Numerical Methods of Electromagnetic Field Theory I (NFT I) Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

6th Lecture / 6. Vorlesung

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EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

Equations of first order /
Gleichungen der ersten Ordnung

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underbrace{\begin{aligned}\frac{\partial}{\partial t}\mu\underline{\mathbf{H}}(\underline{\mathbf{R}},t) &= -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\\ \frac{\partial}{\partial t}\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\end{aligned}}_{f\left(\underline{\mathbf{H}},\underline{\mathbf{E}}\right)}$$

Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

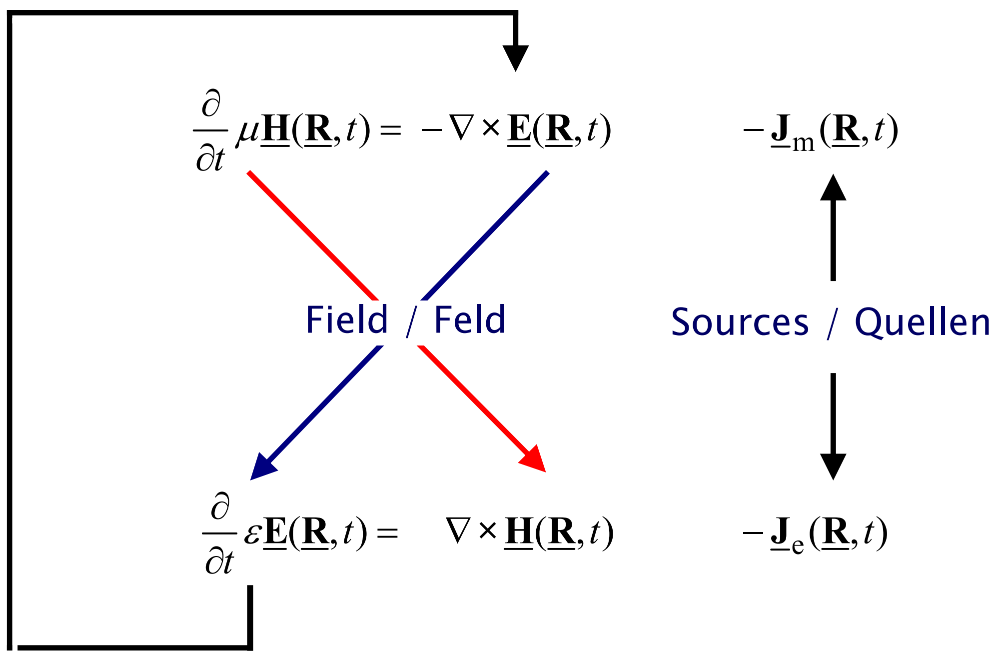
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \nu_0\,\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underbrace{\begin{aligned}\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) &= -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\\ \frac{\partial}{\partial t}\left[\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] &= \nabla\times\left[\nu\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right] - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\end{aligned}}_{f\left(\underline{\mathbf{B}},\underline{\mathbf{E}}\right)}$$

EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms



$$\frac{\partial}{\partial t}\mu\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) \quad -\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

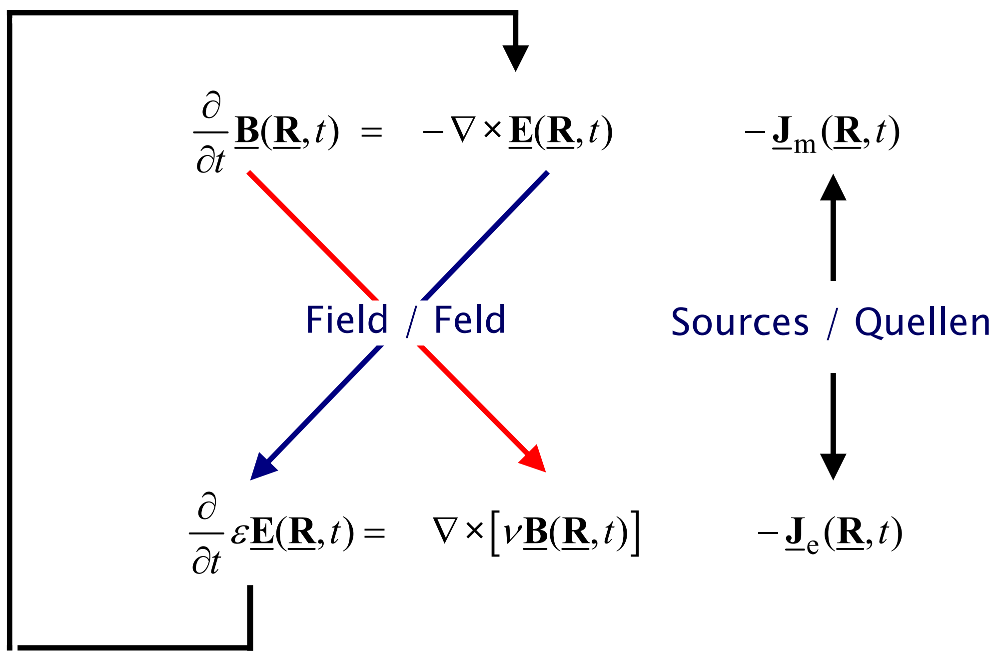
Faraday’s induction law /
Faradaysches Induktionsgesetz

$$\frac{\partial}{\partial t}\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) \quad -\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Ampère-Maxwell‘s circuital law /
Ampère-Maxwellsches
Durchflutungsgesetz

EM Wave Propagation – Finite-Difference Time-Domain (FDTD) /
EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms



$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) \quad -\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Faraday's induction law /
Faradaysches Induktionsgesetz

$$\frac{\partial}{\partial t}\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla\times\left[\nu\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right] \quad -\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Ampère-Maxwell's circuital law /
Ampère-Maxwellsches
Durchflutungsgesetz

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Constitutive Equations for Vacuum / Konstituierende Gleichungen (Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

Ansatz for the electric and magnetic field strength / Ansatz für die elektrische und magnetische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = E_x(z,t)\ \underline{\mathbf{e}}_x$$
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = H_y(z,t)\underline{\mathbf{e}}_y$$

The first two Maxwell's Equations are: / Die ersten beiden Maxwellschen Gleichungen lauten:

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z}E_x(z,t) - \frac{1}{\mu_0}J_{\mathrm{my}}(z,t)$$
$$\frac{\partial}{\partial t}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}H_y(z,t) - \frac{1}{\varepsilon_0}J_{\mathrm{ex}}(z,t)$$

$$\frac{d}{dt}f(t) = \frac{f\left(t+\frac{\Delta t}{2}\right) - f\left(t-\frac{\Delta t}{2}\right)}{\Delta t} + \mathrm{O}[(\Delta t)^2]$$
$$\frac{d}{dz}f(z) = \frac{f\left(z+\frac{\Delta z}{2}\right) - f\left(z-\frac{\Delta z}{2}\right)}{\Delta x} + \mathrm{O}[(\Delta x)^2]$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) \qquad -\frac{1}{\mu_0} J_{my}(z,t)$$

Field / Feld

Sources / Quellen

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) \qquad -\frac{1}{\varepsilon_0} J_{ex}(z,t)$$

1-D EM Wave Propagation – FDTD – Discretization of the 1st Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 1ten Gleichung

Spatial discretization of the 1st equation /
Räumliche Diskretisierung der 1ten Gleichung

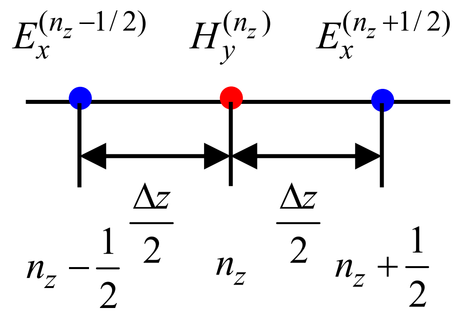
$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$

$$H_y : z \to n_z \Delta z, \qquad n_z = 1,\ldots,N_z$$

$$E_x : z \to \left(n_z + 1/2\right)\Delta z, \quad n_z = 1,\ldots,N_z$$

$$\frac{\partial}{\partial z} E_x(z,t) \;\to\; \left.\frac{\partial}{\partial z} E_x(z,t)\right|_z = \frac{1}{\Delta z}\left[E_x\left(z + \frac{\Delta z}{2}\right) - E_x\left(z - \frac{\Delta z}{2}\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\left[E_x^{(n_z)}\right]$$



$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

1-D EM Wave Propagation – FDTD – Discretization of the 2nd Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 2ten Gleichung

Spatial discretization of the 2nd equation / Räumliche Diskretisierung der 2ten Gleichung

$$\frac{\partial}{\partial t}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}H_y(z,t) - \frac{1}{\varepsilon_0}J_{\mathrm{ex}}(z,t)$$

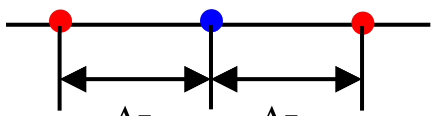
$$H_y : z \rightarrow n_z \Delta z, \qquad n_z = 1,\ldots,N_z$$

$$E_x : z \rightarrow \left(n_z + 1/2\right)\Delta z, \quad n_z = 1,\ldots,N_z$$

$$\frac{\partial}{\partial z}H_y(z,t) \;\rightarrow\; \frac{\partial}{\partial z}H_y(z,t)\Big|_{z+\frac{\Delta z}{2}} = \frac{1}{\Delta z}\left[H_y\left(z+\Delta z\right) - H_y\left(z\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\left[H_y^{(n_z+1/2)}\right]$$

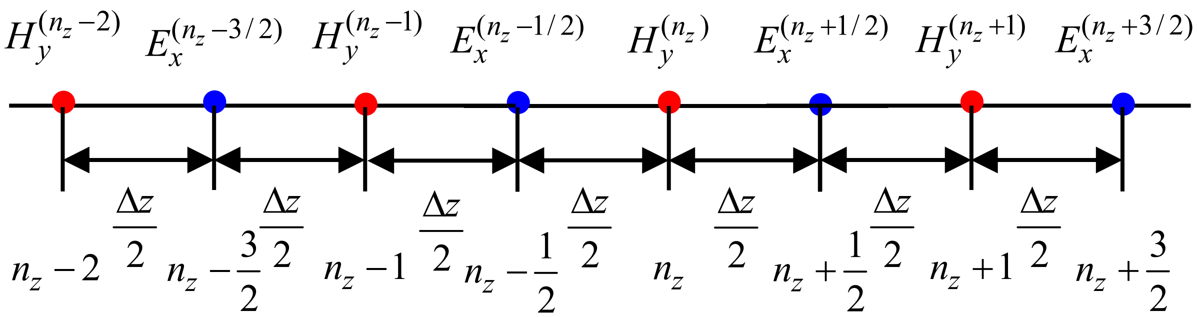
$$H_y^{(n_z)} \qquad E_x^{(n_z+1/2)} \qquad H_y^{(n_z+1)}$$

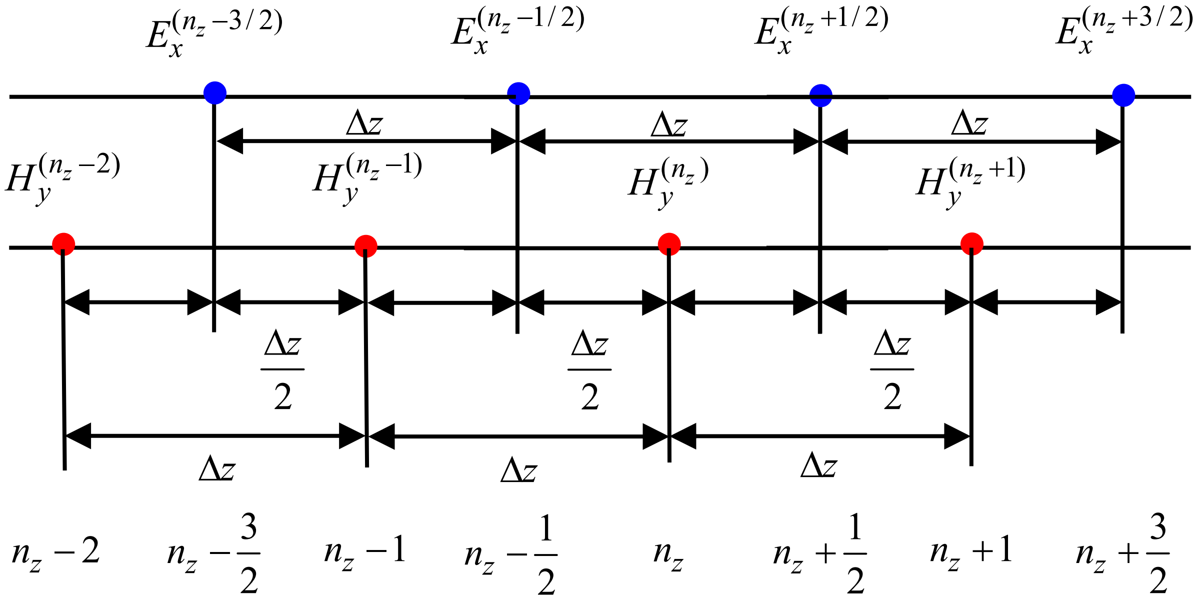


$$n_z \quad \frac{\Delta z}{2} \quad n_z + \frac{1}{2} \quad \frac{\Delta z}{2} \quad n_z + 1$$

$$\frac{\partial}{\partial t}E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0}J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum





