{\partial z}\right)\bullet\left(\underline{\mathrm{e}}_x\frac{\partial}{\partial x}+\underline{\mathrm{e}}_y\frac{\partial}{\partial y}+\underline{\mathrm{e}}_z\frac{\partial}{\partial z}\right)$$

$$=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left(\underline{e}_x \frac{\partial}{\partial x} + \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z}\right) \bullet \left[\left(\underline{e}_x \frac{\partial}{\partial x} + \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

Short-hand notation / Abkürzende Schreibweise

$$\frac{\partial}{\partial x} = \partial_x \qquad \frac{\partial}{\partial y} = \partial_y \qquad \frac{\partial}{\partial z} = \partial_z \qquad \frac{\partial}{\partial t} = \partial_t$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left(\underline{e}_x\partial_x + \underline{e}_y\partial_y + \underline{e}_z\partial_z\right) \bullet \left[\left(\underline{e}_x\partial_x + \underline{e}_y\partial_y + \underline{e}_z\partial_z\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$\left(\underline{e}_x\partial_x + \underline{e}_y\partial_y + \underline{e}_z\partial_z\right)\left[E_x(\underline{\mathbf{R}},t)\underline{e}_x + E_y(\underline{\mathbf{R}},t)\underline{e}_y + E_z(\underline{\mathbf{R}},t)\underline{e}_z\right]$$

$$= \underline{e}_x\underline{e}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{e}_x\underline{e}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{e}_x\underline{e}_z\partial_x E_z(\underline{\mathbf{R}},t)$$

$$+ \underline{e}_y\underline{e}_x\partial_y E_y(\underline{\mathbf{R}},t) + \underline{e}_y\underline{e}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{e}_y\underline{e}_z\partial_y E_z(\underline{\mathbf{R}},t)$$

$$+ \underline{e}_z\underline{e}_x\partial_x E_z(\underline{\mathbf{R}},t) + \underline{e}_z\underline{e}_y\partial_z E_y(\underline{\mathbf{R}},t) + \underline{e}_z\underline{e}_z\partial_z E_z(\underline{\mathbf{R}},t)$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] \\
&= \left(\underline{\mathrm{e}}_x\partial_x + \underline{\mathrm{e}}_y\partial_y + \underline{\mathrm{e}}_z\partial_z\right) \bullet \left[\left(\underline{\mathrm{e}}_x\partial_x + \underline{\mathrm{e}}_y\partial_y + \underline{\mathrm{e}}_z\partial_z\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] \\
&= \left(\underline{\mathrm{e}}_x\partial_x + \underline{\mathrm{e}}_y\partial_y + \underline{\mathrm{e}}_z\partial_z\right) \bullet \Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_x E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_x E_z(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_y E_z(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] \\
&= \underline{\mathrm{e}}_x\partial_x \bullet \Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big] \\
&\quad + \underline{\mathrm{e}}_y\partial_y \bullet \Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big] \\
&\quad + \underline{\mathrm{e}}_z\partial_z \bullet \Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t) \\
&\qquad + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]
\end{aligned}
$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] \\
&= \Big[\ \underline{\mathrm{e}}_x\partial_x^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\partial_x^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\partial_x^2 E_z(\underline{\mathbf{R}},t) \\
&\quad + \Big[\ \underline{\mathrm{e}}_x\partial_y^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\partial_y^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\partial_y^2 E_z(\underline{\mathbf{R}},t) \\
&\quad + \Big[\ \underline{\mathrm{e}}_x\partial_z^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_y\partial_z^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathrm{e}}_z\partial_z^2 E_z(\underline{\mathbf{R}},t) \\
&= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underbrace{\left[E_x(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_x + E_y(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_y + E_z(\underline{\mathbf{R}},t)\underline{\mathrm{e}}_z\right]}_{=\underline{\mathbf{E}}(\underline{\mathbf{R}},t)} \\
&= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t) \\
&= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)
\end{aligned}
$$

3-D Electromagnetic Wave Propagation / 3D elektromagnetische Wellenausbreitung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\left(\partial_x^2+\partial_y^2+\partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\left(\partial_x^2+\partial_y^2+\partial_z^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)=-\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)=-\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

2-D EM Wave Propagation – 2-D TM Case and 2-D TE Case / 2D EM Wellenausbreitung – 2D-TM-Fall und 2D-TE-Fall

We consider the xz plane and assume that the field is independent of y /
Wir betrachten die xz-Ebene und nehmen an, dass das Feld unabhängig von y ist ➡ $\frac{\partial}{\partial y} \equiv 0$

Then it follows for the 3-D wave equations / Es folgt dann für die 3D-Wellengleichungen

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(x,z,t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(x,z,t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(x,z,t)+\frac{1}{\mu_0}\nabla\rho_\mathrm{m}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(x,z,t) = \nabla\times\underline{\mathbf{J}}_\mathrm{m}(x,z,t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(x,z,t)+\frac{1}{\varepsilon_0}\nabla\rho_\mathrm{e}(x,z,t)$$

And we confine the current sources to / Und wir beschränken die Stromquellen auf

$$\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)=\underline{\mathbf{J}}_\mathrm{m}(x,z,t)=J_\mathrm{my}(x,z,t)\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)=\underline{\mathbf{J}}_\mathrm{e}(x,z,t)=J_\mathrm{ey}(x,z,t)\underline{\mathbf{e}}_y$$

This yields for the above given 3-D wave equation /
Dies ergibt für die oben gegebenen 3D-Wellengleichungen

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(x,z,t)=\nabla\times\left[J_\mathrm{ey}(x,z,t)\underline{\mathbf{e}}_y\right]+\varepsilon_0\frac{\partial}{\partial t}J_\mathrm{my}(x,z,t)\underline{\mathbf{e}}_y+\frac{1}{\mu_0}\nabla\rho_\mathrm{m}(x,y,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(x,z,t) = \nabla\times\left[J_\mathrm{my}(x,z,t)\underline{\mathbf{e}}_y\right]+\mu_0\frac{\partial}{\partial t}J_\mathrm{ey}(x,z,t)\underline{\mathbf{e}}_y+\frac{1}{\varepsilon_0}\nabla\rho_\mathrm{e}(x,y,t)$$

2-D EM Wave Propagation – 2-D TM Case and 2-D TE Case / 2D EM Wellenausbreitung – 2D-TM-Fall und 2D-TE-Fall

Curl and divergence of the current sources / Rotation und Divergenz der Stromquellen

$$\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = J_{\mathrm{my}}(x,z,t)\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = J_{\mathrm{ey}}(x,z,t)\underline{\mathbf{e}}_y$$

$$\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ \partial_x & \partial_y & \partial_z \\ 0 & J_{\mathrm{ez}}(x,z,t) & 0 \end{vmatrix}$$

$$= \frac{\partial}{\partial x} J_{\mathrm{ez}}(x,z,t)\underline{\mathbf{e}}_z - \frac{\partial}{\partial z} J_{\mathrm{ez}}(x,z,t)\underline{\mathbf{e}}_x$$

$$= -\frac{\partial}{\partial z} J_{\mathrm{ez}}(x,z,t)\underline{\mathbf{e}}_x + \frac{\partial}{\partial x} J_{\mathrm{ez}}(x,z,t)\underline{\mathbf{e}}_z$$

$$\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ \partial_x & \partial_y & \partial_z \\ 0 & J_{\mathrm{mz}}(x,z,t) & 0 \end{vmatrix}$$

$$= \frac{\partial}{\partial x} J_{\mathrm{mz}}(x,z,t)\underline{\mathbf{e}}_z - \frac{\partial}{\partial z} J_{\mathrm{mz}}(x,z,t)\underline{\mathbf{e}}_x$$

$$= -\frac{\partial}{\partial z} J_{\mathrm{mz}}(x,z,t)\underline{\mathbf{e}}_x + \frac{\partial}{\partial x} J_{\mathrm{mz}}(x,z,t)\underline{\mathbf{e}}_z$$

$$\nabla\bullet J_{\mathrm{my}}(x,z,t)\underline{\mathbf{e}}_y = \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\bullet J_{\mathrm{my}}(x,z,t)\underline{\mathbf{e}}_y = \frac{\partial}{\partial y} J_{\mathrm{my}}(x,z,t) = 0$$

$$\nabla\bullet J_{\mathrm{ey}}(x,z,t)\underline{\mathbf{e}}_y = \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\bullet J_{\mathrm{ey}}(x,z,t)\underline{\mathbf{e}}_y = \frac{\partial}{\partial y} J_{\mathrm{ey}}(x,z,t) = 0$$

The divergence of the current sources is in this special case zero, because the currents are constant in y direction. / Die Divergenz der Stromquellen ist in diesem speziellen Fall null, da die Ströme in y-Richtung konstant sind.

2-D EM Wave Propagation – 2-D TM Case and 2-D TE Case / 2D EM Wellenausbreitung – 2D-TM-Fall und 2D-TE-Fall

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{m}}(x,z,\omega) = \mathrm{j}\omega\rho_{\mathrm{m}}(x,z,\omega)$$

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{e}}(x,z,\omega) = \mathrm{j}\omega\rho_{\mathrm{e}}(x,z,\omega)$$

$\Rightarrow$

$$\rho_{\mathrm{m}}(x,z,\omega) = \frac{1}{\mathrm{j}\omega}\nabla \bullet \underline{\mathbf{J}}_{\mathrm{m}}(x,z,\omega)$$

$$\rho_{\mathrm{e}}(x,z,\omega) = \frac{1}{\mathrm{j}\omega}\nabla \bullet \underline{\mathbf{J}}_{\mathrm{e}}(x,z,\omega)$$

$$\nabla \bullet J_{\mathrm{m}y}(x,z,\omega)\underline{\mathbf{e}}_y = 0$$

$$\nabla \bullet J_{\mathrm{e}y}(x,z,\omega)\underline{\mathbf{e}}_y = 0$$

$$\rho_{\mathrm{m}}(\underline{\mathbf{R}},\omega) = 0$$

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}},\omega) = 0$$

$\Rightarrow$

$$\rho_{\mathrm{m}}(\underline{\mathbf{R}},t) = 0$$

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}},t) = 0$$

2-D EM Wave Propagation – 2-D TM Case and 2-D TE Case / 2D EM Wellenausbreitung – 2D-TM-Fall und 2D-TE-Fall

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(x,z,t)=-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(x,z,t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(x,z,t)=\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(x,z,t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(x,z,t)$$

$$\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)=\frac{\partial}{\partial x}J_{\mathrm{e}z}(x,z,t)\underline{\mathbf{e}}_z-\frac{\partial}{\partial z}J_{\mathrm{e}z}(x,z,t)\underline{\mathbf{e}}_x$$

$$\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)=\frac{\partial}{\partial x}J_{\mathrm{m}z}(x,z,t)\underline{\mathbf{e}}_z-\frac{\partial}{\partial z}J_{\mathrm{m}z}(x,z,t)\underline{\mathbf{e}}_x$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(x,z,t)=-\frac{\partial}{\partial x}J_{\mathrm{e}y}(x,z,t)\underline{\mathbf{e}}_z+\frac{\partial}{\partial z}J_{\mathrm{e}y}(x,z,t)\underline{\mathbf{e}}_x+\varepsilon_0\frac{\partial}{\partial t}J_{\mathrm{m}y}(x,z,t)\underline{\mathbf{e}}_y$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(x,z,t)=\frac{\partial}{\partial x}J_{\mathrm{m}y}(x,z,t)\underline{\mathbf{e}}_z-\frac{\partial}{\partial z}J_{\mathrm{m}y}(x,z,t)\underline{\mathbf{e}}_x+\mu_0\frac{\partial}{\partial t}J_{\mathrm{e}y}(x,z,t)\underline{\mathbf{e}}_y$$

2-D EM Wave Propagation – 2-D TM Case and 2-D TE Case / 2D EM Wellenausbreitung – 2D-TM-Fall und 2D-TE-Fall

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(x,z,t)=-\frac{\partial}{\partial x}J_{\mathrm{ey}}(x,z,t)\underline{\mathbf{e}}_z+\frac{\partial}{\partial z}J_{\mathrm{ey}}(x,z,t)\underline{\mathbf{e}}_x+\varepsilon_0\frac{\partial}{\partial t}J_{\mathrm{my}}(x,z,t)\underline{\mathbf{e}}_y$$

Decoupled equations / Entkoppelte Gleichungen ➡

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_x(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_x(x,z,t)=\frac{\partial}{\partial z}J_{\mathrm{ey}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_y(x,z,t)=\varepsilon_0\frac{\partial}{\partial t}J_{\mathrm{my}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_z(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_z(x,z,t)=-\frac{\partial}{\partial x}J_{\mathrm{ey}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(x,z,t)=-\frac{\partial}{\partial z}J_{\mathrm{my}}(x,z,t)\underline{\mathbf{e}}_x+\mu_0\frac{\partial}{\partial t}J_{\mathrm{ey}}(x,z,t)\underline{\mathbf{e}}_y+\frac{\partial}{\partial x}J_{\mathrm{my}}(x,z,t)\underline{\mathbf{e}}_z$$

Decoupled equations / Entkoppelte Gleichungen ➡

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_x(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_x(x,z,t)=-\frac{\partial}{\partial z}J_{\mathrm{my}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_y(x,z,t)=\mu_0\frac{\partial}{\partial t}J_{\mathrm{ey}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_z(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_z(x,z,t)=\frac{\partial}{\partial x}J_{\mathrm{my}}(x,z,t)$$

2-D EM Wave Propagation - 2-D TM Case and 2-D TE Case / 2D EM Wellenausbreitung - 2D-TM-Fall und 2D-TE-Fall

➡ Separation in 2-D → TM and TE case / Separation in 2D → TM- und TE- Fall

TM: transversal magnetic / transversal magnetisch
TE: transversal electric / transversal elektrisch

TM_y case / TM_y-Fall

$E_y(x,z,t)$
$H_x(x,z,t)$
$H_z(x,z,t)$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_y(x,z,t) = \mu_0\frac{\partial}{\partial t}J_{\mathrm{ey}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_x(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_x(x,z,t) = \frac{\partial}{\partial z}J_{\mathrm{ey}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_z(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_z(x,z,t) = -\frac{\partial}{\partial x}J_{\mathrm{ey}}(x,z,t)$$

TE_y case / TE_y-Fall

$H_y(x,z,t)$
$E_x(x,z,t)$
$E_z(x,z,t)$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_y(x,z,t) = \varepsilon_0\frac{\partial}{\partial t}J_{\mathrm{my}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_x(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_x(x,z,t) = -\frac{\partial}{\partial z}J_{\mathrm{my}}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_z(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_z(x,z,t) = \frac{\partial}{\partial x}J_{\mathrm{my}}(x,z,t)$$

2-D EM Wave Propagation – 2-D TM Case / 2D EM Wellenausbreitung – 2D-TM-Fall

Separation in 2-D → TM case / Separation in 2D → TM-Fall

TM: transversal magnetic / transversal magnetisch

TM_y case / TM_y-Fall

$E_y(x,z,t)$

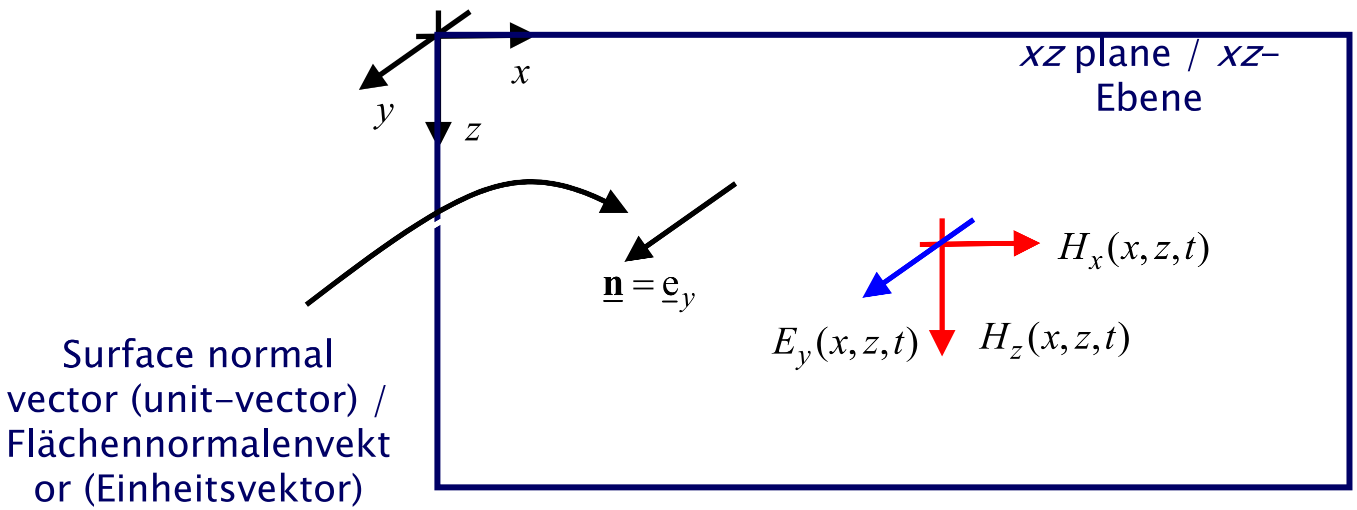
$H_x(x,z,t)$

$H_z(x,z,t)$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_y(x,z,t) = \mu_0\frac{\partial}{\partial t}J_{ey}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_x(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_x(x,z,t) = \frac{\partial}{\partial z}J_{ey}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_z(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_z(x,z,t) = -\frac{\partial}{\partial x}J_{ey}(x,z,t)$$



2-D EM Wave Propagation – 2-D TE Case / 2D EM Wellenausbreitung – 2D-TE-Fall

➡ Separation in 2-D → TE case / Separation in 2D → TE- Fall

TE: transversal electric / transversal elektrisch

TE$_y$ case / TE$_y$-Fall

$H_y(x,z,t)$

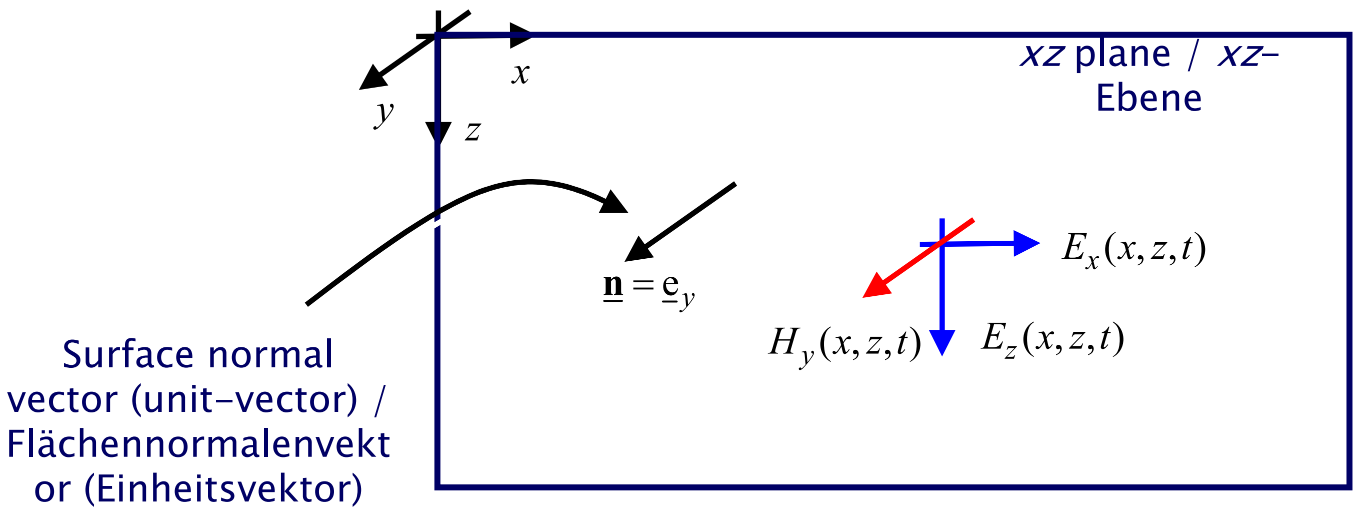
$E_x(x,z,t)$

$E_z(x,z,t)$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)H_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}H_y(x,z,t)=\varepsilon_0\frac{\partial}{\partial t}J_{my}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_x(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_x(x,z,t)=-\frac{\partial}{\partial z}J_{my}(x,z,t)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_z(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_z(x,z,t)=\frac{\partial}{\partial x}J_{my}(x,z,t)$$



FD Method – 2-D TM Wave Equation / FD-Methode – 2D-TM-Wellengleichung

Central FD Operators / Zentrale FD-Operatoren

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_y(x,z,t) = \mu_0\frac{\partial}{\partial t}J_{\mathrm{ey}}(x,z,t)$$

Central FD Operators / Zentrale FD-Operatoren

$$\frac{\partial^2}{\partial x^2}E_y(x,z,t)=\frac{E_y(x+\Delta x,z,t)-2E_y(x,z,t)+E_y(x-\Delta x,z,t)}{(\Delta x)^2}+\bigcirc[(\Delta x)^2]$$

$$\frac{\partial^2}{\partial z^2}E_y(x,z,t)=\frac{E_y(x,z+\Delta z,t)-2E_y(x,z,t)+E_y(x,z-\Delta z,t)}{(\Delta z)^2}+\bigcirc[(\Delta z)^2]$$

$$\frac{\partial^2}{\partial t^2}E_y(x,z,t)=\frac{E_y(x,z,t+\Delta t)-2E_y(x,z,t)+E_y(x,z,t-\Delta t)}{(\Delta t)^2}+\bigcirc[(\Delta t)^2]$$

Backward FD Operator / Rückwärts-FD-Operator

$$\frac{\partial}{\partial t}J_{\mathrm{ey}}(x,z,t)=\frac{J_{\mathrm{ey}}(x.z,t)-J_{\mathrm{ey}}(x,z,t-\Delta t)}{\Delta t}+\bigcirc(\Delta t)$$

FD Method – 2-D TM Wave Equation / FD-Methode – 2D-TM-Wellengleichung

2-D TM wave equation / 2D-TM-Wellengleichung

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_y(x,z,t) = \mu_0\frac{\partial}{\partial t}J_{ey}(x,z,t)$$

Explicit FD algorithm in the time domain of 2nd order in space and time / Expliziter FD-Algorithmus im Zeitbereich 2ter Ordnung in Raum und Zeit

$$\frac{E_y(x+\Delta x,z,t)-2E_y(x,z,t)+E_y(x-\Delta x,z,t)}{(\Delta x)^2}+\frac{E_y(x,z+\Delta z,t)-2E_y(x,z,t)+E_y(x,z-\Delta z,t)}{(\Delta z)^2}$$

$$-\frac{1}{c_0^2}\frac{E_y(x,z,t+\Delta t)-2E_y(x,z,t)+E_y(x,z,t-\Delta t)}{(\Delta t)^2}$$

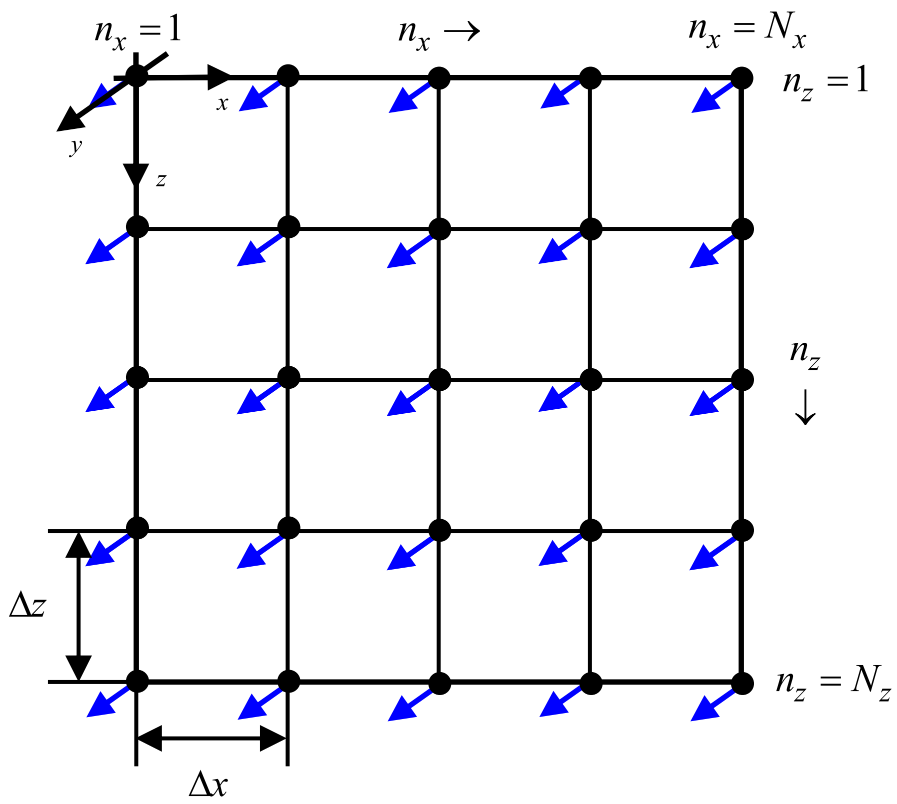
$$=\mu_0\frac{J_{ey}(x,z,t)-J_{ey}(x,z,t-\Delta t)}{\Delta t}+\bigcirc[(\Delta x)^2]+\bigcirc[(\Delta z)^2]+\bigcirc[(\Delta t)^2]$$

FD Method – 2-D TM Wave Equation – 2-D FD Grid / FD-Methode – 2D-TM-Wellengleichung – 2D-FD-Gitter

2-D FD grid / 2D-FD-Gitter

$$E_y(x,z,t) \rightarrow E_y^{(n_x,n_z,n_t)} \rightarrow E_y^{(n,n_t)}$$

- $E_y(x,z,t) = E_y^{(n_x,n_z,n_t)}$
  $= E_y^{(n,n_t)}$



$n_x = 1, \ldots, N_x$

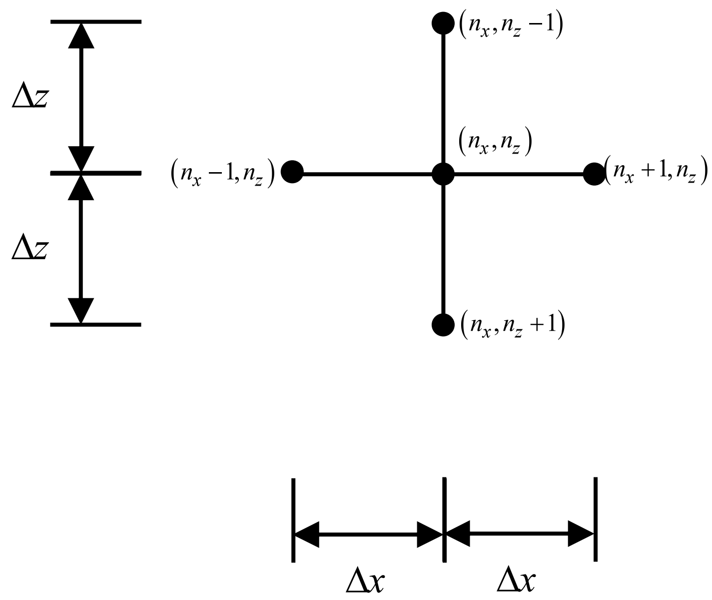
$n_z = 1, \ldots, N_z$

Global grid node numbering / Globale Gitterknotennummerierung

$$n = n_x + N_x(n_z - 1) \qquad n = 1, \ldots, N \qquad N = N_x N_z$$

FD Method – 2-D TM Wave Equation – 2-D FD Stencil / FD-Methode – 2D-TM-Wellengleichung – 2D-FD-Schablone

2-D FD stencil in space /
2D-FD-Schablone im Raum



FD Method – 2-D TM Wave Equation / FD-Methode – 2D-TM-Wellengleichung

Explicit 2-D FD algorithm in the time domain of 2nd order in space and time / Expliziter 2D-FD-Algorithmus im Zeitbereich 2ter Ordnung in Raum und Zeit

$$
\begin{aligned}
E_y(x,z,t+\Delta t) &= 2E_y(x,z,t) - E_y(x,z,t-\Delta t) \\
&+ c_0^2 \frac{(\Delta t)^2}{(\Delta x)^2}\left[E_y(x+\Delta x,z,t) - 2E_y(x,z,t) + E_y(x-\Delta x,x,t)\right] \\
&+ c_0^2 \frac{(\Delta t)^2}{(\Delta z)^2}\left[E_y(x,z+\Delta z,t) - 2E_y(x,z,t) + E_y(x,z-\Delta z,t)\right] \\
&+ c_0^2 \mu_0 \Delta t\left[J_{\mathrm{ey}}(x,z,t) - J_{\mathrm{ey}}(x,z,t-\Delta t)\right] + \bigcirc[(\Delta x)^2] + \bigcirc[(\Delta z)^2] + \bigcirc[(\Delta t)^2]
\end{aligned}
$$

Marching-on-in-time algorithm / „Marschieren in der Zeit"-Algorithmus

$$
\begin{aligned}
x &\to n_x \Delta x, \quad n_x = 1,\ldots,N_x \\
z &\to n_z \Delta z, \quad n_z = 1,\ldots,N_z \\
t &\to n_t \Delta t, \quad n_t = 1,\ldots,N_t
\end{aligned}
$$

$$
\begin{aligned}
E_y(x,z,t) &\to E_y^{(n_x,n_z,n_t)} \\
J_{\mathrm{ey}}(x,z,t) &\to J_{\mathrm{ey}}^{(n_x,n_z,n_t)}
\end{aligned}
$$

FD Method – 2-D TM Wave Equation / FD-Methode – 2D-TM-Wellengleichung

Explicit 2-D FD algorithm in the time domain of 2nd order in space and time / Expliziter 2D FD-Algorithmus im Zeitbereich 2ter Ordnung in Raum und Zeit

$$
\begin{aligned}
E_y^{(n_x,n_z,n_t+1)} &= 2E_y^{(n_x,n_z,n_t)} - E_y^{(n_x,n_z,n_t-1)} \\
&+ \left(\frac{c_0\Delta t}{\Delta x}\right)^2 \left[E_y^{(n_x+1,n_z,n_t)} - 2E_y^{(n_x,n_z,n_t)} + E_y^{(n_x-1,n_z,n_t)} \right] \\
&+ \left(\frac{c_0\Delta t}{\Delta z}\right)^2 \left[E_y^{(n_x,n_z+1,n_t)} - 2E_y^{(n_x,n_z,n_t)} + E_y^{(n_x,n_z-1,n_t)} \right] \\
&+ c_0^2\mu_0\Delta t \left[J_{\mathrm{ey}}^{(n_x,n_z,n_t)} - J_{\mathrm{ey}}^{(n_x,n_z,n_t-1)} \right] + O[(\Delta x)^2] + O[(\Delta z)^2] + O[(\Delta t)^2]
\end{aligned}
$$

Homogeneous 2-D FD grid of quadratic cells / Homogenes 2D- FD-Gitter aus quadratischen Zellen

$$\Delta x = \Delta z$$

$$
\begin{aligned}
E_y^{(n_x,n_z,n_t+1)} &= 2E_y^{(n_x,n_z,n_t)} - E_y^{(n_x,n_z,n_t-1)} \\
&+ \left(\frac{c_0\Delta t}{\Delta x}\right)^2 \left[E_y^{(n_x,n_z+1,n_t)} + E_y^{(n_x+1,n_z,n_t)} - 4E_y^{(n_x,n_z,n_t)} + E_y^{(n_x-1,n_z,n_t)} + E_y^{(n_x,n_z-1,n_t)} \right] \\
&+ c_0^2\mu_0\Delta t \left[J_{\mathrm{ey}}^{(n_x,n_z,n_t)} - J_{\mathrm{ey}}^{(n_x,n_z,n_t-1)} \right] + O[(\Delta x)^2] + O[(\Delta t)^2]
\end{aligned}
$$