1–D EM Wave Propagation – Finite–Difference Time–Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

$$\Downarrow$$

$$\frac{d}{dz} f(z) = \frac{1}{\Delta x}\left[f\left(z + \frac{\Delta z}{2}\right) - f\left(z - \frac{\Delta z}{2}\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\Downarrow$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2)}(t)$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = ?$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = ?$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

$$\frac{d}{dt} f(t) = \frac{1}{\Delta t}\left[f\left(t+\frac{\Delta t}{2}\right) - f\left(t-\frac{\Delta t}{2}\right)\right] + \mathrm{O}[(\Delta t)^2]$$

Staggered grid in time / Versetztes Gitter in der Zeit

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = \frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} + \mathrm{O}[(\Delta t)^2]$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = \frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t-1/2)}}{\Delta t} + \mathrm{O}[(\Delta t)^2]$$

$$\frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t-1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t+1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

Explicit 1-D FDTD algorithm on a staggered grid in space and time /
Expliziter 1D-FDTD-Algorithmus auf einem versetzten Gitter im Raum und Zeit

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t+1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2,n_t)}$$

FDTD: Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

1-D EM Wave Propagation – 1-D FDTD / 1D EM Wellenausbreitung – 1D FDTD

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

Explicit 1-D FDTD algorithm of leap-frog type on a staggered grid in space and time /
Expliziter 1D-FDTD-Algorithmus vom „Bocksprung"-Typ auf einem versetzten Gitter im Raum und Zeit

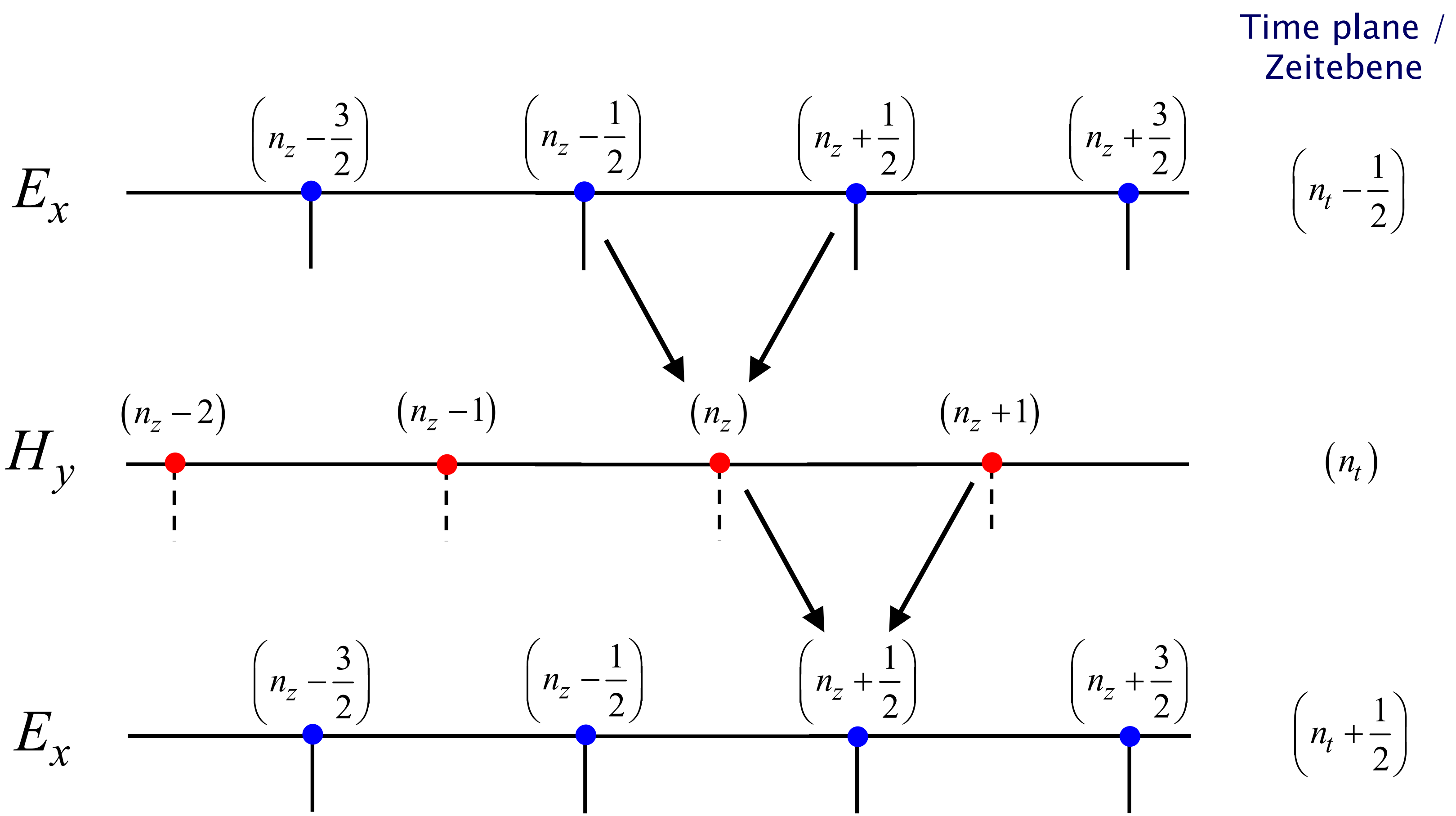
$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

FDTD: Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

1–D EM Wave Propagation – 1–D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1–D FDTD – Versetztes Gitter im Raum

Interleaving of the E_x and H_y field components in space and time in the 1–D FDTD formulation / Überlappung der E_x– und H_y–Feldkomponente in der 1D–FDTD–Formulierung im Raum und in der Zeit



1–D EM Wave Propagation – FDTD – Normalization / 1D EM Wellenausbreitung – FDTD – Normierung

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{my}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{ex}^{(n_z+1/2,n_t)}$$

$$\Delta t = \Delta t_{\text{ref}} \widehat{\Delta t} \qquad \Delta t_{\text{ref}} = \frac{\Delta x_{\text{ref}}}{c_{\text{ref}}} \qquad \Delta t = \frac{\Delta x_{\text{ref}}}{c_{\text{ref}}} \widehat{\Delta t}$$

$$\Delta z = \Delta x_{\text{ref}} \widehat{\Delta z} \qquad c = c_{\text{ref}} \hat{c} \qquad \varepsilon = \varepsilon_{\text{ref}} \hat{\varepsilon} \qquad \mu = \mu_{\text{ref}} \hat{\mu} \qquad \mu_{\text{ref}} = \mu_0$$

$$E_x = E_{\text{ref}} \hat{E}_x$$

$$H_y = H_{\text{ref}} \hat{H}_y \qquad H_{\text{ref}} = \frac{E_{\text{ref}}}{c_{\text{ref}} \mu_{\text{ref}}} = \frac{\sqrt{\varepsilon_{\text{ref}} \mu_{\text{ref}}}}{\mu_{\text{ref}}} E_{\text{ref}} = \sqrt{\frac{\varepsilon_{\text{ref}}}{\mu_{\text{ref}}}} E_{\text{ref}} = \frac{E_{\text{ref}}}{Z_{\text{ref}}}$$

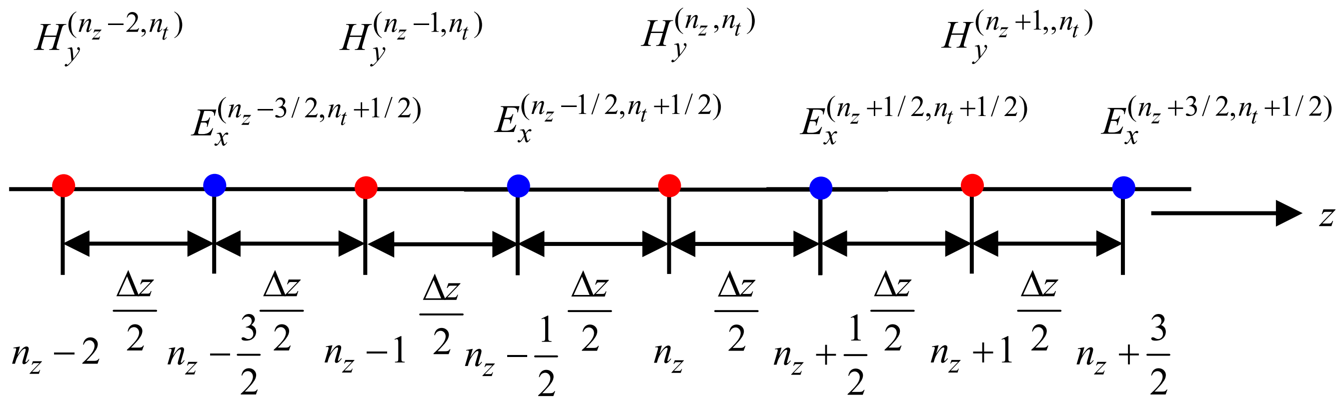
$$J_{\text{ex}} = J_{\text{e ref}} \hat{J}_{\text{ex}} \qquad J_{\text{e ref}} = \frac{\varepsilon_{\text{ref}}}{\Delta t_{\text{ref}}} E_{\text{ref}}$$

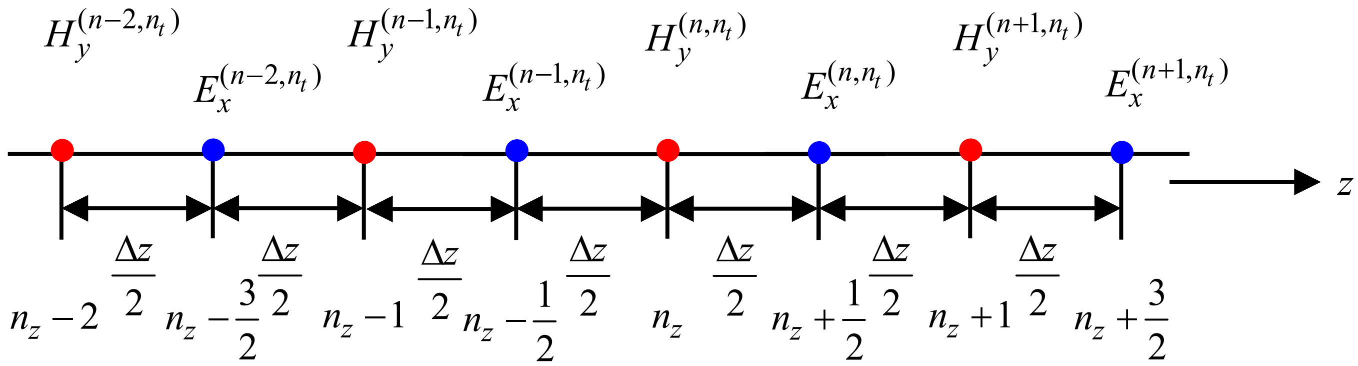
$$J_{\text{mx}} = J_{\text{m ref}} \hat{J}_{\text{mx}} \qquad J_{\text{m ref}} = \frac{\mu_{\text{ref}}}{\Delta t_{\text{ref}}} H_{\text{ref}} = \frac{E_{\text{ref}}}{\Delta t_{\text{ref}} c_{\text{ref}}}$$

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t} \hat{J}_{my}^{(n_z,n_t-1/2)}$$