FD Method – 2-D TM Wave Equation / FD-Methode – 2D-TM-Wellengleichung

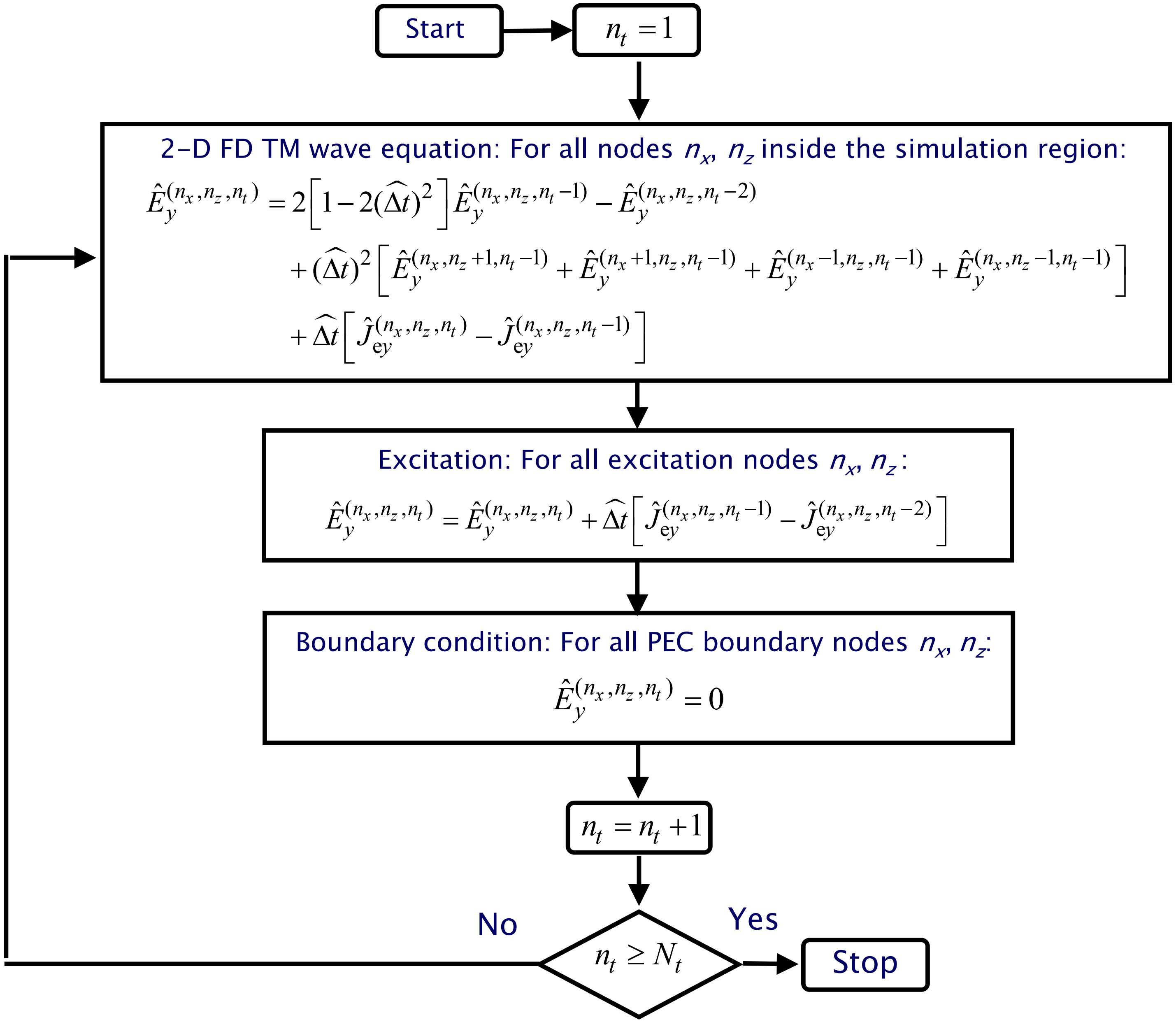
Explicit FD algorithm in the time domain of 2nd order in space and time / Expliziter FD-Algorithmus im Zeitbereich 2ter Ordnung in Raum und Zeit

$$\begin{aligned}\hat{E}_y^{(n_x,n_z,n_t+1)} &= 2\hat{E}_y^{(n_x,n_z,n_t)} - \hat{E}_y^{(n_x,n_z,n_t-1)} \\ &+ (\widehat{\Delta t})^2 \left[\hat{E}_y^{(n_x,n_z+1,n_t)} + \hat{E}_y^{(n_x+1,n_z,n_t)} - 4\hat{E}_y^{(n_x,n_z,n_t)} + \hat{E}_y^{(n_x-1,n_z,n_t)} + \hat{E}_y^{(n_x,n_z-1,n_t)} \right] \\ &+ \widehat{\Delta t} \left[\hat{J}_{\mathrm{ey}}^{(n_x,n_z,n_t)} - \hat{J}_{\mathrm{ey}}^{(n_x,n_z,n_t-1)} \right]\end{aligned}$$

$$\text{for / für} \quad \begin{cases} 1 \le n_x \le N_x \\ 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{cases}$$

$$\begin{aligned}\hat{E}_y^{(n_x,n_z,n_t)} &= 2\left[1 - 2(\widehat{\Delta t})^2\right]\hat{E}_y^{(n_x,n_z,n_t-1)} - \hat{E}_y^{(n_x,n_z,n_t-2)} \\ &+ (\widehat{\Delta t})^2 \left[\hat{E}_y^{(n_x,n_z+1,n_t)} + \hat{E}_y^{(n_x+1,n_z,n_t)} + \hat{E}_y^{(n_x-1,n_z,n_t)} + \hat{E}_y^{(n_x,n_z-1,n_t)} \right] \\ &+ \widehat{\Delta t} \left[\hat{J}_{\mathrm{ey}}^{(n_x,n_z,n_t)} - \hat{J}_{\mathrm{ey}}^{(n_x,n_z,n_t-1)} \right]\end{aligned}$$

FD Method – 2–D FD Wave Equation – TM Case – Flow Chart / FD–Methode – 2D FD–Wellengleichung – TM–Fall – Flussdiagramm



FD Method – 2-D TM Wave Equation – Example / FD-Methode – 2D-TM-Wellengleichung – Beispiel

Scalar 2-D TM wave equation / Skalare 2D-TM-Wellengleichung

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y(x,z,t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}E_y(x,z,t)=\mu_0\frac{\partial}{\partial t}J_{ey}(x,z,t)\quad \text{for / für}\quad \begin{cases}0\le x\le X\\ 0\le z\le Z\\ 0\le t\le T\end{cases}$$

Initial condition / Anfangsbedingung

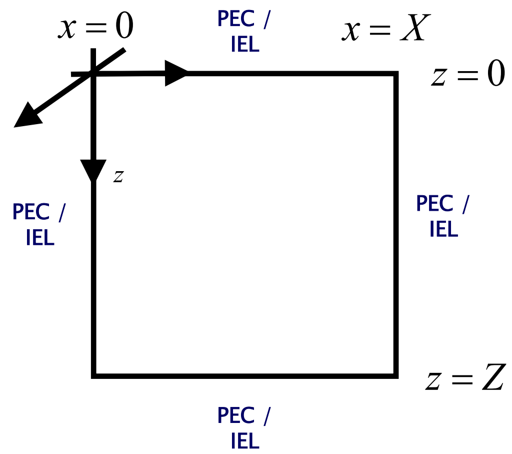
$$E_y(x,z,t)=J_{ey}(x,z,t)=0 \qquad t\le 0$$

$$J_{ey}(x,z,t)=\delta(x-x_0)\,\delta(z-z_0)\,f(t) \qquad t>0$$

Causality / Kausalität

Boundary conditions for a perfectly electrically conducting (PEC) boundary / Randbedingung für einen ideal elektrisch leitenden (IEL) Rand

$$\left.\begin{aligned}E_y(0,z,t)&=0\\ E_y(X,z,t)&=0\end{aligned}\right\}\ \forall z,t\forall t \quad \text{and / und} \quad \left.\begin{aligned}E_y(x,0,t)&=0\\ E_y(x,Z,t)&=0\end{aligned}\right\}\ \forall x,t\forall t$$



Hyperbolic initial-boundary-value problem / Hyperbolisches Anfangs-Randwert-Problem

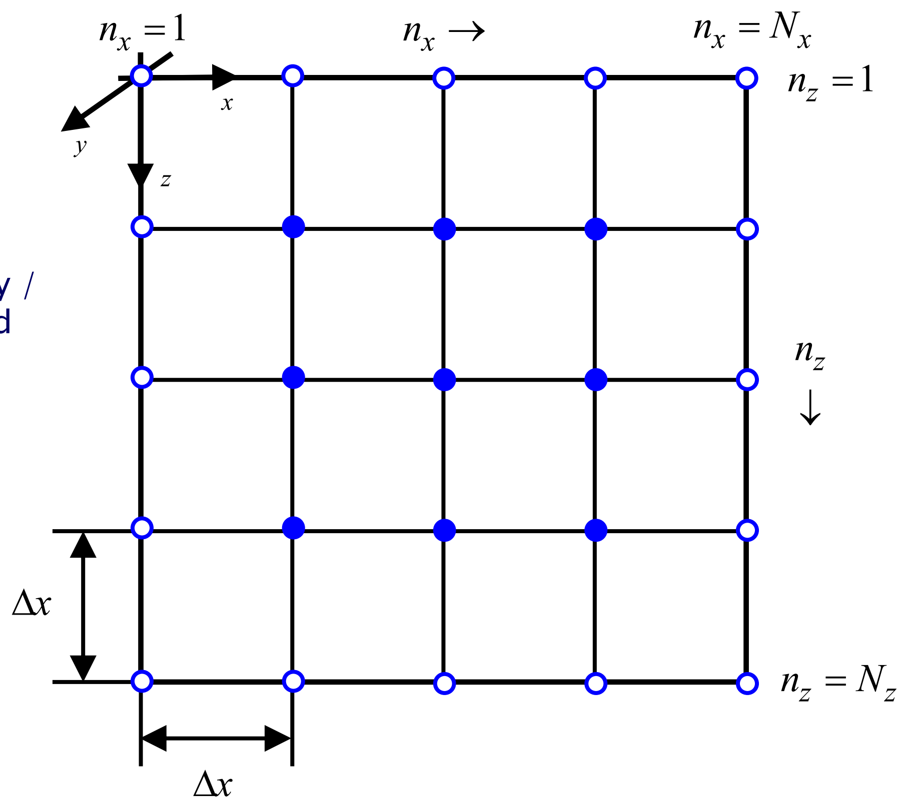
FD Method – 2-D TM Wave Equation – Example / FD-Methode – 2D-TM-Wellengleichung – Beispiel

Nodes in the simulation region / Knoten im Simulationsgebiet

● $E_y^{(n,n_t)} \neq 0$

Nodes at the PEC boundary / Knoten auf dem IEL-Rand

○ $E_y^{(n,n_t)} = 0$



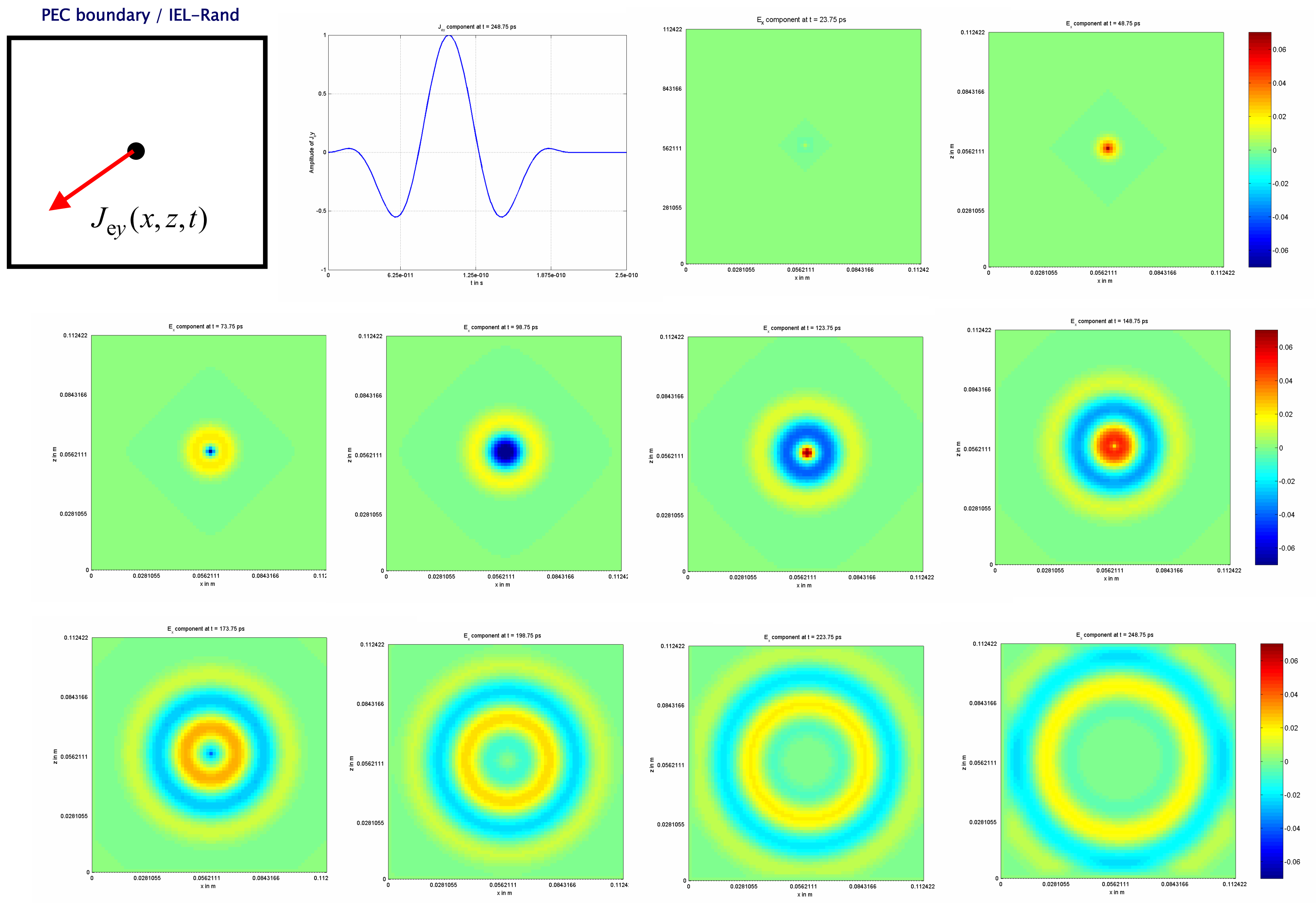
$n_x = 1, \ldots, N_x$

$n_z = 1, \ldots, N_z$

Global grid node numbering / Globale Gitterknotennummerierung

$n = n_x + N_x(n_z - 1) \qquad n = 1, \ldots, N \qquad N = N_x N_z$

FD Method – 2-D TM Wave Equation – Example/ FD-Methode – 2D-TM-Wellengleichung – Beispiel



FD Method – 2-D FD Wave Equation – TM Case – Validation / FD-Methode – 2D FD-Wellengleichung – TM-Fall – Validierung

Domain integral representation / (Gebiets-) Integraldarstellung

1-D case / 1D-Fall

$$E_x(z,\omega) = \mathrm{j}\omega\mu_0 \int_{z'=-\infty}^{\infty} G(z-z',\omega)\, J_{\mathrm{ex}}(z',\omega)\, \mathrm{d}z'$$

2-D case / 2D-Fall

$$E_y(x,z,\omega) = \mathrm{j}\omega\mu_0 \int_{z'=-\infty}^{\infty} \int_{x'=-\infty}^{\infty} G(x-x',z-z',\omega)\, J_{\mathrm{ey}}(x',z',\omega)\, \mathrm{d}x'\mathrm{d}z'$$

Green's function / Greensche Funktion

$$G(z,\omega) = \frac{c_0}{2}\left[\mathrm{j}\,\mathrm{PV}\frac{1}{\omega_0} + \pi\delta(z)\right] \mathrm{e}^{\mathrm{j}k_0|z|}$$

$$G(z,t) = \frac{c_0}{2} u\left(t - \frac{|z|}{c_0}\right)$$

$$G(x,z,\omega) = \frac{\mathrm{j}}{4}\mathrm{H}_0^{(1)}\left(\frac{\omega}{c_0}\sqrt{x^2+z^2}\right)$$

$$G(x,z,t) = \frac{c_0}{2\pi}\frac{1}{\sqrt{c_0^2 t^2 - (x^2+z^2)}}\, u\left(t - \frac{\sqrt{x^2+z^2}}{c_0}\right)$$

FD Method – 2-D FD Wave Equation – TM Case – Validation / FD-Methode – 2D FD-Wellengleichung – TM-Fall – Validierung

2-D Domain integral representation / 2D-(Gebiets-)Integraldarstellung

$$E_y(r,\omega) = \mathrm{j}\omega\mu_0 \int_{r'=0}^{\infty} G(r-r',\omega)\, J_{\mathrm{e}y}(r',\omega)\, \mathrm{d}r'$$

$$G(r,\omega) = \frac{\mathrm{j}}{4}\mathrm{H}_0^{(1)}\left(\frac{\omega}{c_0} r\right)$$

$$G(r,t) = \frac{c_0}{2\pi}\frac{1}{\sqrt{c_0^2 t^2 - r^2}}\, u\left(t - \frac{r}{c_0}\right)$$

$$E_y(\underline{\mathbf{r}},\omega) = \mathrm{j}\omega\mu_0 \int_{\underline{\mathbf{r}}'} G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)\, J_{\mathrm{e}y}(\underline{\mathbf{r}}',\omega)\, \mathrm{d}\underline{\mathbf{r}}'$$

$$G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega) = \frac{\mathrm{j}}{4}\mathrm{H}_0^{(1)}\left(\frac{\omega}{c_0}\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|\right)$$

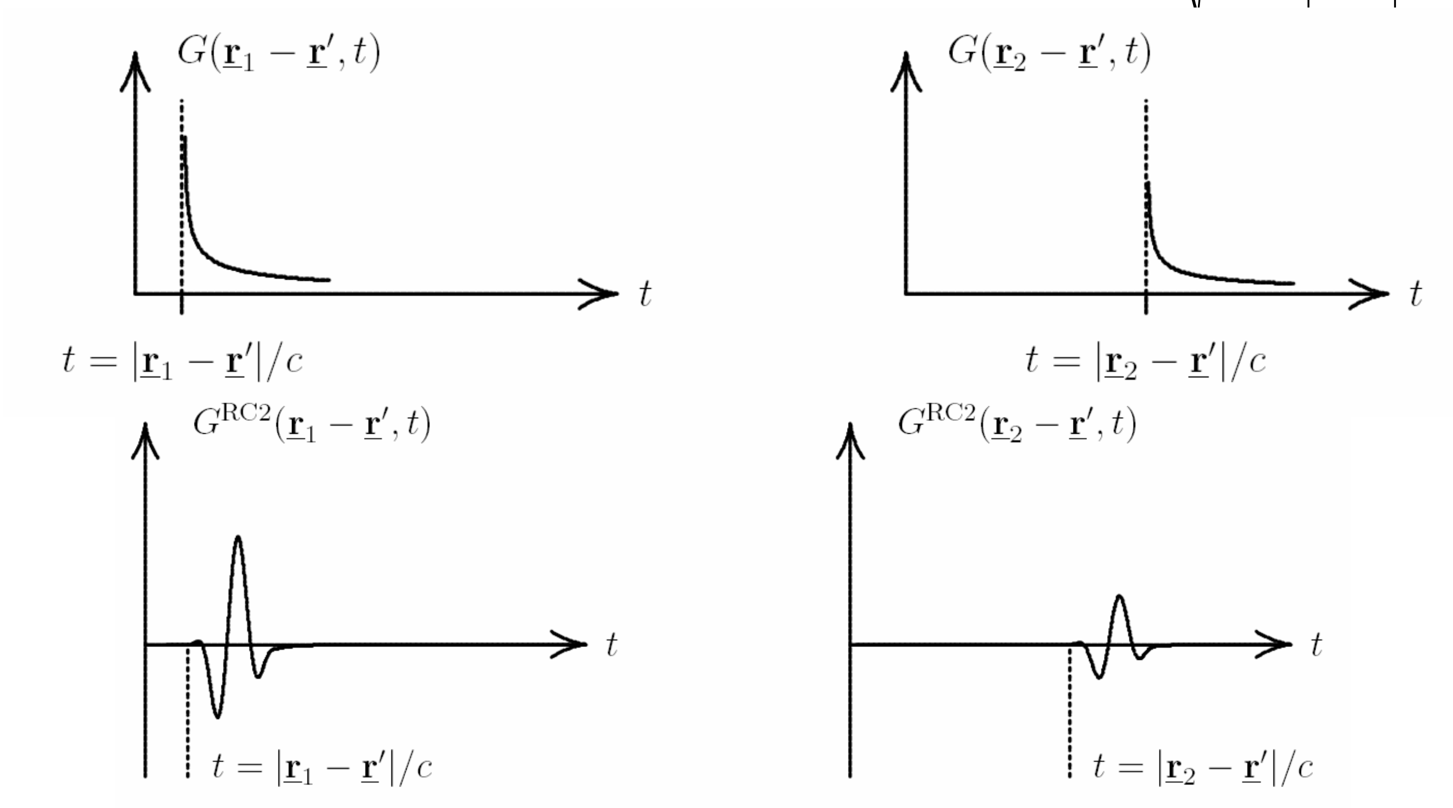
$$G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',t) = \frac{c_0}{2\pi}\frac{1}{\sqrt{c_0^2 t^2 - \left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|^2}}\, u\left(t - \frac{\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|}{c_0}\right)$$

FD Method – 2-D FD Wave Equation – TM Case – Validation / FD-Methode – 2D FD-Wellengleichung – TM-Fall – Validierung

2-D Domain integral representation / 2D-(Gebiets-)Integraldarstellung

$$G^{\mathrm{RC2}}(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)=\mathrm{RC2}(\omega)\,\frac{\mathrm{j}}{4}\mathrm{H}_0^{(1)}\left(\frac{\omega}{c_0}\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|\right)$$

$$G^{\mathrm{RC2}}(\underline{\mathbf{r}}-\underline{\mathbf{r}}',t)=\mathrm{RC2}(t)*_t\frac{c_0}{2\pi}\frac{1}{\sqrt{c_0^2t^2-\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|^2}}\,u\left(t-\frac{\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|}{c_0}\right)$$



$$G^{\mathrm{RC2}}(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)\simeq\frac{1}{4}\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}\sqrt{\frac{2c}{\pi}}\,\frac{\mathrm{RC2}(\omega)}{\sqrt{\omega}}\,\frac{\mathrm{e}^{\mathrm{j}k\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|}}{\sqrt{\left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|}}$$

EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

Equations of first order /
Gleichungen der ersten Ordnung

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$f\left(\underline{\mathbf{H}},\underline{\mathbf{E}}\right)$$

Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \nu_0\,\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

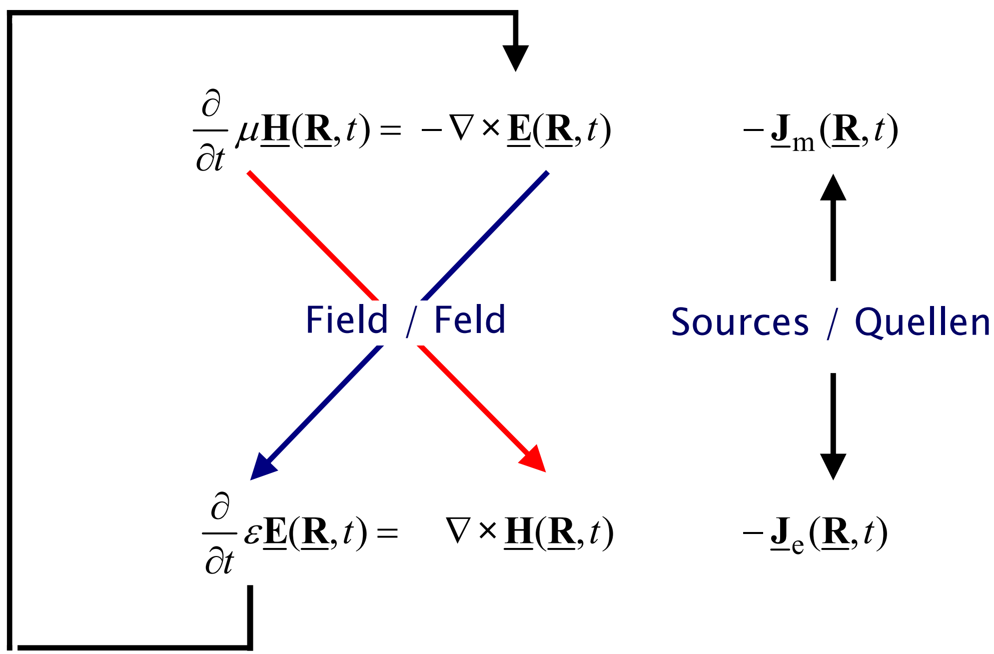
$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\left[\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] = \nabla\times\left[\nu\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right] - \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$f\left(\underline{\mathbf{B}},\underline{\mathbf{E}}\right)$$

EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms



Faraday's induction law /
Faradaysches Induktionsgesetz

Ampère–Maxwell's circuital law /
Ampère–Maxwellsches
Durchflutungsgesetz

EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) \qquad -\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Faraday's induction law / Faradaysches Induktionsgesetz

Field / Feld

Sources / Quellen

$$\frac{\partial}{\partial t}\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla\times\left[\nu\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right] \qquad -\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Ampère-Maxwell's circuital law / Ampère-Maxwellsches Durchflutungsgesetz

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

Ansatz for the electric and
magnetic field strength /
Ansatz für die elektrische und
magnetische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = E_x(z,t)\;\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = H_y(z,t)\,\underline{\mathbf{e}}_y$$

$$\frac{\partial}{\partial t}H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z}E_x(z,t) - \frac{1}{\mu_0}J_{\mathrm{my}}(z,t)$$

$$\frac{\partial}{\partial t}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}H_y(z,t) - \frac{1}{\varepsilon_0}J_{\mathrm{ex}}(z,t)$$

$$\frac{d}{dt}f(t) = \frac{f\left(t+\frac{\Delta t}{2}\right) - f\left(t-\frac{\Delta t}{2}\right)}{\Delta t} + \mathrm{O}[(\Delta t)^2]$$

$$\frac{d}{dz}f(z) = \frac{f\left(z+\frac{\Delta z}{2}\right) - f\left(z-\frac{\Delta z}{2}\right)}{\Delta x} + \mathrm{O}[(\Delta x)^2]$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) \qquad -\frac{1}{\mu_0} J_{my}(z,t)$$

Field / Feld

Sources / Quellen

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) \qquad -\frac{1}{\varepsilon_0} J_{ex}(z,t)$$

1-D EM Wave Propagation – FDTD – Discretization of the 1st Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 1ten Gleichung

Spatial discretization of the 1st equation / Räumliche Diskretisierung der 1ten Gleichung

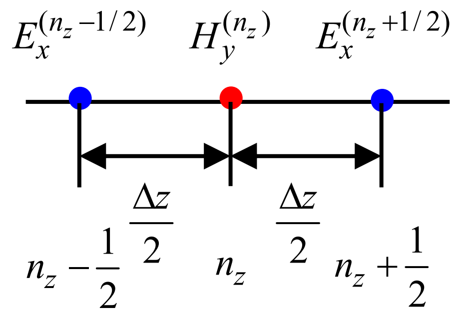
$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$

$$H_y : z \to n_z \Delta z, \qquad n_z = 1,\ldots,N_z$$

$$E_x : z \to \left(n_z + 1/2\right)\Delta z, \quad n_z = 1,\ldots,N_z$$

$$\frac{\partial}{\partial z} E_x(z,t) \;\to\; \frac{\partial}{\partial z} E_x(z,t)\bigg|_z = \frac{1}{\Delta z}\left[E_x\left(z+\frac{\Delta z}{2}\right) - E_x\left(z-\frac{\Delta z}{2}\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\left[E_x^{(n_z)}\right]$$



$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

1-D EM Wave Propagation – FDTD – Discretization of the 2nd Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 2ten Gleichung

Spatial discretization of the 2nd equation /
Räumliche Diskretisierung der 2ten Gleichung

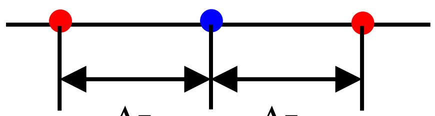
$$\frac{\partial}{\partial t}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}H_y(z,t) - \frac{1}{\varepsilon_0}J_{\text{ex}}(z,t)$$

$$H_y : z \to n_z\Delta z, \qquad n_z = 1,\ldots,N_z$$

$$E_x : z \to \left(n_z + 1/2\right)\Delta z, \quad n_z = 1,\ldots,N_z$$

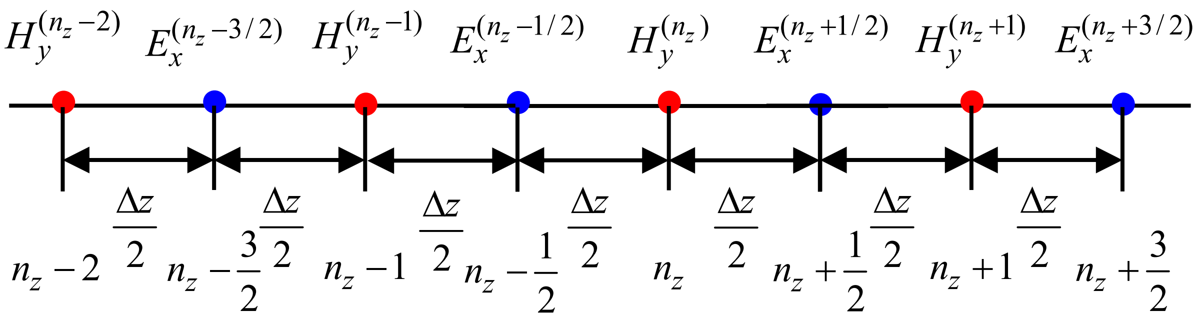
$$\frac{\partial}{\partial z}H_y(z,t) \;\to\; \left.\frac{\partial}{\partial z}H_y(z,t)\right|_{z+\frac{\Delta z}{2}} = \frac{1}{\Delta z}\left[H_y\left(z+\Delta z\right) - H_y\left(z\right)\right] + \mathrm{O}[(\Delta z)^2]$$

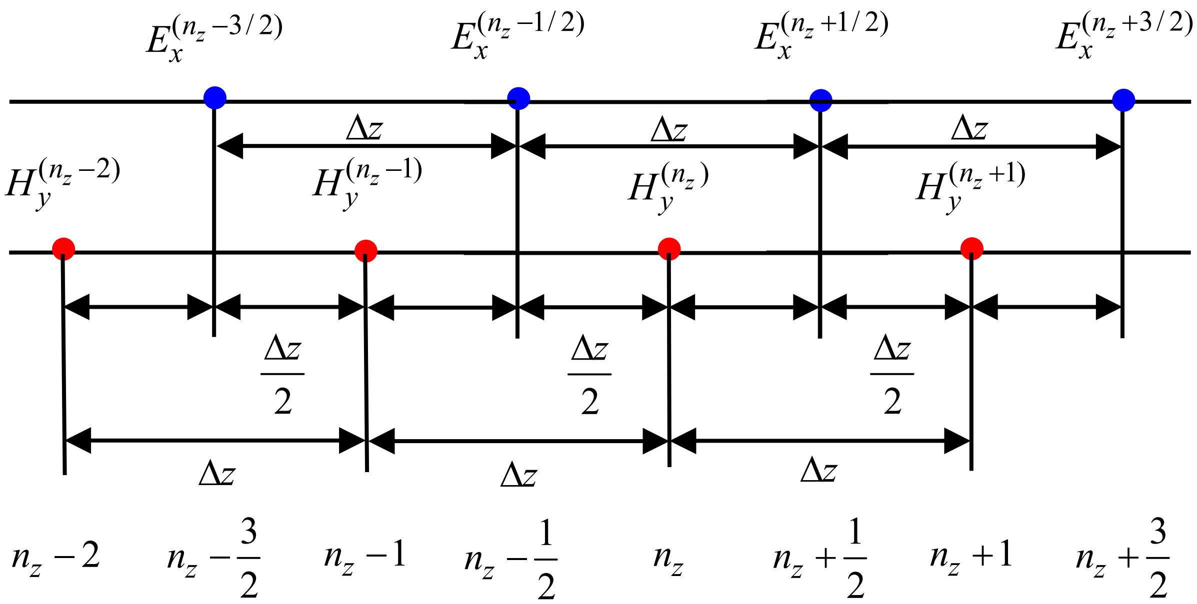
$$\left[H_y^{(n_z+1/2)}\right]$$



$$\frac{\partial}{\partial t}E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0\Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0}J_{\text{ey}}^{(n_z+1/2)}(t)$$