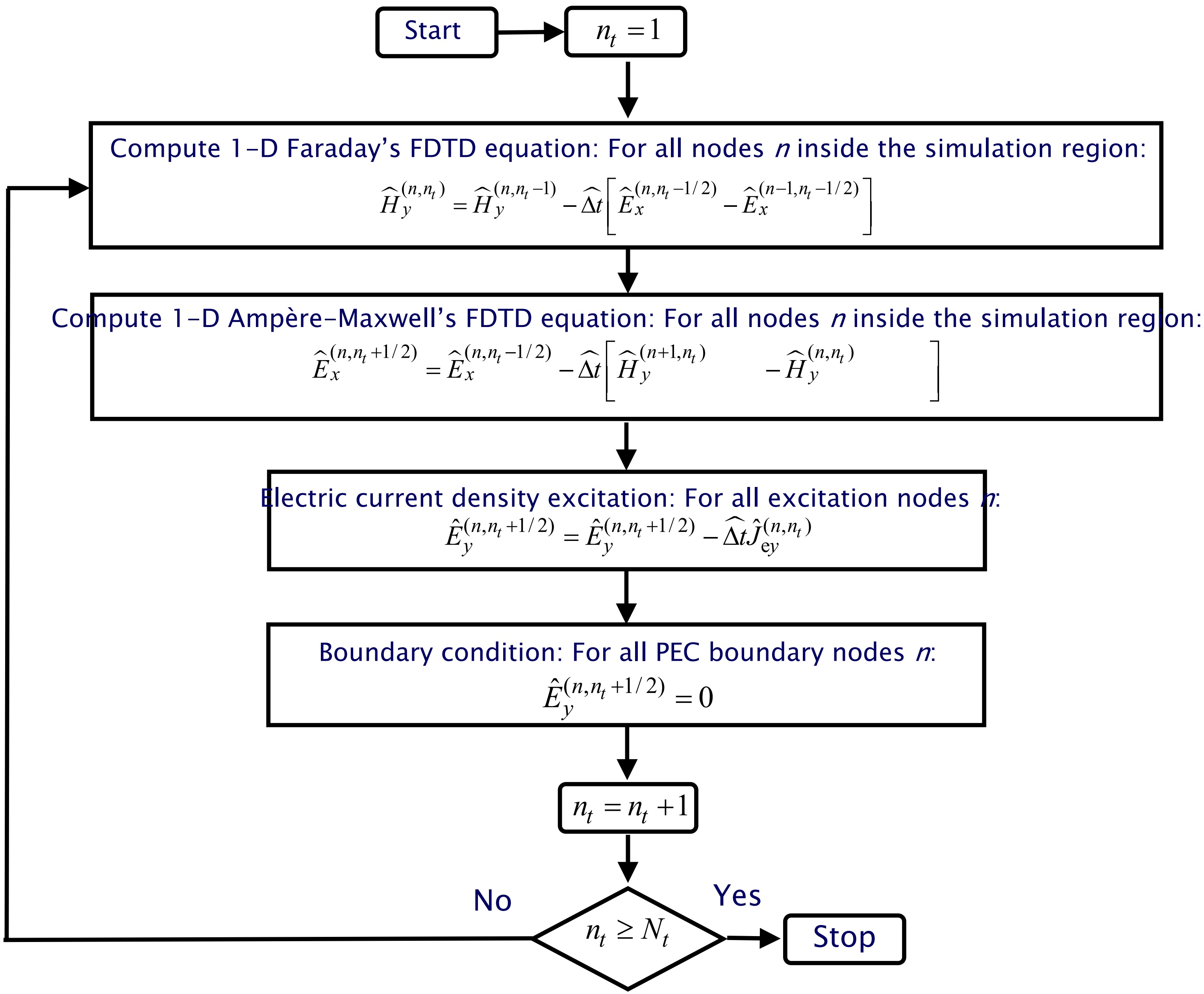
$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\, \hat{J}_{ex}^{(n_z+1/2,n_t)}$$

1–D FDTD – Staggered Grid in Space – Global Node Numbering / 1D–FDTD – Versetztes Gitter im Raum – Globale Knotennummerierung

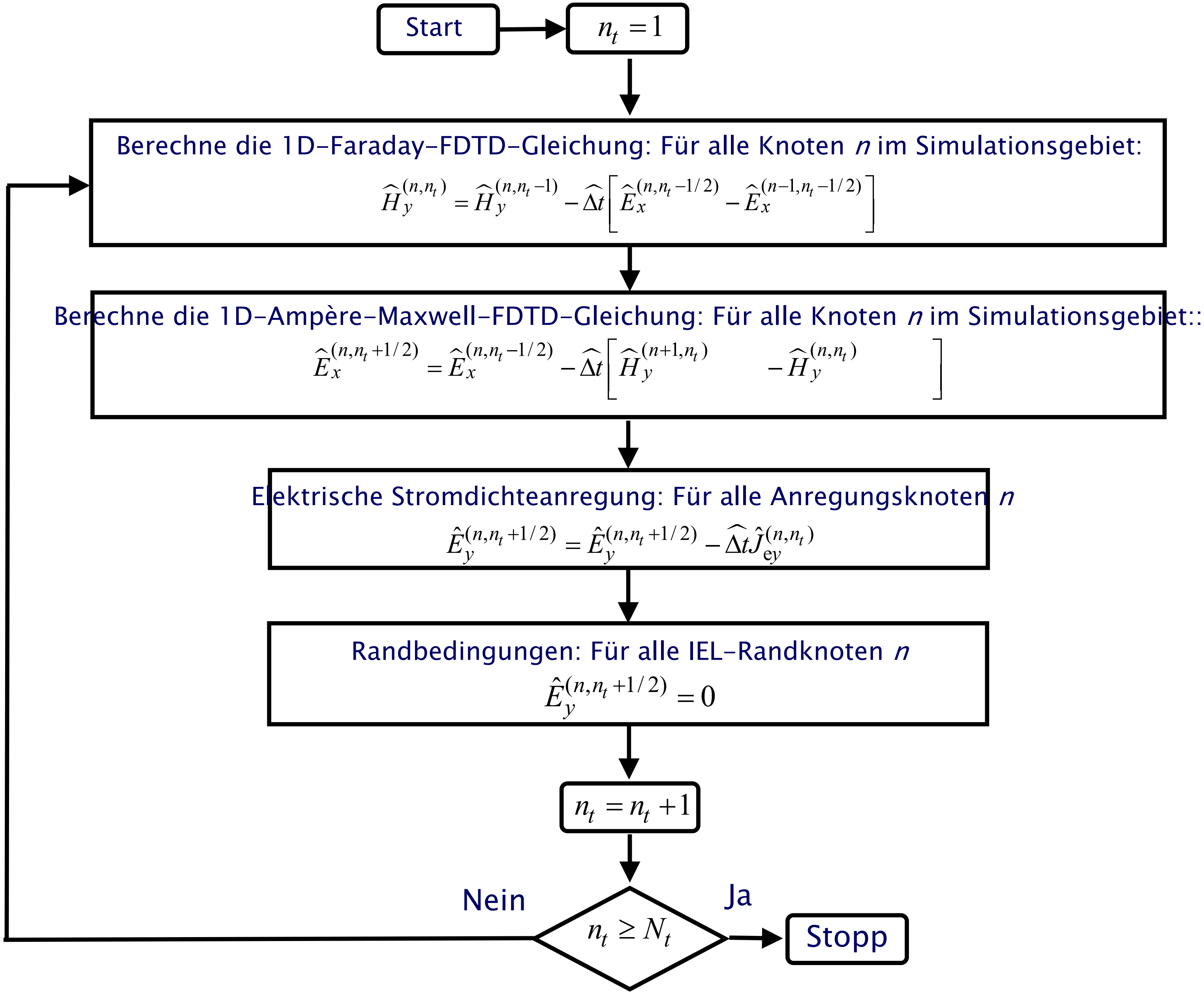




1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



1–D FDTD Algorithm – Flow Chart / 1D–FDTD–Algorithmus – Flussdiagramm



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Maxwell's equations / Maxwellsche Gleichungen

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{my}(z,t) \quad \text{for / für} \quad \begin{cases} 0 \le z \le Z \\ 0 \le t \le T \end{cases}$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{ex}(z,t)$$

Initial condition / Anfangsbedingung

$$H_y(z,t) = J_{my}(z,t) = 0 \qquad t \le 0$$

$$E_x(z,t) = J_{ex}(z,t) = 0 \qquad t \le 0$$

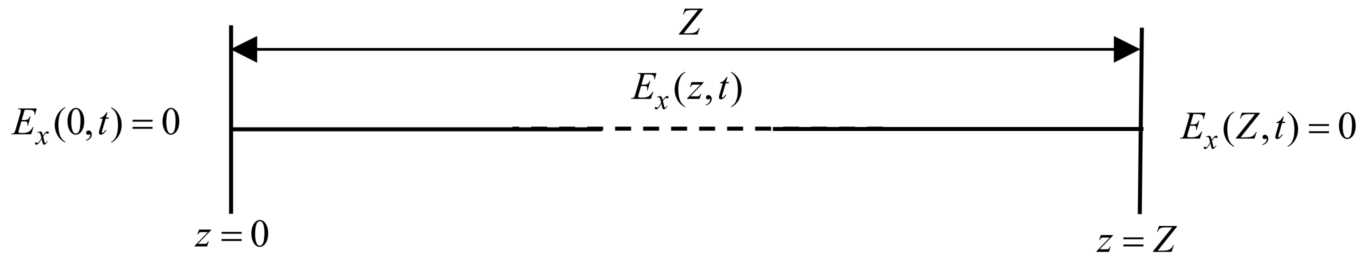
$$J_{ex}(z,t) = K_{e0}(z_0)\,\delta(z - z_0)\, f(t) \qquad t > 0$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{matrix} E_x(0,t) = 0 \\ E_x(Z,t) = 0 \end{matrix}\right\} \quad \forall t$$

Hyperbolic initial-boundary-value problem / Hyperbolisches Anfangs-Randwert-Problem



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Discrete 1-D FDTD equations / Diskrete 1D-FDTD-Gleichungen

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\widehat{J}_{\text{my}}^{(n_z,n_t-1/2)} \quad \text{for / für} \quad \begin{cases} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{cases}$$

$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\widehat{J}_{\text{ex}}^{(n_z+1/2,n_t)}$$

Initial condition / Anfangsbedingung

$$H_y^{(n_z,n_t)} = J_{\text{my}}^{(n_z,n_t)} = 0 \qquad n_t \le 1$$

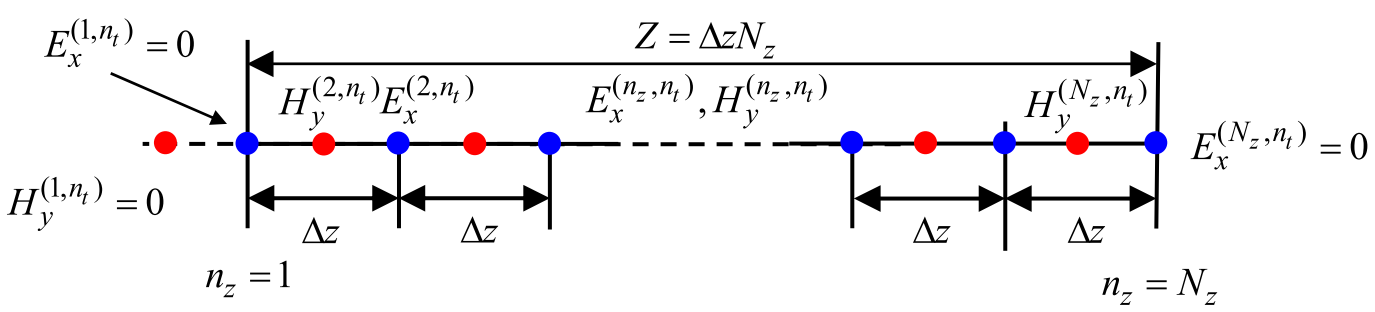
$$E_x^{(n_z,n_t)} = J_{\text{ex}}^{(n_z,n_t)} = 0 \qquad n_t \le 1$$

$$J_{\text{ex}}^{(n_z,n_t)} = K_{\text{ex}}^{(n_{z0})}\,\delta^{(n_z-n_{z0})}\,f^{(n_t)} \qquad n_t > 1$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