1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum





1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

$$\Downarrow$$

$$\frac{d}{dz} f(z) = \frac{1}{\Delta x}\left[f\left(z+\frac{\Delta z}{2}\right) - f\left(z-\frac{\Delta z}{2}\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\Downarrow$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t) \right] - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2)}(t)$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = ?$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = ?$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

$$\frac{d}{dt} f(t) = \frac{1}{\Delta t}\left[f\left(t+\frac{\Delta t}{2}\right) - f\left(t-\frac{\Delta t}{2}\right)\right] + O[(\Delta t)^2]$$

Staggered grid in time / Versetztes Gitter in der Zeit

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = \frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} + O[(\Delta t)^2]$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = \frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t-1/2)}}{\Delta t} + O[(\Delta t)^2]$$

$$\frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t-1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0}J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t+1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0\Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0}J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

Explicit 1-D FDTD algorithm on a staggered grid in space and time /
Expliziter 1D-FDTD-Algorithmus auf einem versetzten Gitter im Raum und Zeit

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0\Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0}J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t+1/2)} - \frac{\Delta t}{\varepsilon_0\Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0}J_{\mathrm{ey}}^{(n_z+1/2,n_t)}$$

FDTD: Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

1-D EM Wave Propagation – 1-D FDTD / 1D EM Wellenausbreitung – 1D FDTD

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{my}(z,t)$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{ex}(z,t)$$

Explicit 1-D FDTD algorithm of leap-frog type on a staggered grid in space and time /
Expliziter 1D-FDTD-Algorithmus vom „Bocksprung"-Typ auf einem versetzten Gitter im Raum und Zeit

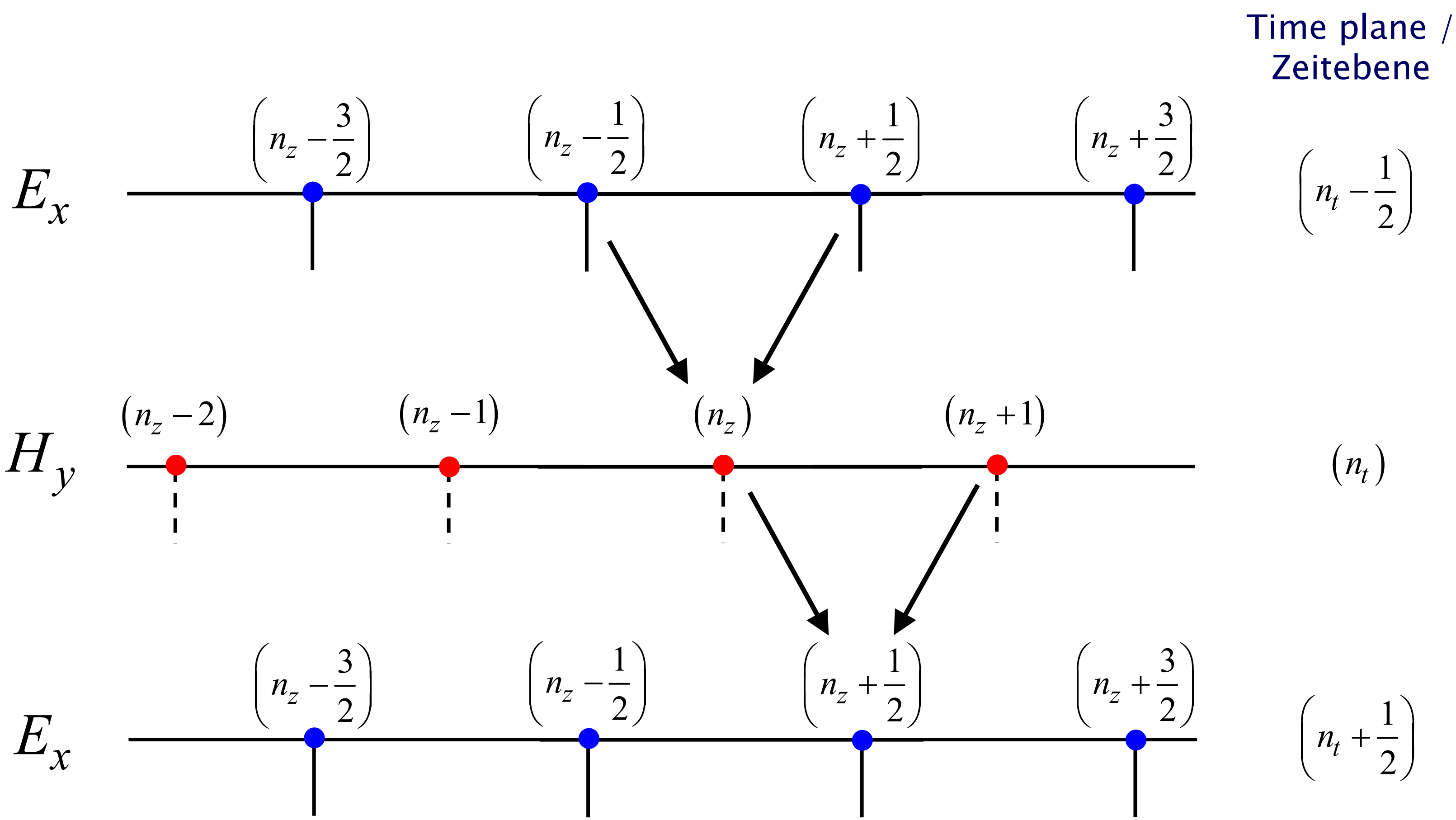
$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{my}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{ex}^{(n_z+1/2,n_t)}$$

FDTD: Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

1–D EM Wave Propagation – 1–D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1–D FDTD – Versetztes Gitter im Raum

Interleaving of the E_x and H_y field components in space and time in the 1–D FDTD formulation / Überlappung der E_x– und H_y–Feldkomponente in der 1D–FDTD–Formulierung im Raum und in der Zeit



1-D EM Wave Propagation – FDTD – Normalization / 1D EM Wellenausbreitung – FDTD – Normierung

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

$$\Delta t = \Delta t_{\mathrm{ref}}\,\widehat{\Delta t} \qquad \Delta t_{\mathrm{ref}} = \frac{\Delta x_{\mathrm{ref}}}{c_{\mathrm{ref}}} \qquad \Delta t = \frac{\Delta x_{\mathrm{ref}}}{c_{\mathrm{ref}}}\widehat{\Delta t}$$

$$\Delta z = \Delta x_{\mathrm{ref}}\,\widehat{\Delta z} \qquad c = c_{\mathrm{ref}}\hat{c} \qquad \varepsilon = \varepsilon_{\mathrm{ref}}\,\hat{\varepsilon} \qquad \mu = \mu_{\mathrm{ref}}\,\hat{\mu} \qquad \mu_{\mathrm{ref}} = \mu_0$$

$$E_x = E_{\mathrm{ref}}\hat{E}_x$$

$$H_y = H_{\mathrm{ref}}\hat{H}_y \qquad H_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{c_{\mathrm{ref}}\mu_{\mathrm{ref}}} = \frac{\sqrt{\varepsilon_{\mathrm{ref}}\mu_{\mathrm{ref}}}}{\mu_{\mathrm{ref}}}E_{\mathrm{ref}} = \sqrt{\frac{\varepsilon_{\mathrm{ref}}}{\mu_{\mathrm{ref}}}}E_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{Z_{\mathrm{ref}}}$$

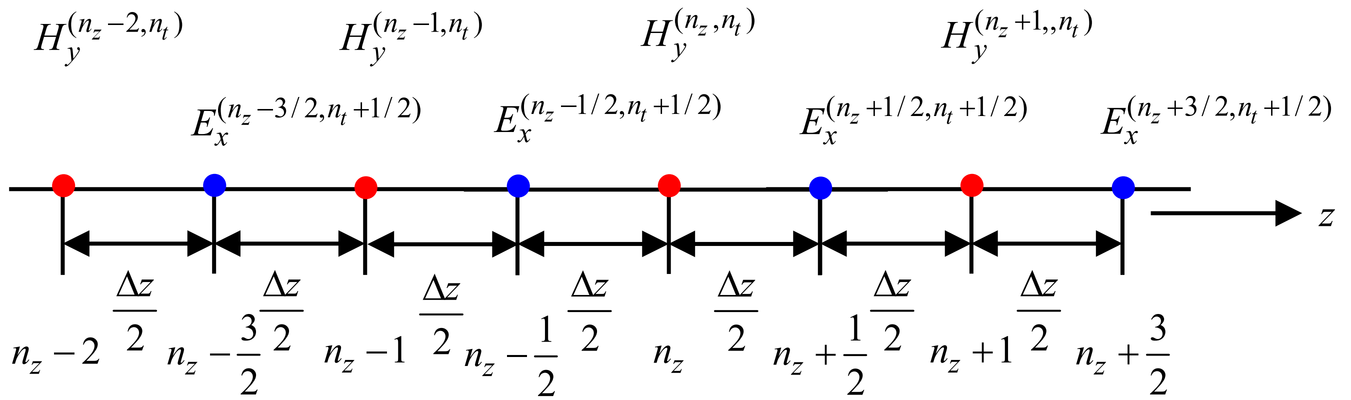
$$J_{\mathrm{ex}} = J_{\mathrm{e\,ref}}\hat{J}_{\mathrm{ex}} \qquad J_{\mathrm{e\,ref}} = \frac{\varepsilon_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}}E_{\mathrm{ref}}$$

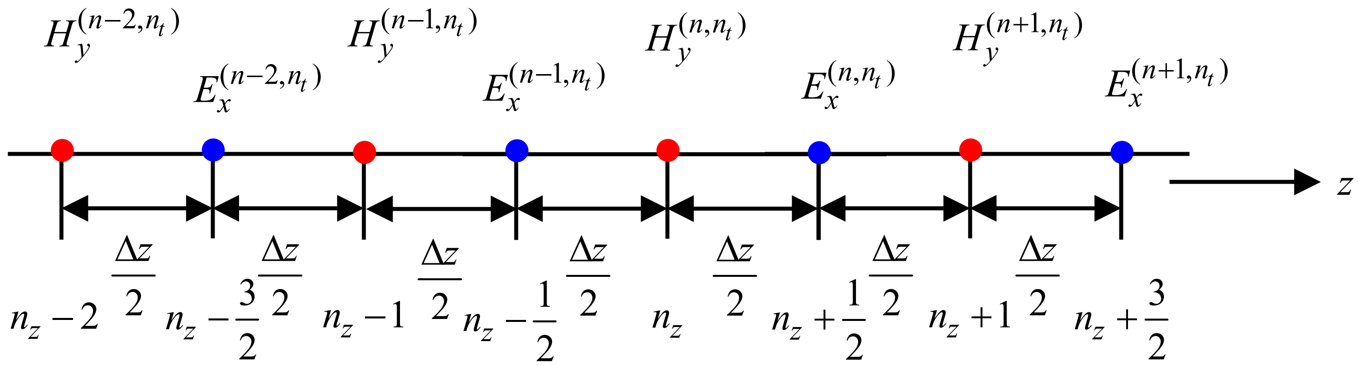
$$J_{\mathrm{mx}} = J_{\mathrm{m\,ref}}\hat{J}_{\mathrm{mx}} \qquad J_{\mathrm{m\,ref}} = \frac{\mu_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}}H_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}c_{\mathrm{ref}}}$$

$$\hat{H}_y^{(n_z,n_t)} = \hat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\hat{E}_x^{(n_z+1/2,n_t-1/2)} - \hat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\hat{J}_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

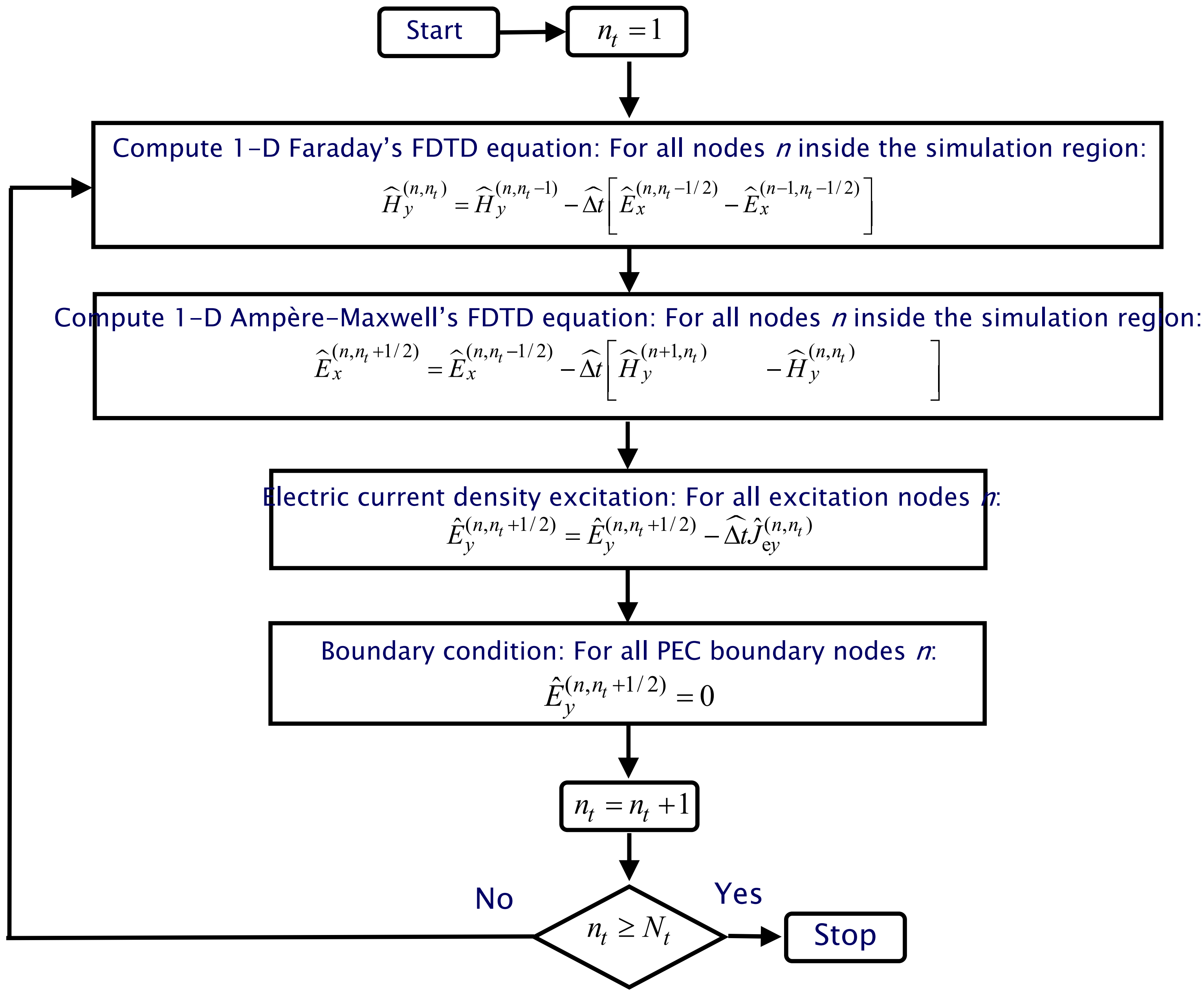
$$\hat{E}_x^{(n_z+1/2,n_t+1/2)} = \hat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\hat{H}_y^{(n_z+1,n_t)} - \hat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\hat{J}_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

1-D FDTD – Staggered Grid in Space – Global Node Numbering / 1D-FDTD – Versetztes Gitter im Raum – Globale Knotennummerierung

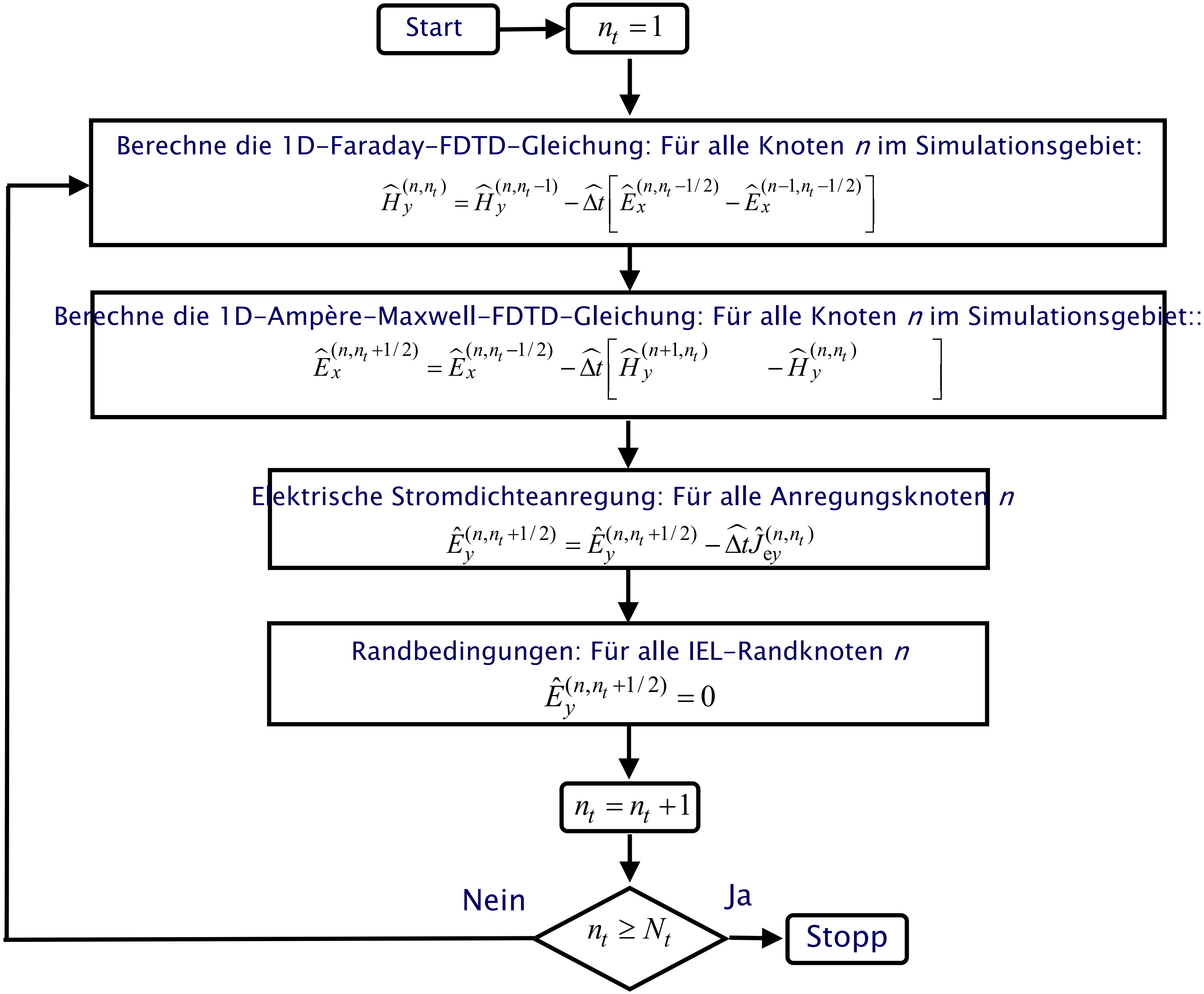




1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



1–D FDTD Algorithm – Flow Chart / 1D–FDTD–Algorithmus – Flussdiagramm



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Maxwell's equations / Maxwellsche Gleichungen

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t) \quad \text{for / für} \quad \begin{cases} 0 \le z \le Z \\ 0 \le t \le T \end{cases}$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

Initial condition / Anfangsbedingung

$$H_y(z,t) = J_{\mathrm{my}}(z,t) = 0 \qquad t \le 0$$

$$E_x(z,t) = J_{\mathrm{ex}}(z,t) = 0 \qquad t \le 0$$

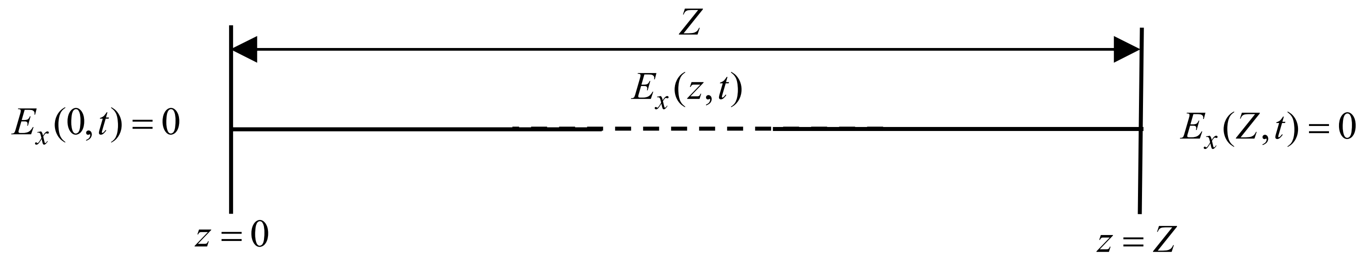
$$J_{\mathrm{ex}}(z,t) = K_{\mathrm{e0}}(z_0)\,\delta(z - z_0)\,f(t) \qquad t > 0$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x(0,t) &= 0 \\ E_x(Z,t) &= 0 \end{aligned}\right\} \quad \forall t$$

Hyperbolic initial-boundary-value problem / Hyperbolisches Anfangs-Randwert-Problem



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Discrete 1-D FDTD equations / Diskrete 1D-FDTD-Gleichungen

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\widehat{J}_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\widehat{J}_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

for / für $\begin{cases} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{cases}$

Initial condition / Anfangsbedingung

$$H_y^{(n_z,n_t)} = J_{\mathrm{my}}^{(n_z,n_t)} = 0 \qquad n_t \le 1$$

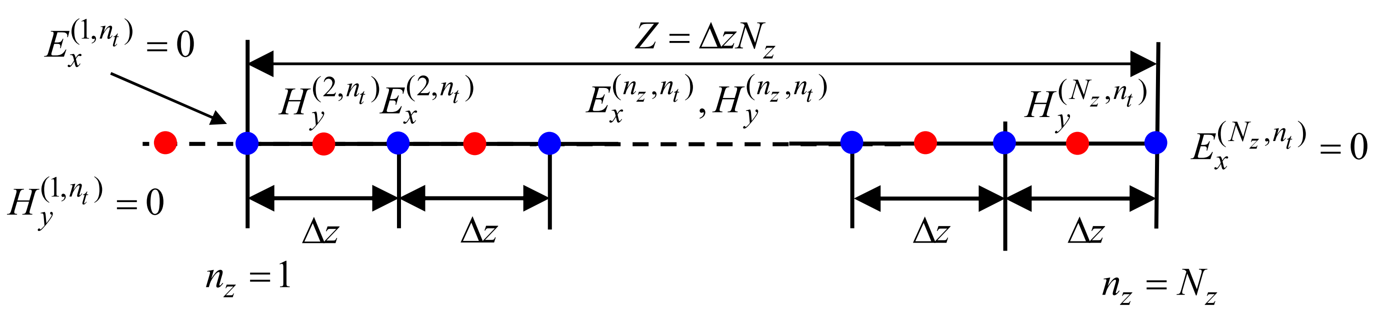
$$E_x^{(n_z,n_t)} = J_{\mathrm{ex}}^{(n_z,n_t)} = 0 \qquad n_t \le 1$$

$$J_{\mathrm{ex}}^{(n_z,n_t)} = K_{\mathrm{ex}}^{(n_{z0})}\,\delta^{(n_z-n_{z0})}\,f^{(n_t)} \qquad n_t > 1$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

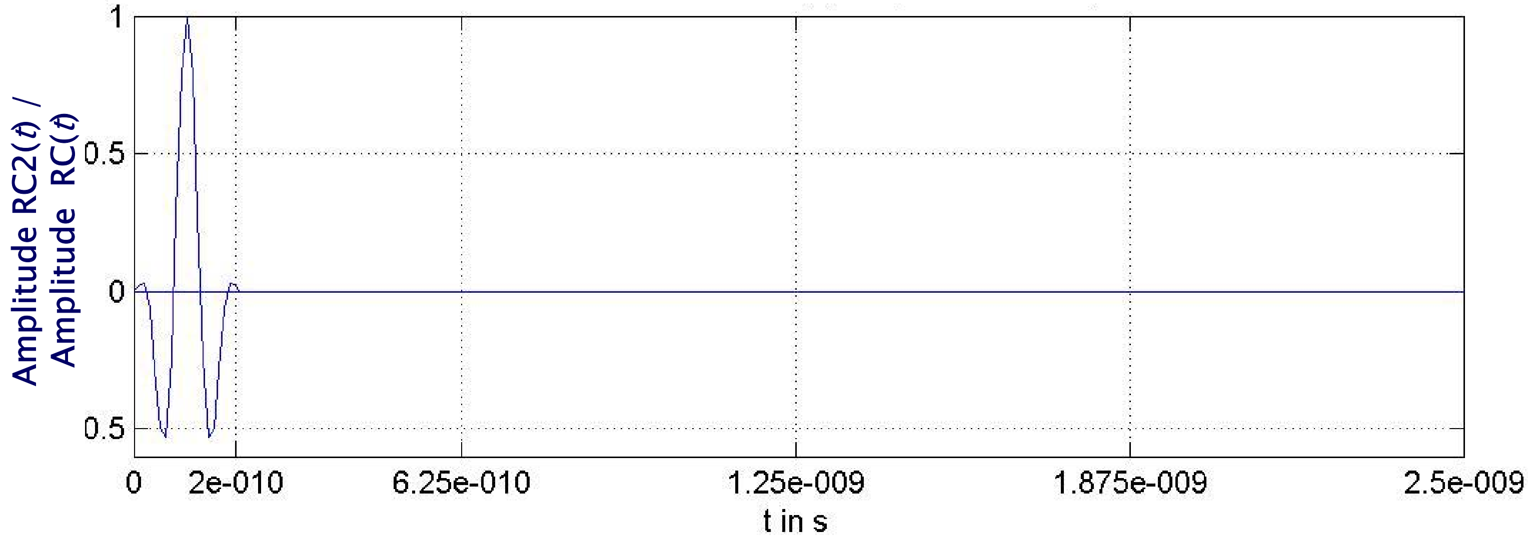
$$\left.\begin{array}{l} E_x^{(1,n_t)} = 0 \\ E_x^{(N_z,n_t)} = 0 \end{array}\right\} \quad 1 \le n_t \le N_t$$



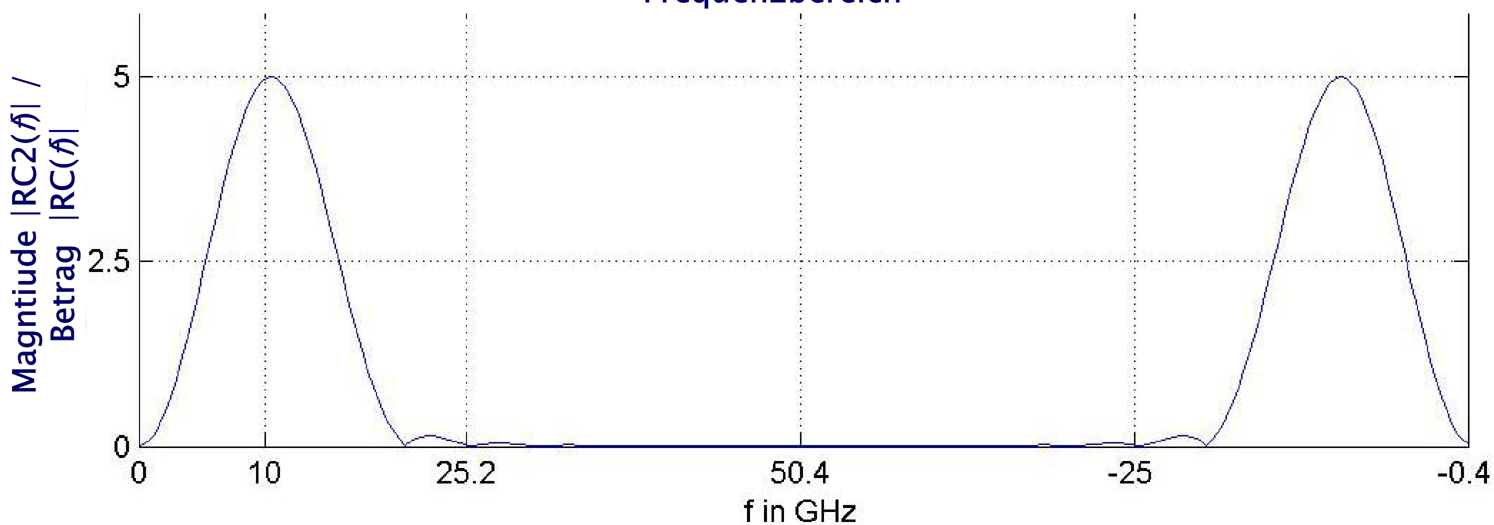
Discrete hyperbolic initial-boundary-value problem / Diskretes hyperbolisches Anfangs-Randwert-Problem

FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

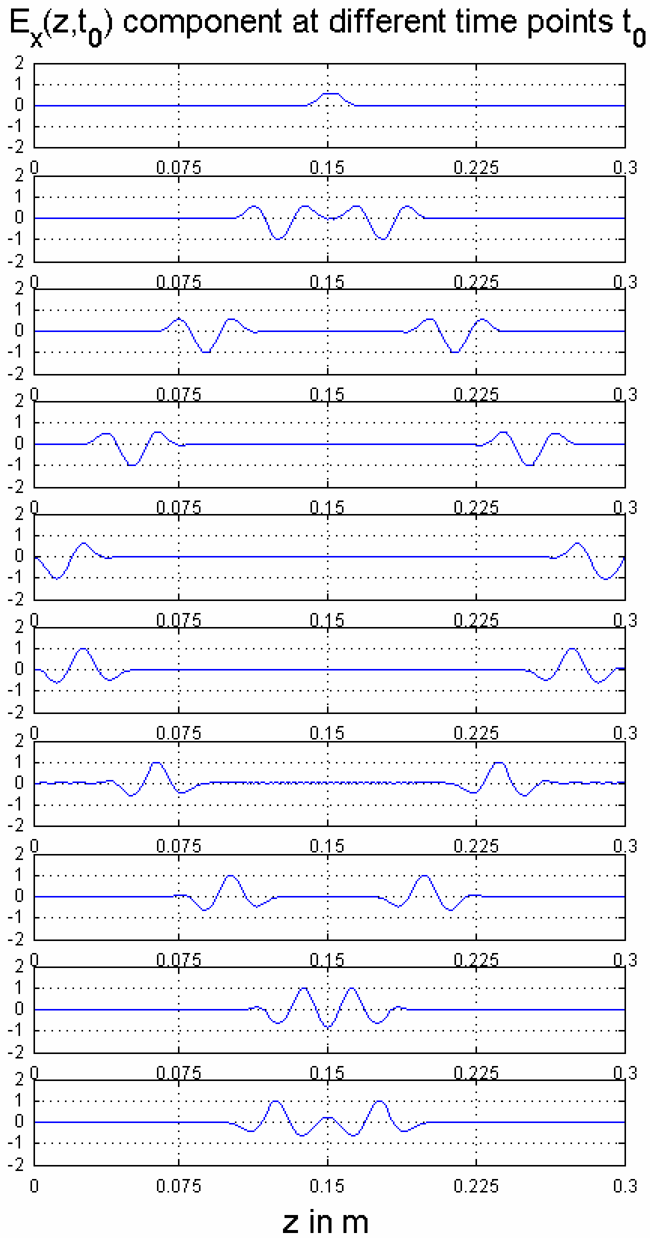
Excitation pulse: RC2(*t*) – Time Domain / Anregungsfunktion: RC2(*t*) – Zeitbereich