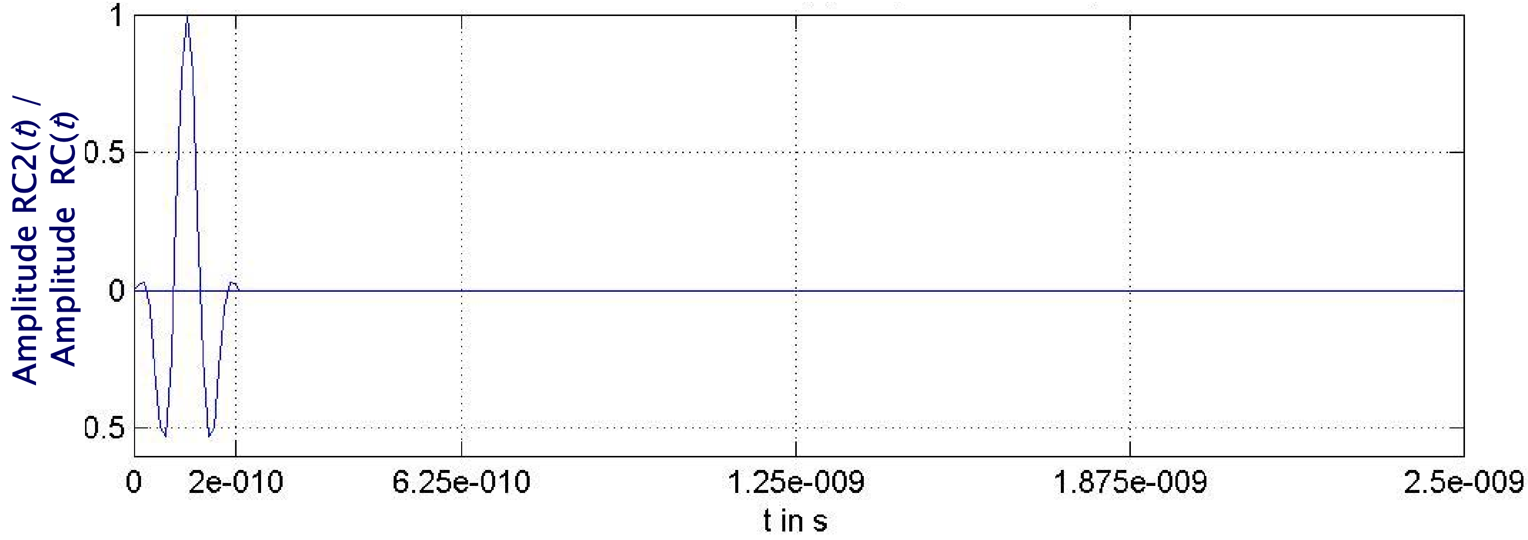
$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$



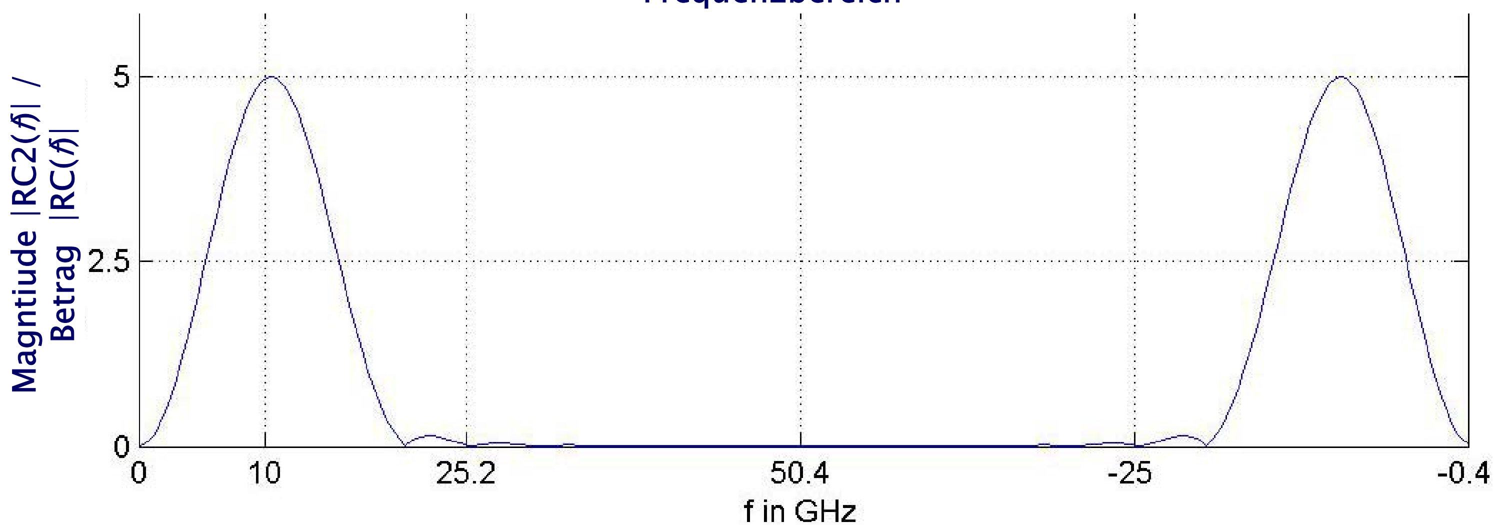
Discrete hyperbolic initial-boundary-value problem / Diskretes hyperbolisches Anfangs-Randwert-Problem

FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

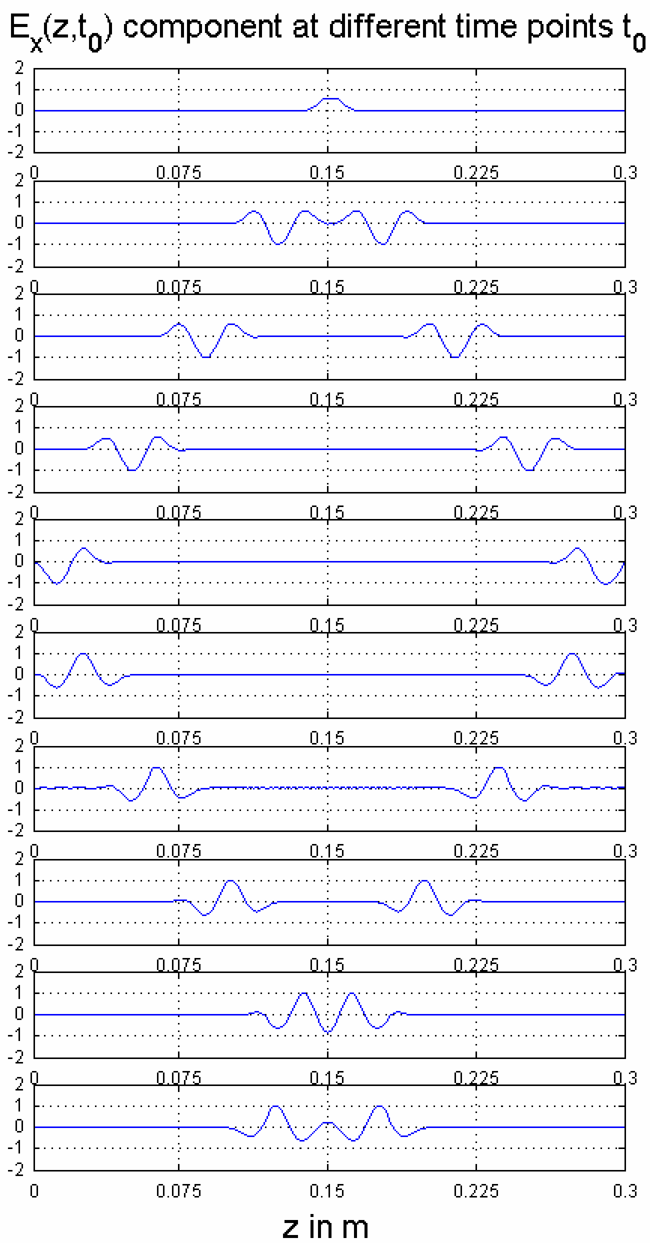
Excitation pulse: RC2(*t*) – Time Domain / Anregungsfunktion: RC2(*t*) – Zeitbereich

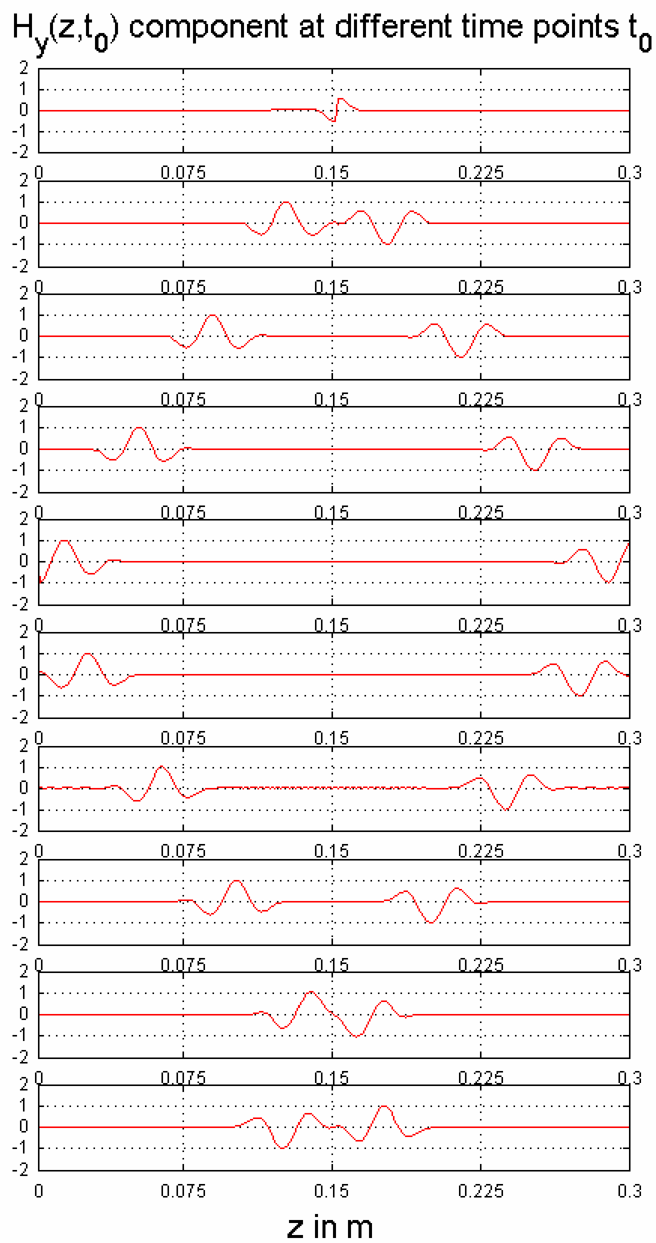


Excitation pulse: RC2(*f*) – Frequency Domain / Anregungsfunktion: RC(*f*) – Frequenzbereich

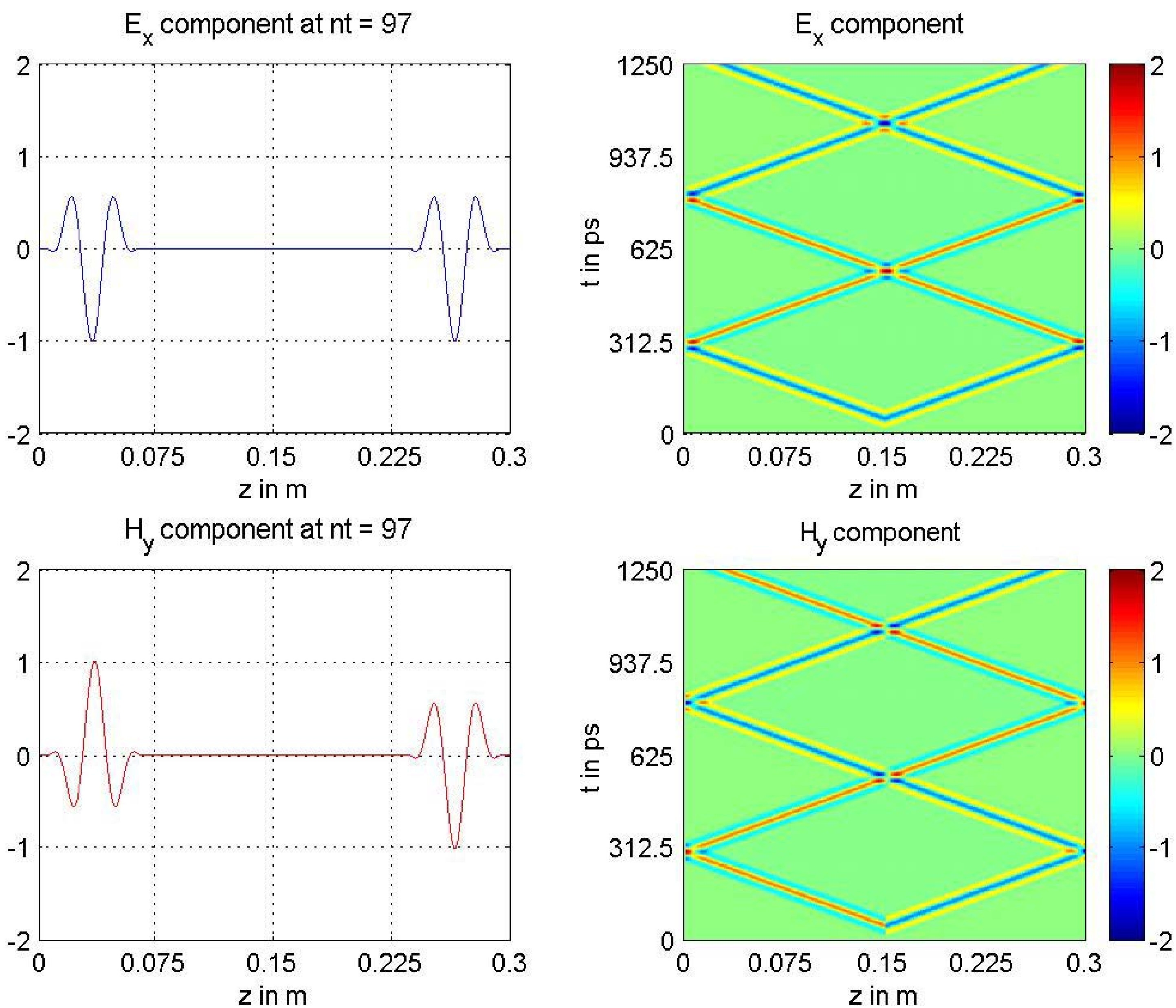


FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen





FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



Implementation of Boundary Conditions / Implementierung von Randbedingungen

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

Absorbing/open boundary condition / Absorbierende/offene Randbedingung

Space-time-extrapolation of the first order / Raum-Zeit-Extrapolation der ersten Ordnung

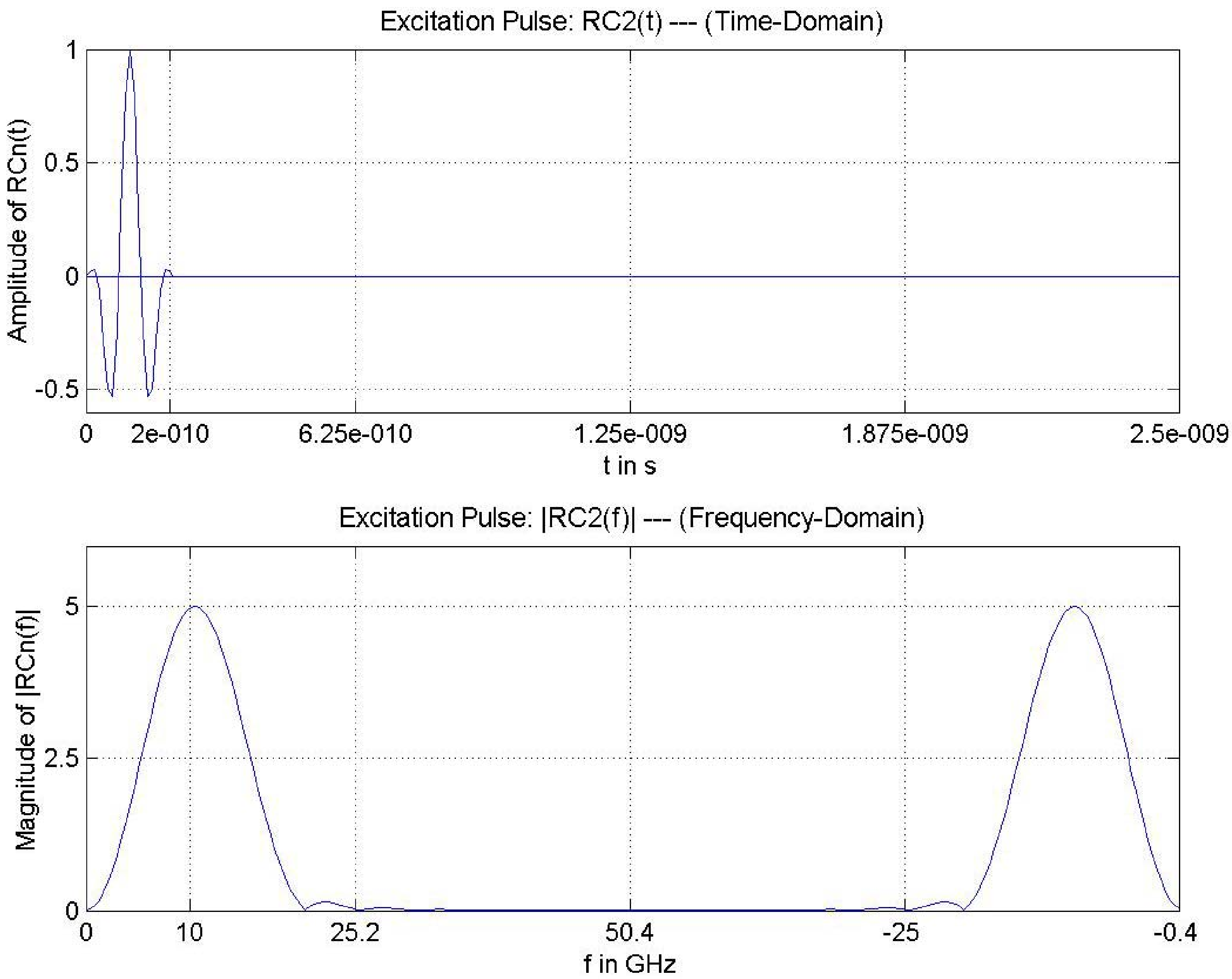
For / Für $\widehat{\Delta t} = 0.5$

a plane wave needs two time steps, 2 n_t, to travel over one grid cell with the size Δz / braucht eine ebene Welle zwei Zeitschritte, 2 n_t, um sich über eine Gitterzelle der Größe Δz auszubreiten

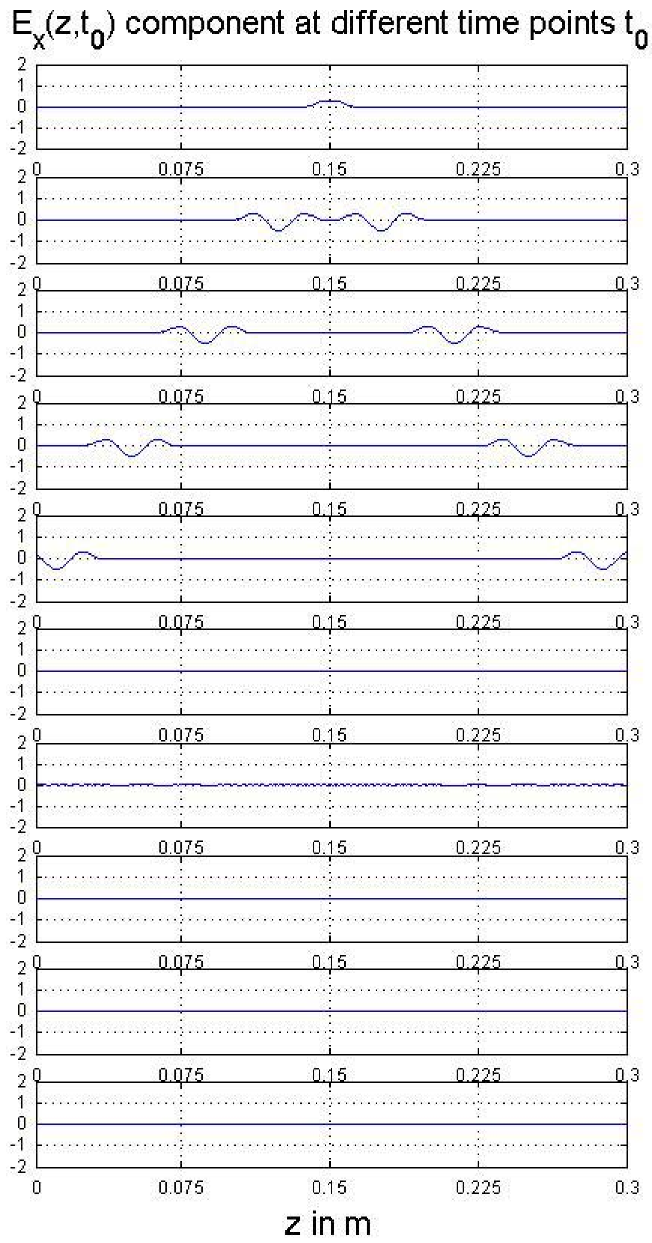
$$\left.\begin{aligned} E_x^{(1,n_t)} &= E_x^{(2,n_t-2)} \\ E_x^{(N_z,n_t)} &= E_x^{(N_z-1,n_t-2)} \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

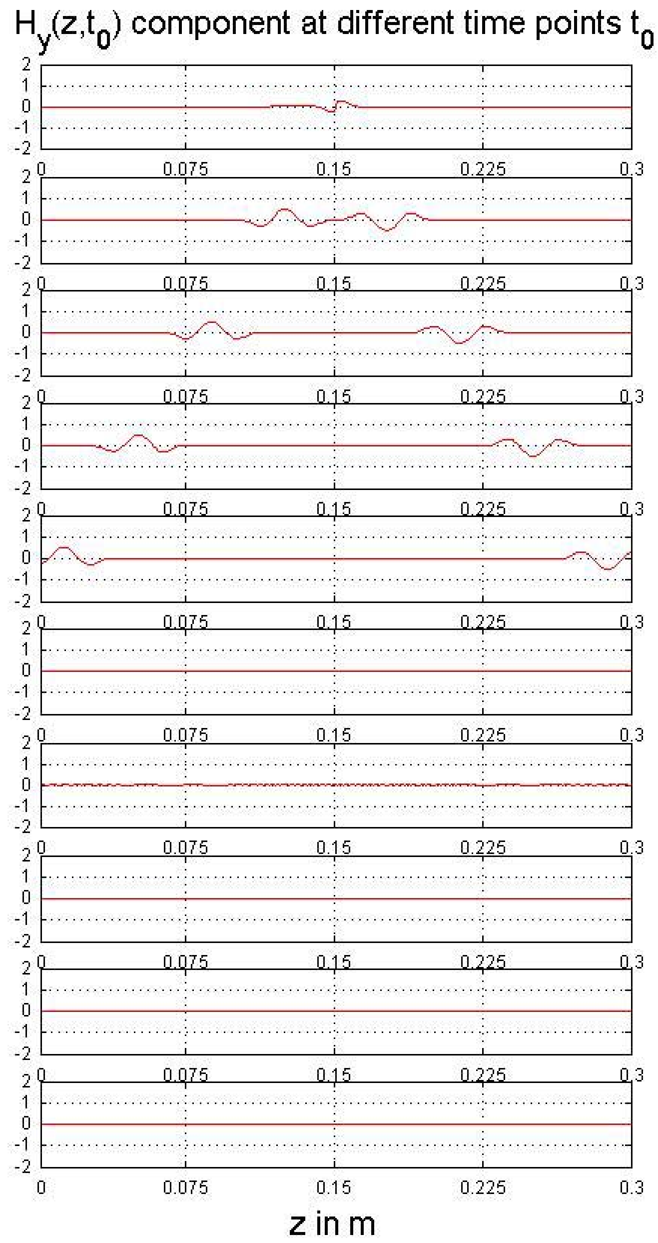
Space-time-extrapolation of the first order / Raum-Zeit-Extrapolation der ersten Ordnung

FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

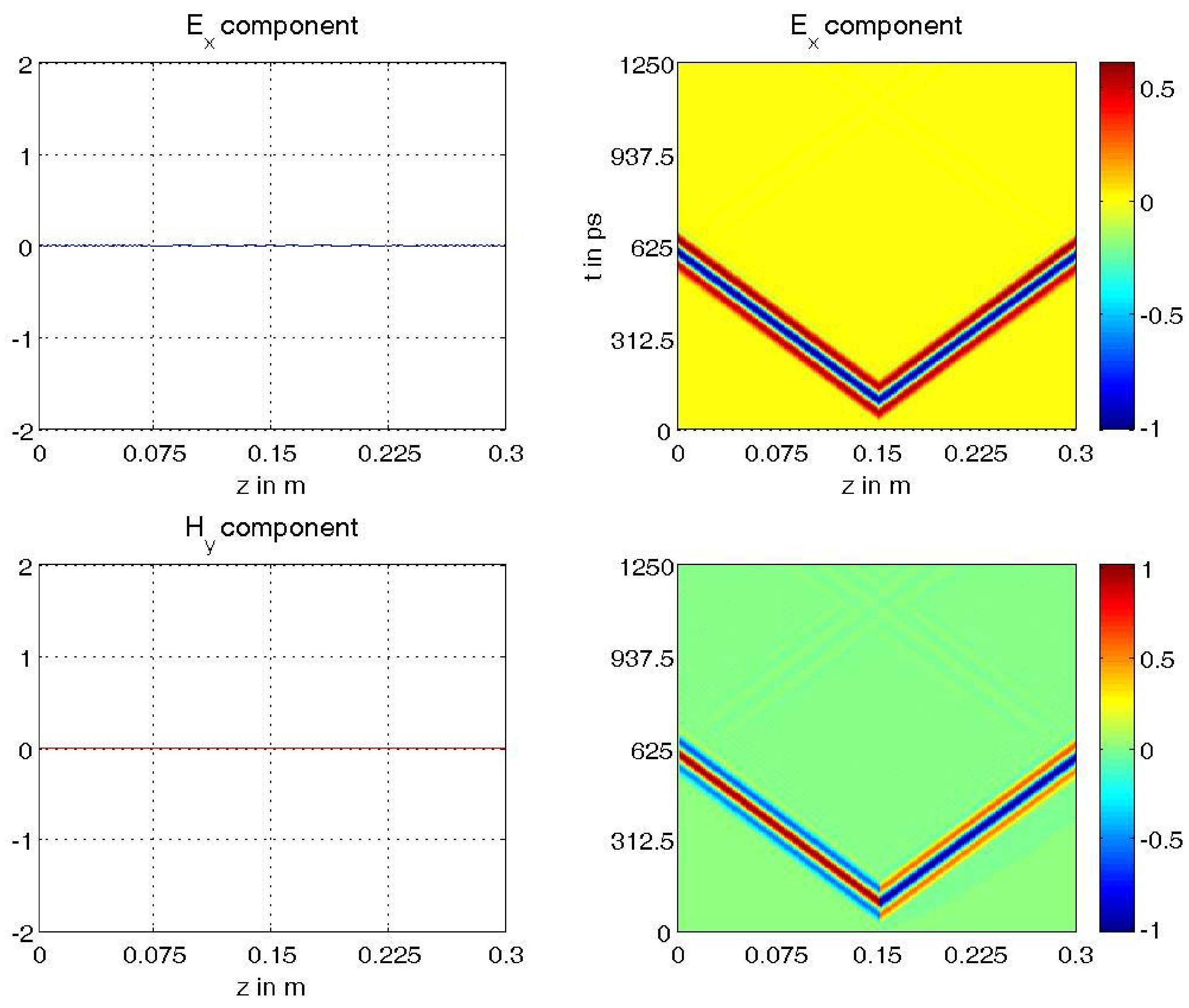


FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

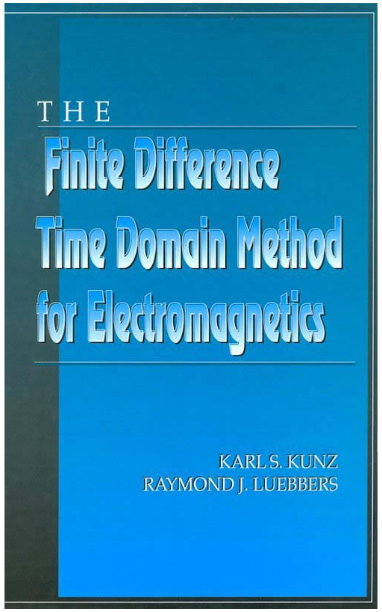




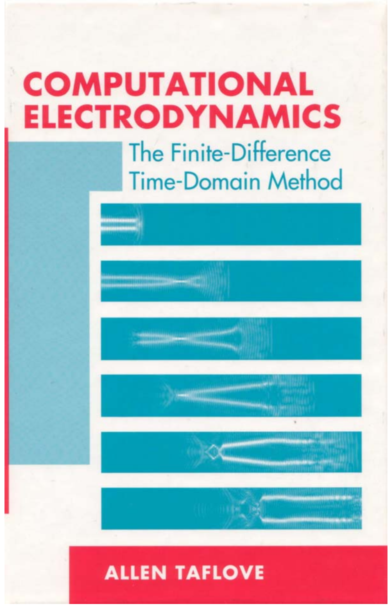
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



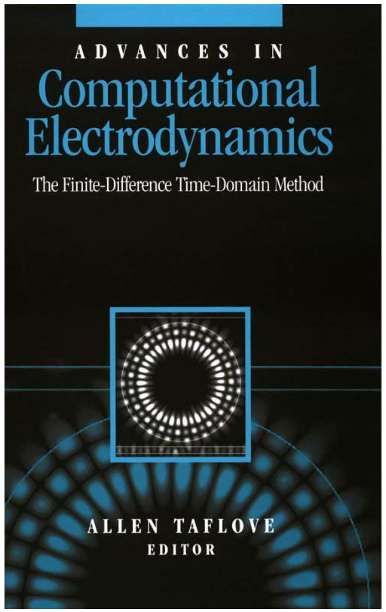
FDTD Books / FDTD-Bücher



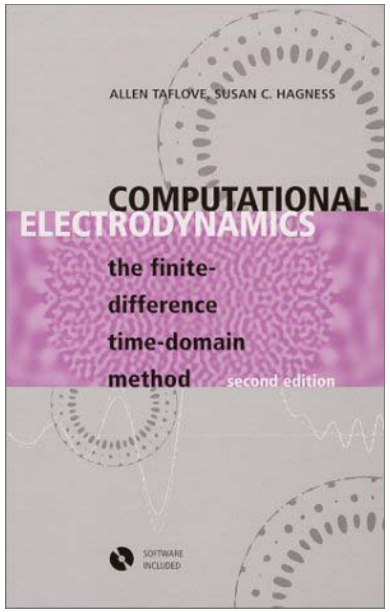
Kunz, K. S., Luebbers, R. J.: *The Finite Difference Time Domain Method for Electromagnetics*. 1993



Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Artech House, Boston, 1995.

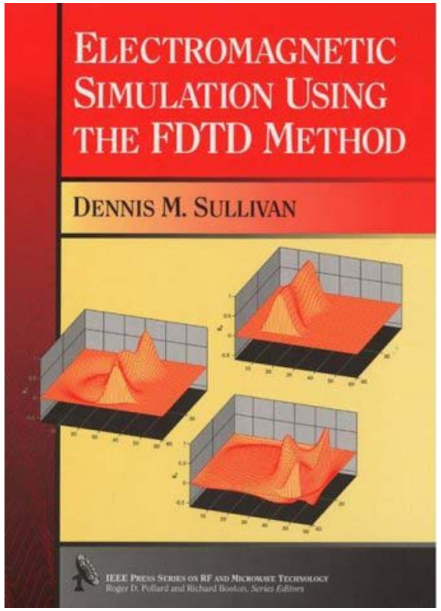


Taflove, A. (Editor): *Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Artech House, 1998.



Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. 2nd Editon, Artech House, Boston, 2000.

FDTD Books / FDTD-Bücher



Sullivan, D. M.: *Electromagnetic Simulation Using the FDTD Method*. IEEE Press, New York, 2000.

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

The first two Maxwell's Equations are in differential form /
Die ersten beiden Maxwellschen Gleichungen lauten in Differentialform:

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

In Cartesian Coordinates we find for the Curl operator applied to E and H /
Im Kartesischen Koordinatensystem finden wir für den Rotationsoperator angewendet auf E und H:

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(\underline{\mathbf{R}},t) & E_y(\underline{\mathbf{R}},t) & E_z(\underline{\mathbf{R}},t) \end{vmatrix}$$

$$= \left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(\underline{\mathbf{R}},t) & H_y(\underline{\mathbf{R}},t) & H_z(\underline{\mathbf{R}},t) \end{vmatrix}$$

$$= \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z$$

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

If we insert the last expressions into the first two Maxwell's equations are in differential form read /
Wenn wir die letzten Ausdrücke in the ersten beiden Maxwellschen Gleichungen in Differentialform einsetzen, erhalten wir:

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\left[B_x(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + B_y(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + B_z(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$

$$= -\left\{\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z\right\}$$

$$-\left[J_{\mathrm{m}x}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + J_{\mathrm{m}y}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + J_{\mathrm{m}z}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\left[D_x(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + D_y(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + D_z(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$

$$= \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z$$

$$-\left[J_{\mathrm{e}x}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + J_{\mathrm{e}y}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + J_{\mathrm{e}z}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$

Six decoupled scalar equations! /
Sechs entkoppelte skalare Gleichungen!

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

If we insert the last expressions into the first two Maxwell's equations are in differential form we read / Wenn wir die letzten Ausdrücke in die ersten beiden Maxwellschen Gleichungen in Differentialform einsetzen, erhalten wir:

$$\frac{\partial}{\partial t} B_x(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{m}x}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t} B_y(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{m}y}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t} B_z(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right] - J_{\mathrm{m}z}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t} D_x(\underline{\mathbf{R}},t) = \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{e}x}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t} D_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{e}y}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t} D_z(\underline{\mathbf{R}},t) = \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right] - J_{\mathrm{e}z}(\underline{\mathbf{R}},t)$$

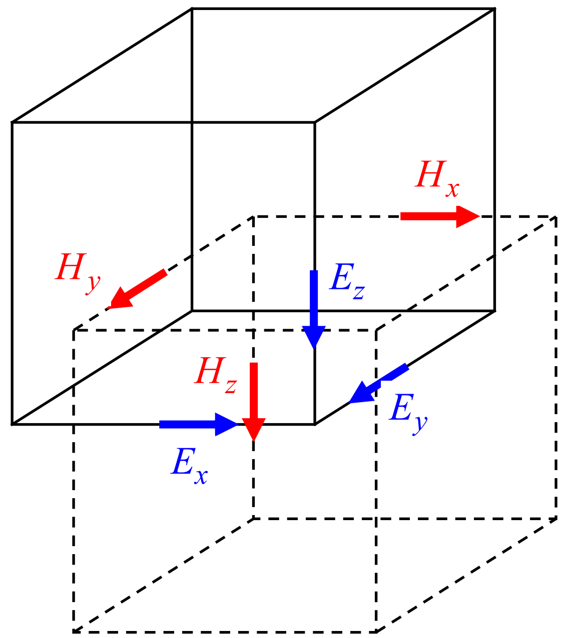
3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

Constitutive equation for homogeneous isotropic materials / Konstituierende Gleichungen für homogene isotrope Materialien:

$$B_x(\underline{\mathbf{R}},t)=\mu H_x(\underline{\mathbf{R}},t) \qquad D_x(\underline{\mathbf{R}},t)=\mu E_x(\underline{\mathbf{R}},t)$$
$$B_y(\underline{\mathbf{R}},t)=\mu H_y(\underline{\mathbf{R}},t) \qquad D_y(\underline{\mathbf{R}},t)=\mu E_y(\underline{\mathbf{R}},t)$$
$$B_z(\underline{\mathbf{R}},t)=\mu H_z(\underline{\mathbf{R}},t) \qquad D_z(\underline{\mathbf{R}},t)=\mu E_z(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu H_x(\underline{\mathbf{R}},t)=-\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y}-\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right]-J_{\mathrm{mx}}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\mu H_y(\underline{\mathbf{R}},t)=-\left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z}-\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right]-J_{\mathrm{my}}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\mu H_z(\underline{\mathbf{R}},t)=-\left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x}-\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right]-J_{\mathrm{mz}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_x(\underline{\mathbf{R}},t)=\left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y}-\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right]-J_{\mathrm{ex}}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\varepsilon E_y(\underline{\mathbf{R}},t)=\left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z}-\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right]-J_{\mathrm{ey}}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\varepsilon E_z(\underline{\mathbf{R}},t)=\left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x}-\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right]-J_{\mathrm{ez}}(\underline{\mathbf{R}},t)$$



$$H_{x_i}=J_{\mathrm{mx}_i},\ i=1,2,3$$
$$E_{x_i}=J_{\mathrm{ex}_i},\ i=1,2,3$$

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

$$\frac{\partial}{\partial t} H_x(\underline{\mathbf{R}},t) = \dot{H}_x(\underline{\mathbf{R}},t)$$

$$\mu \dot{H}_x(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{mx}(\underline{\mathbf{R}},t)$$