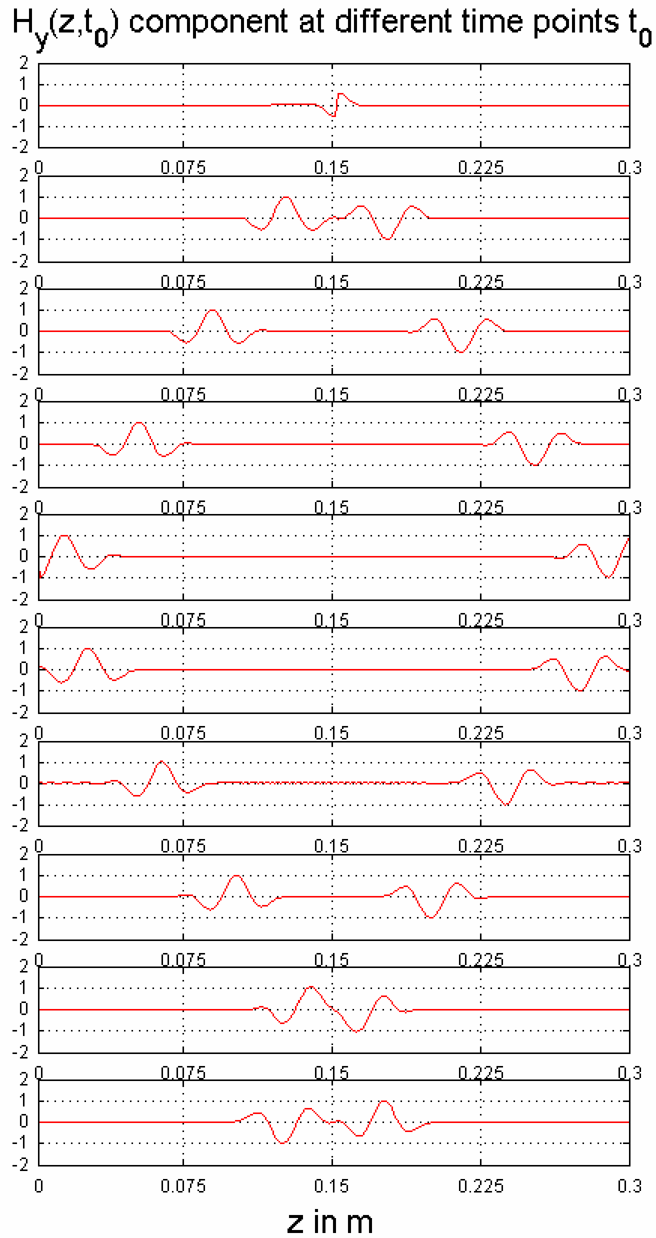


Excitation pulse: RC2(*f*) – Frequency Domain / Anregungsfunktion: RC(*f*) – Frequenzbereich

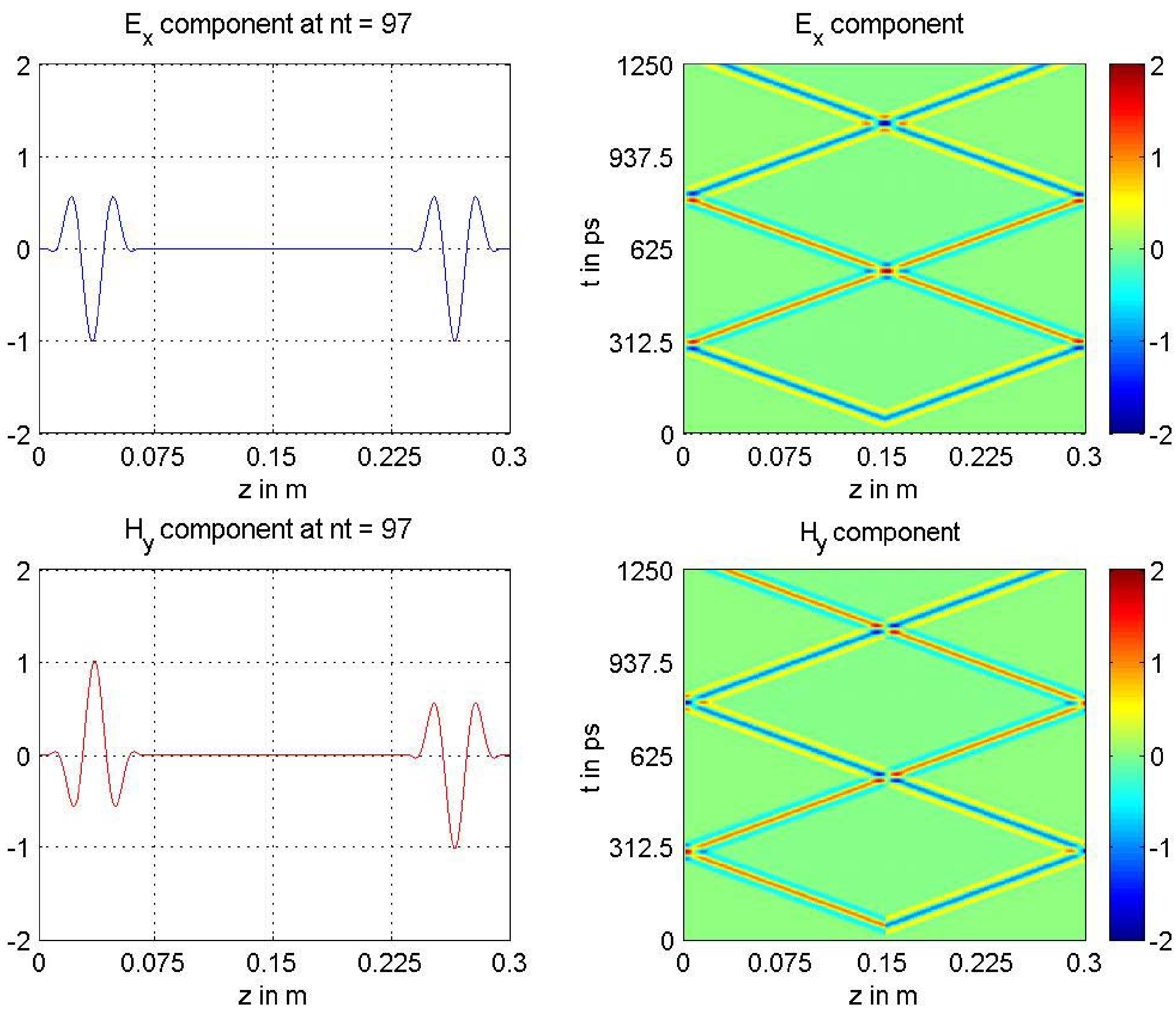


FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen





FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



Implementation of Boundary Conditions / Implementierung von Randbedingungen

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

Absorbing/open boundary condition / Absorbierende/offene Randbedingung

Space–time–extrapolation of the first order / Raum–Zeit–Extrapolation der ersten Ordnung

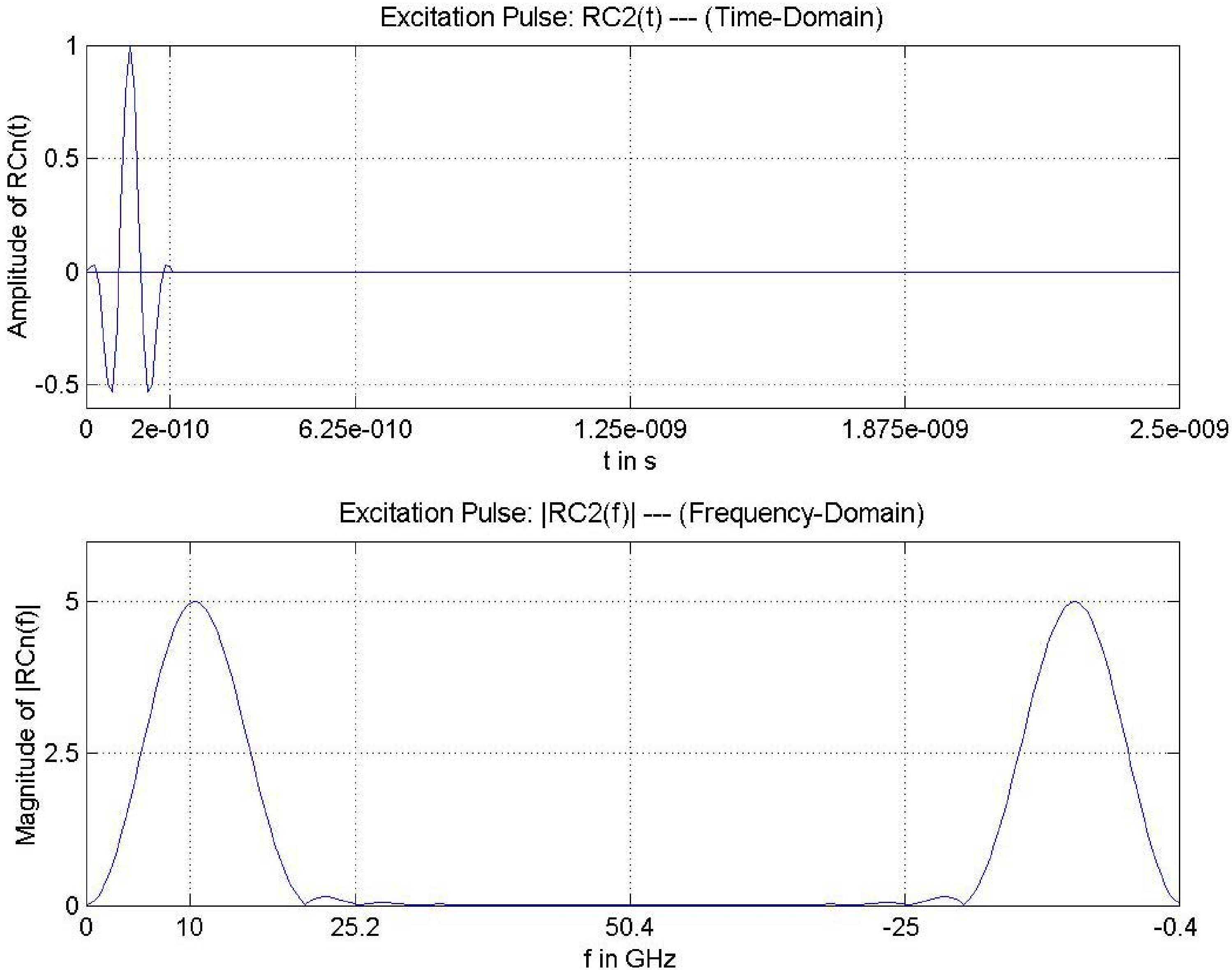
For / Für $\widehat{\Delta t} = 0.5$

a plane wave needs two time steps, 2 n_t, to travel over one grid cell with the size Δz / braucht eine ebene Welle zwei Zeitschritte, 2 n_t, um sich über eine Gitterzelle der Größe Δz auszubreiten

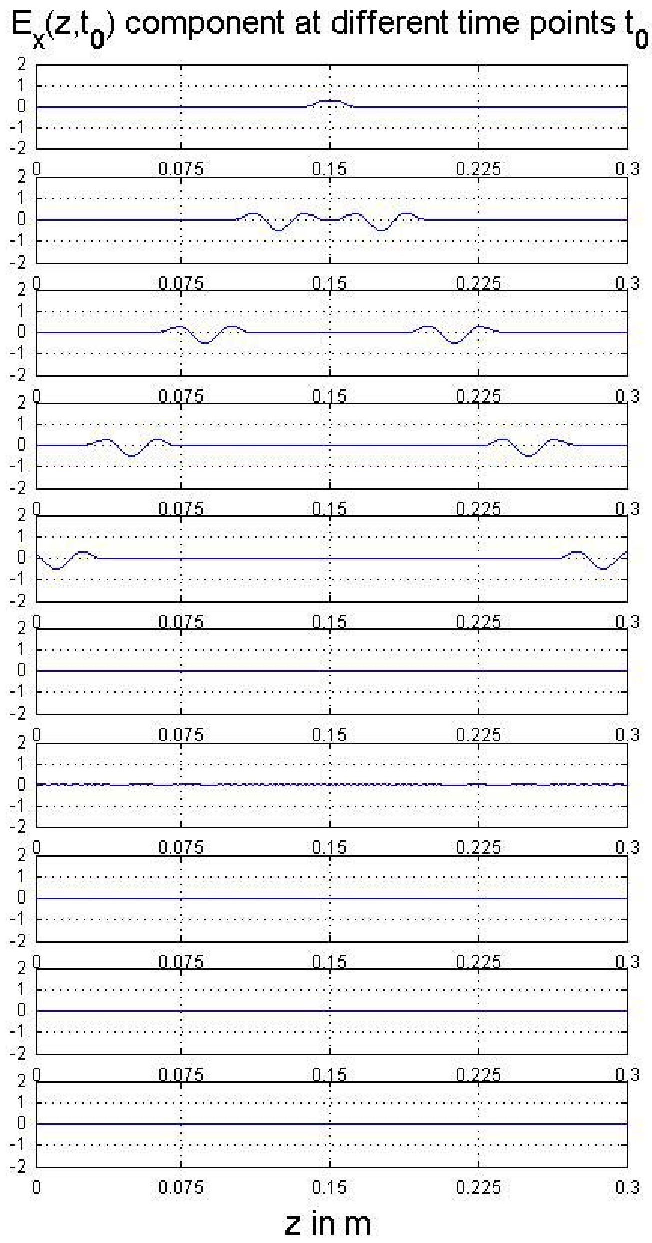
$$\left.\begin{aligned} E_x^{(1,n_t)} &= E_x^{(2,n_t-2)} \\ E_x^{(N_z,n_t)} &= E_x^{(N_z-1,n_t-2)} \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

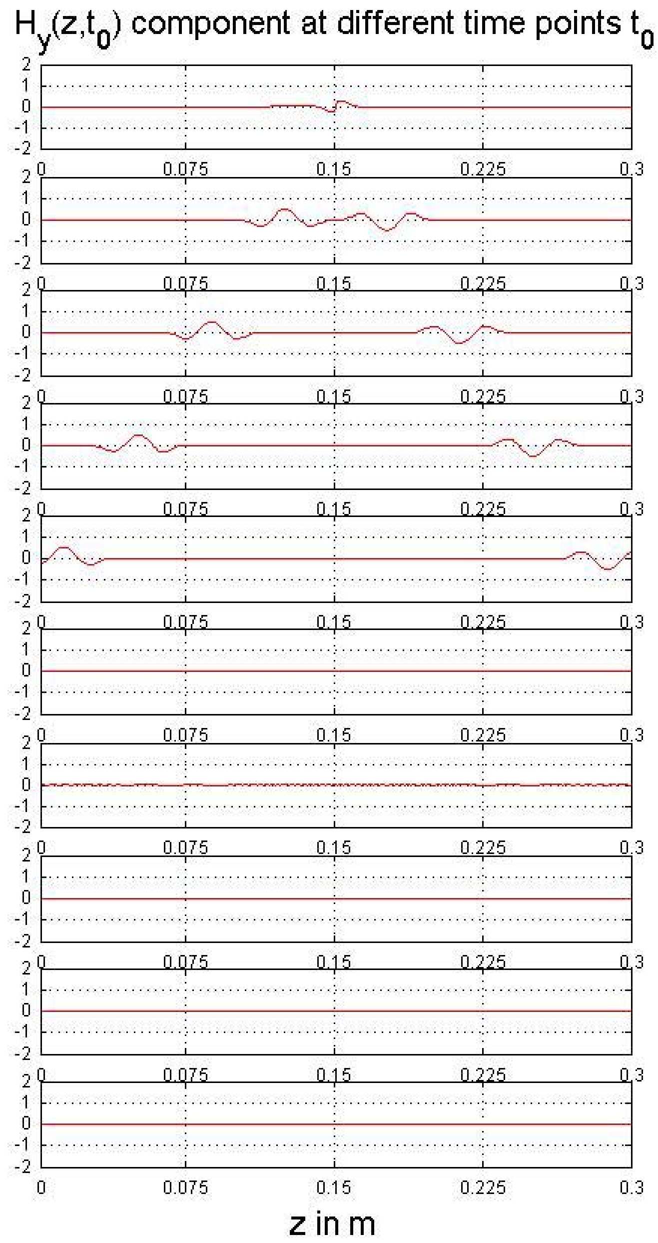
Space–time–extrapolation of the first order / Raum–Zeit–Extrapolation der ersten Ordnung

FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

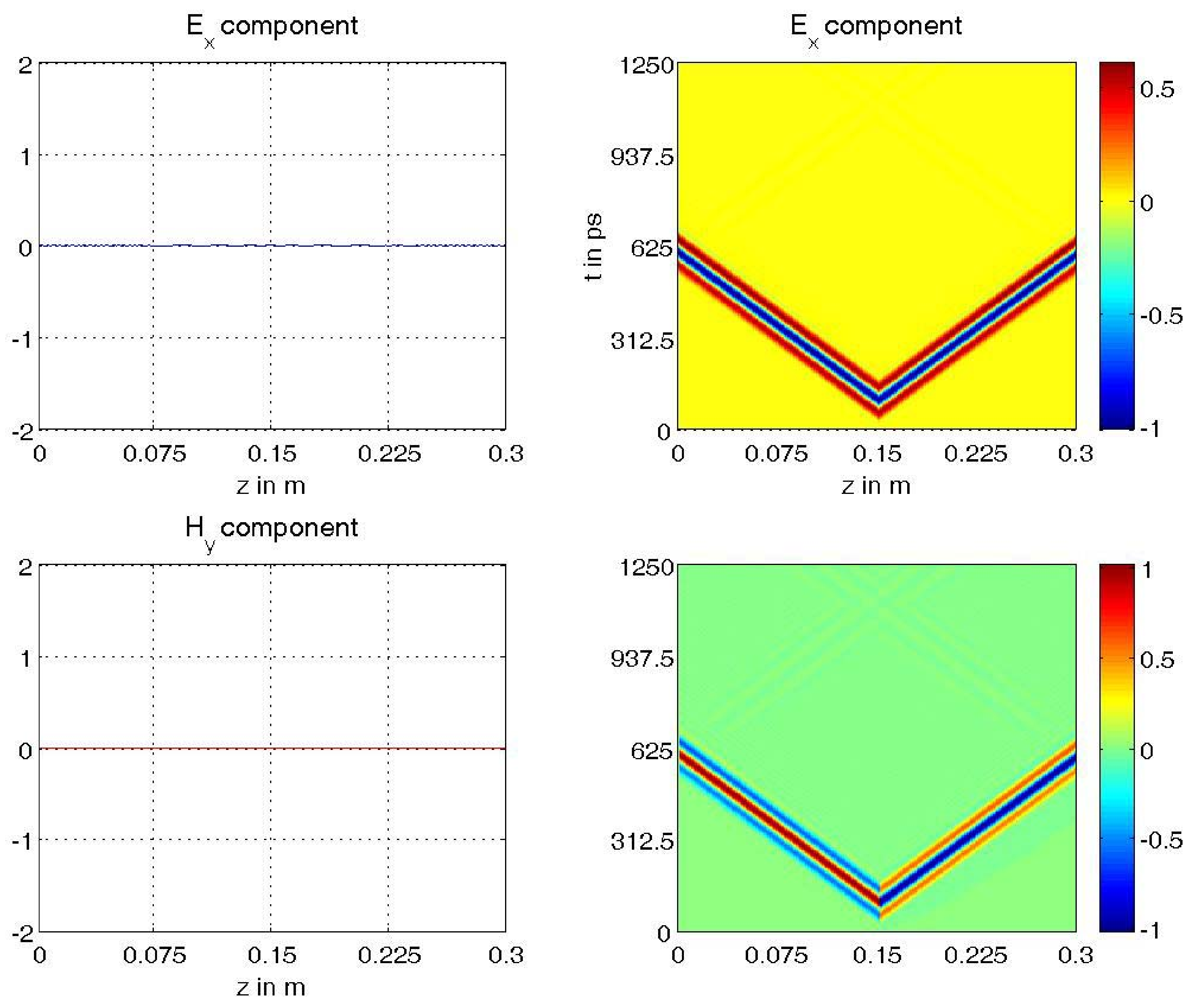


FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

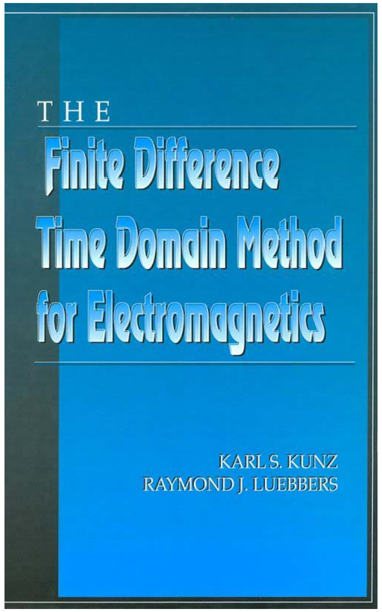




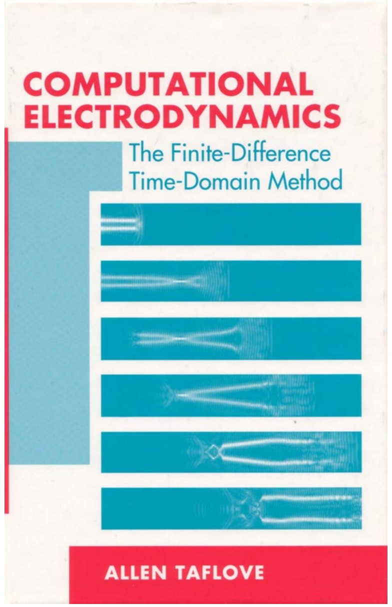
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



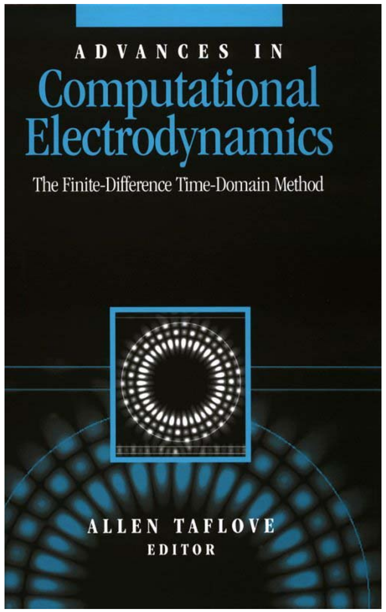
FDTD Books / FDTD-Bücher



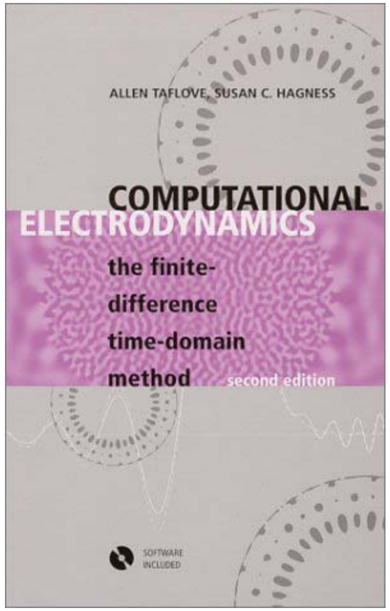
Kunz, K. S., Luebbers, R. J.: *The Finite Difference Time Domain Method for Electromagnetics*. 1993



Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method.* Artech House, Boston, 1995.

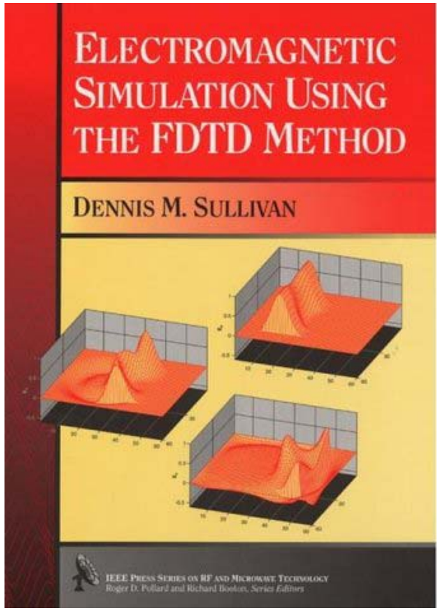


Taflove, A. (Editor): *Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Artech House, 1998.



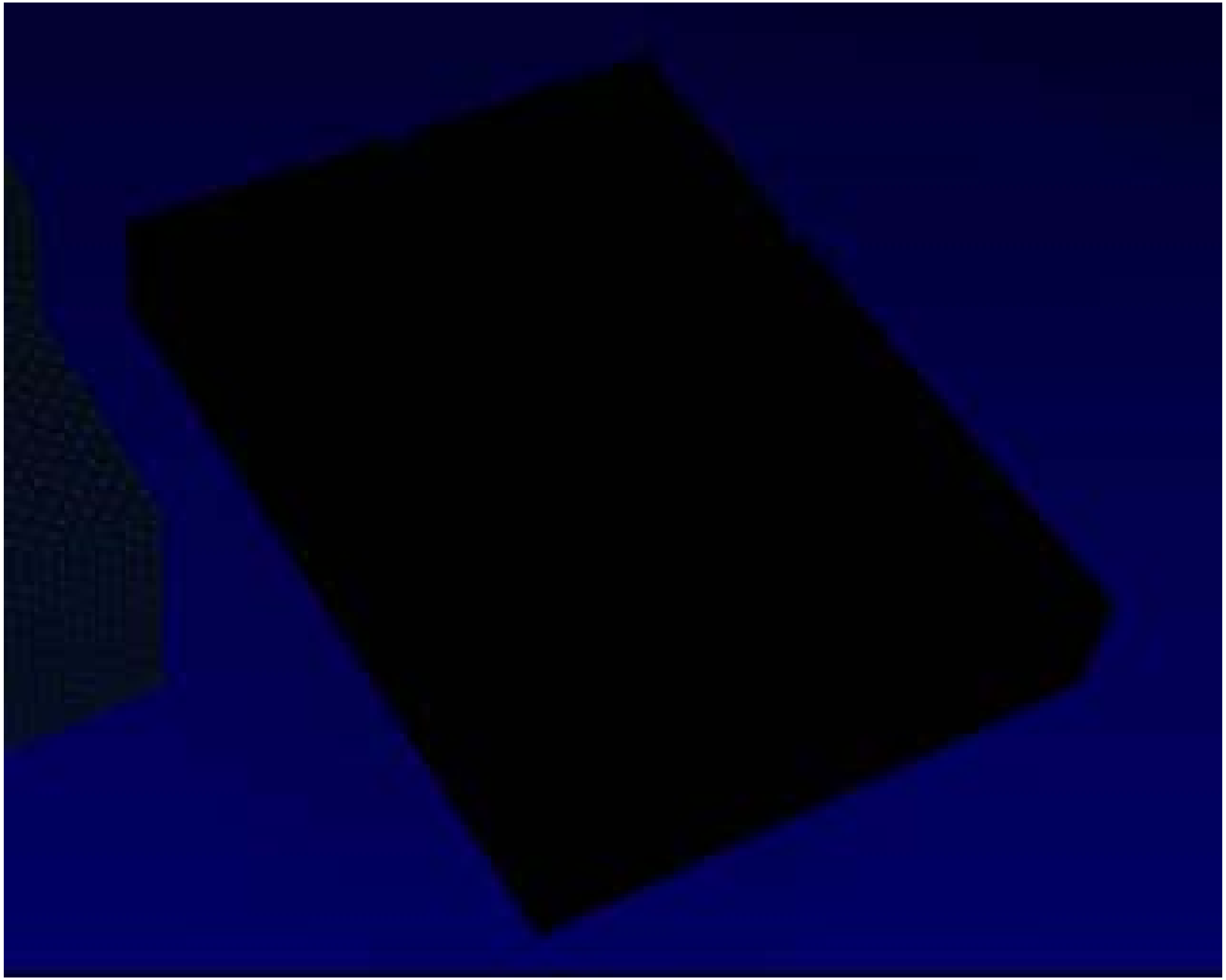
Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. 2nd Editon, Artech House, Boston, 2000.

FDTD Books / FDTD-Bücher

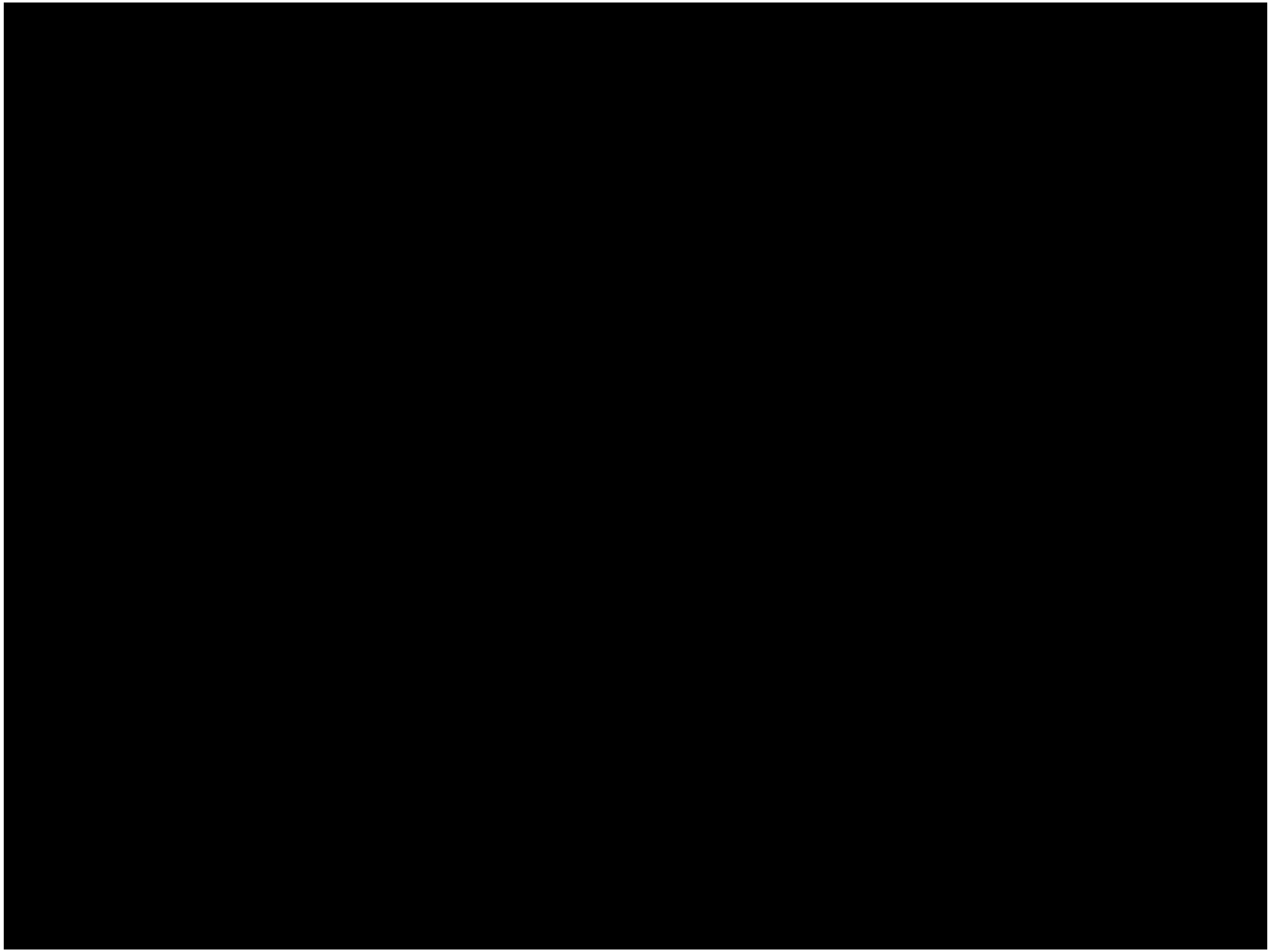


Sullivan, D. M.: *Electromagnetic Simulation Using the FDTD Method*. IEEE Press, New York, 2000.

2-D TM FDTD – Photonic Crystals /
2D-TM-FDTD – Photonische Kristalle



2-D TM FDTD – Photonic Crystals /
2D-TM-FDTD – Photonische Kristalle



End of Lecture 5 /
Ende der 5. Vorlesung