$$\mu \dot{H}_x(\underline{\mathbf{R}},t) = \dot{H}_x^{(m)}(t)$$

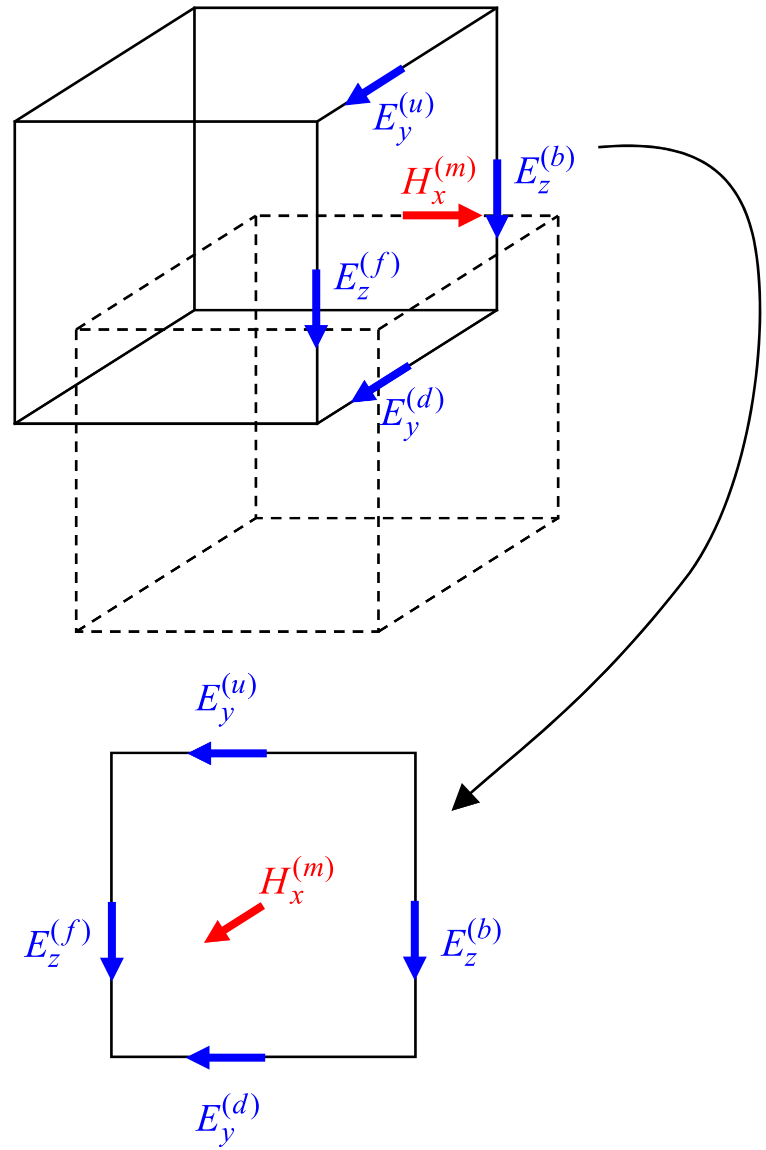
$$J_{mx}(\underline{\mathbf{R}},t) = J_{mx}^{(m)}(t)$$

$$\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} = \frac{E_z^{(f)}(t) - E_z^{(b)}(t)}{\Delta y} + \bigcirc\left[(\Delta y)^2\right]$$

$$\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z} = \frac{E_y^{(d)}(t) - E_y^{(u)}(t)}{\Delta z} + \bigcirc\left[(\Delta z)^2\right]$$

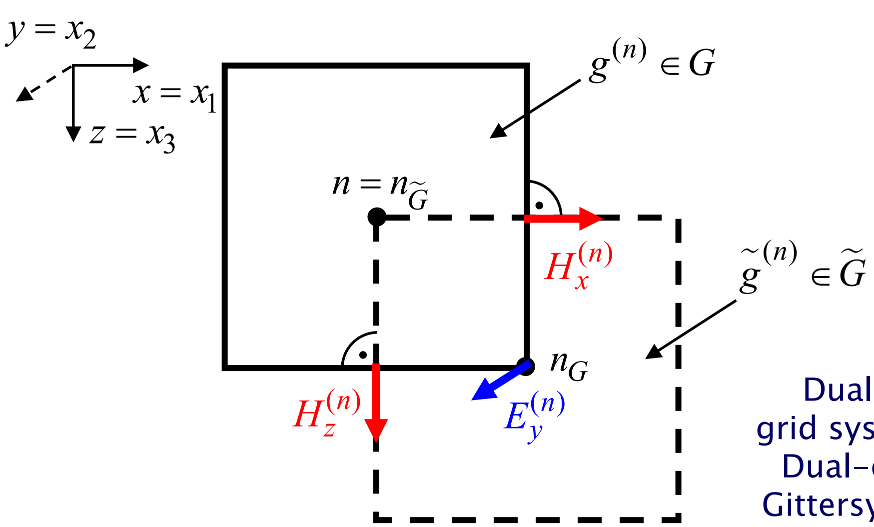
$$\mu \dot{H}_x^{(m)}(t) = -\underbrace{\frac{E_z^{(f)}(t) - E_z^{(b)}(t)}{\Delta y} + \frac{E_y^{(d)}(t) - E_y^{(u)}(t)}{\Delta z}} - J_{mx}^{(m)}(t)$$

A part of the discrete curl operator / Ein Teil des diskreten Rotationsoperators

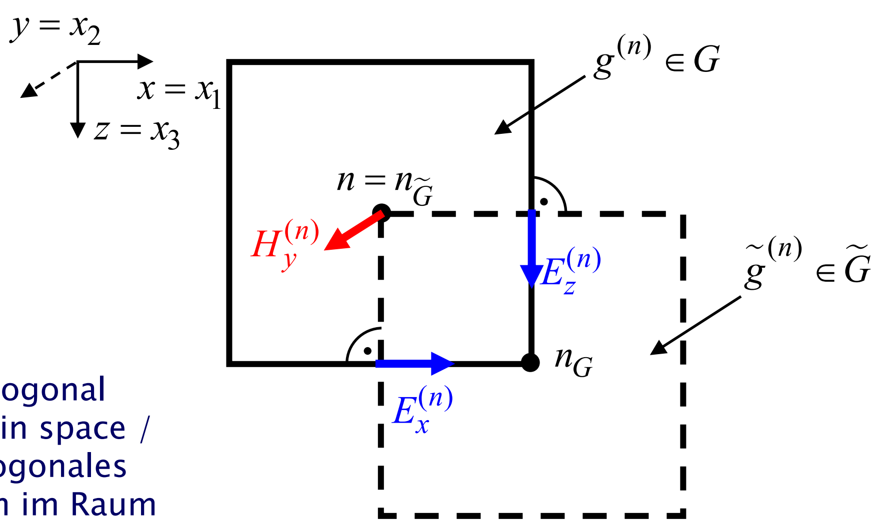


2-D EM Wave Propagation – 2-D FDTD – TM and TE Case / 2D EM Wellenausbreitung – 2D-FDTD – TM- und TE-Fall

2-D TM Case / 2D-TM-Fall



2-D TE Case / 2D-TE-Fall



Dual orthogonal grid system in space / Dual-orthogonales Gittersystem im Raum

$$G \perp \tilde{G}$$

TM:

$$\frac{\partial}{\partial t}\mu H_x(\underline{\mathbf{R}},t) = \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z} - J_{\mathrm{mx}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu H_z(\underline{\mathbf{R}},t) = -\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - J_{\mathrm{mz}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{ey}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + z\underline{\mathbf{e}}_z$$

TE:

$$\frac{\partial}{\partial t}\mu H_y(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{my}}(\underline{\mathbf{R}},t)$$

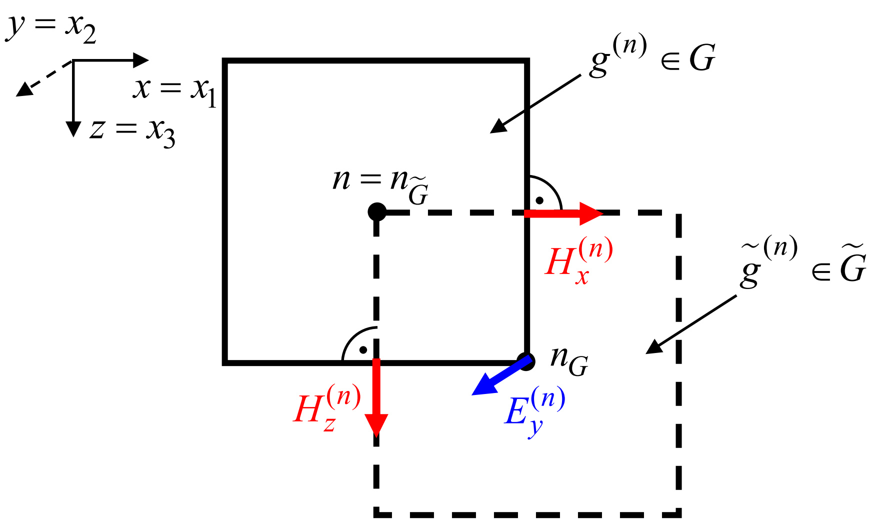
$$\frac{\partial}{\partial t}\varepsilon E_x(\underline{\mathbf{R}},t) = -\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z} - J_{\mathrm{ex}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_z(\underline{\mathbf{R}},t) = \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - J_{\mathrm{ez}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + z\underline{\mathbf{e}}_z$$

2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

2-D TM Case / 2D-TM-Fall



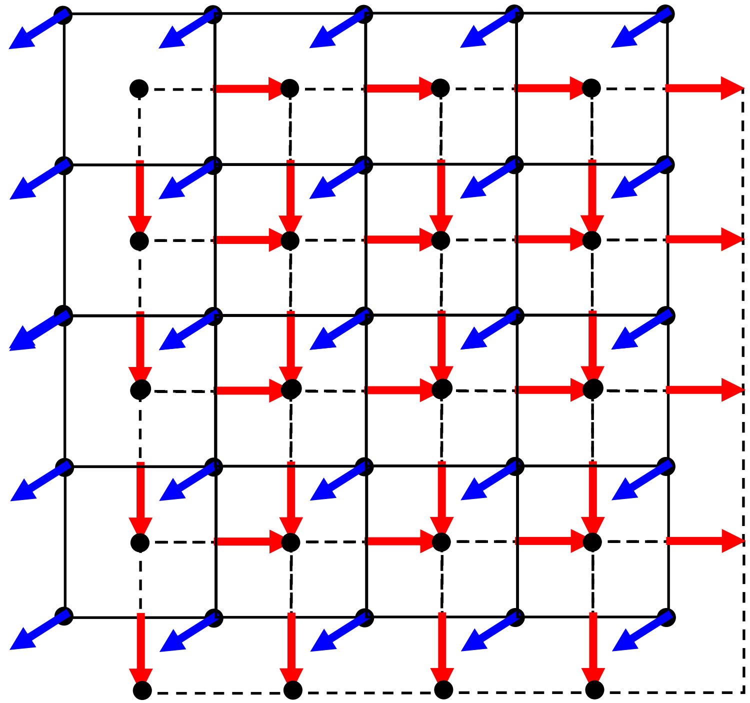
$$\frac{\partial}{\partial t}\mu H_x(\underline{\mathbf{R}},t) = \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z} - J_{mx}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu H_z(\underline{\mathbf{R}},t) = -\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - J_{mz}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{ey}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + z\underline{\mathbf{e}}_z$$

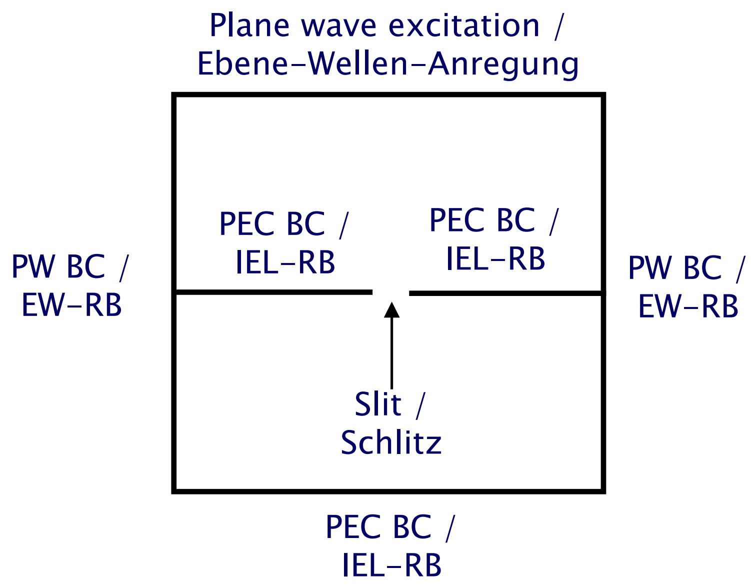
Two-dimensional staggered grid system in the 2-D TM case / Zweidimensionales versetztes Gittersystem im 2D-TM-Fall



Implementation of Boundary Conditions / Implementierung von Randbedingungen

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

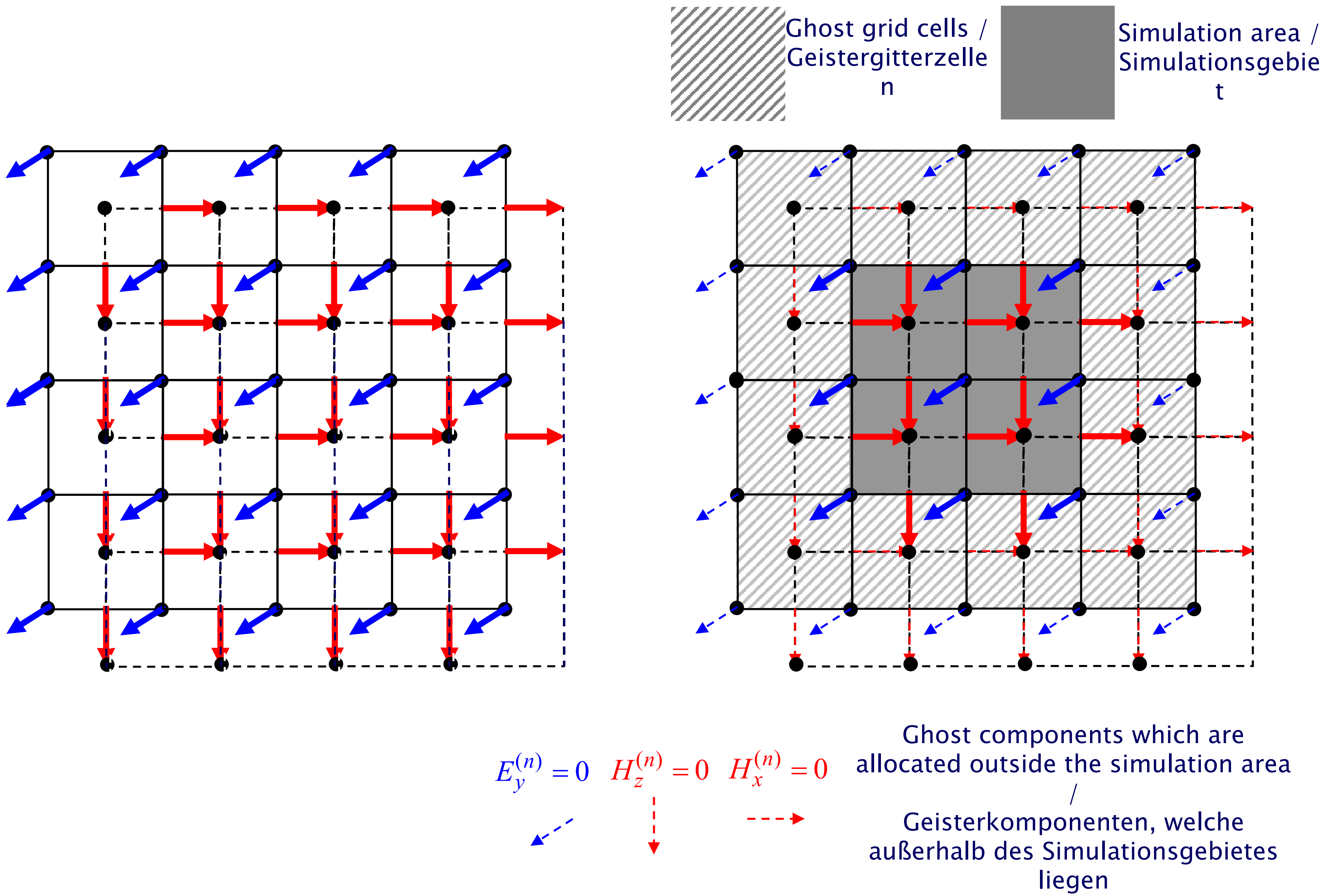
$$\left.\begin{aligned} E_y^{(\bullet,\bullet,n_t)} &= 0 \\ E_y^{(\bullet,\bullet,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$



Plane wave boundary condition for a vertical incident plane wave / Ebene-Wellen-Randbedingung für eine vertikal einfallende ebene Welle

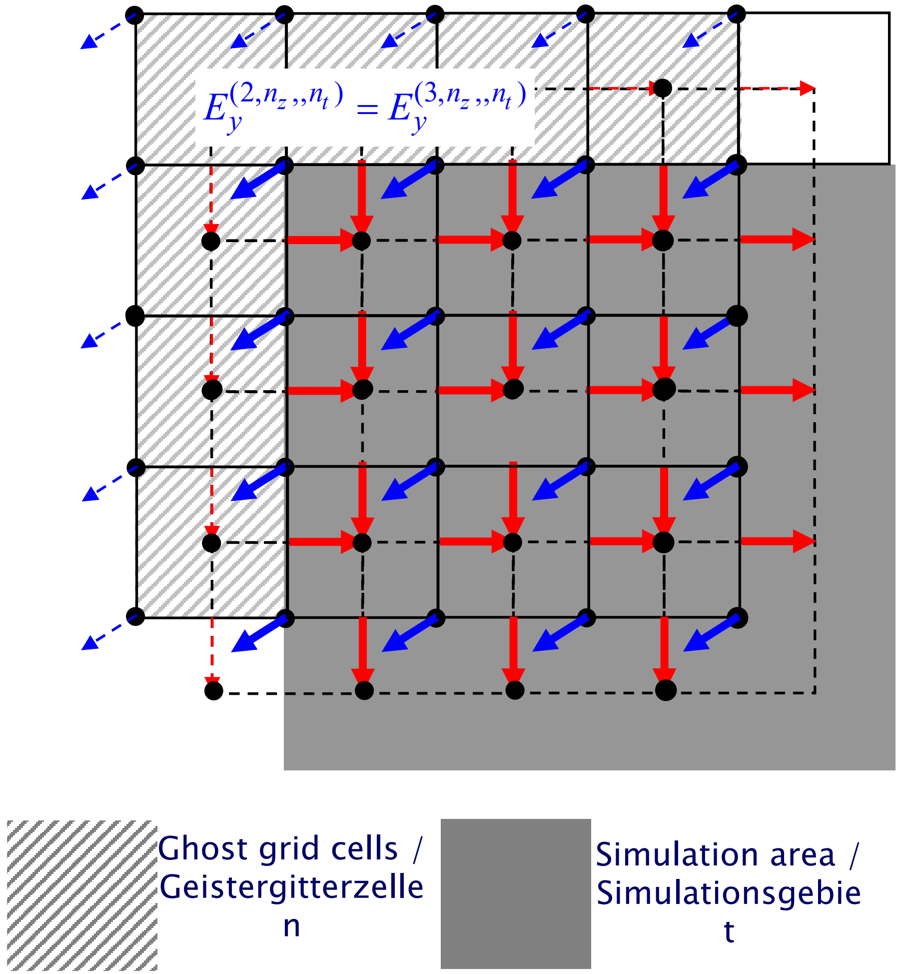
$$\left.\begin{aligned} E_y^{(2,n_z,n_t)} &= E_y^{(3,n_z,n_t)} \\ E_y^{(N_x-1,n_z,n_t)} &= E_y^{(N_x-2,n_z,n_t-2)} \end{aligned}\right\} \quad \begin{aligned} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{aligned}$$

2-D EM Wave Propagation – 2-D FDTD – TM Case/
2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

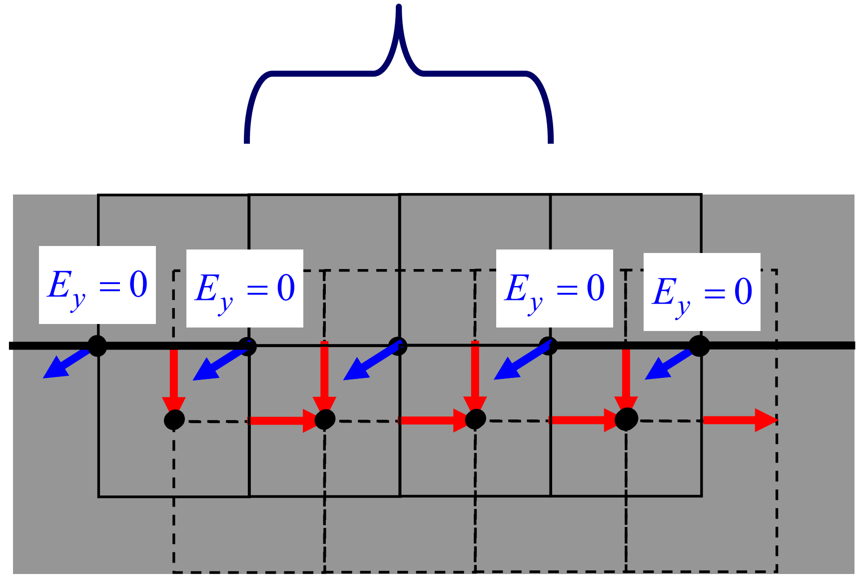


2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

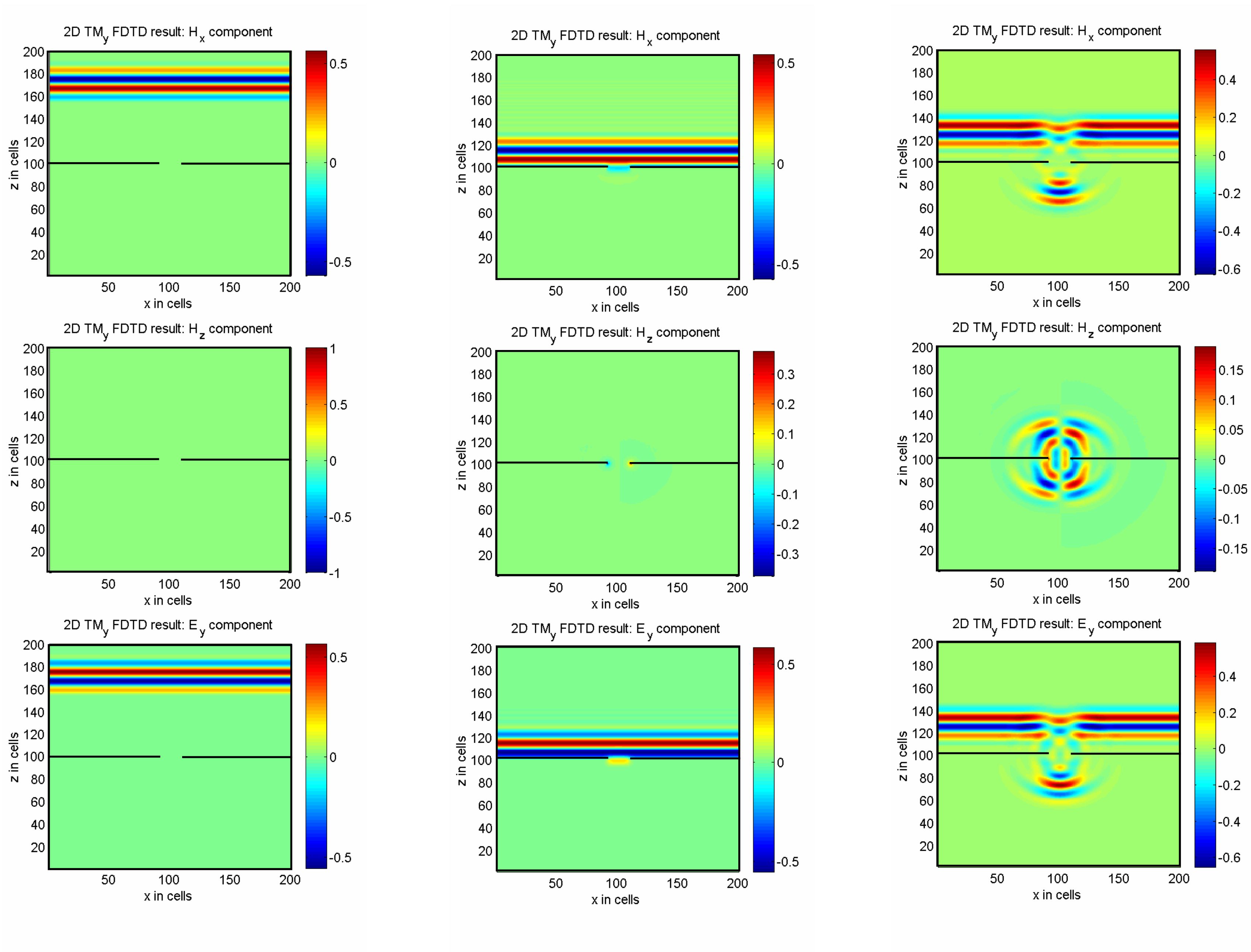
Plane wave excitation / Ebene-Wellen-Anregung



Slit / Schlitz



2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung an einem Spalt



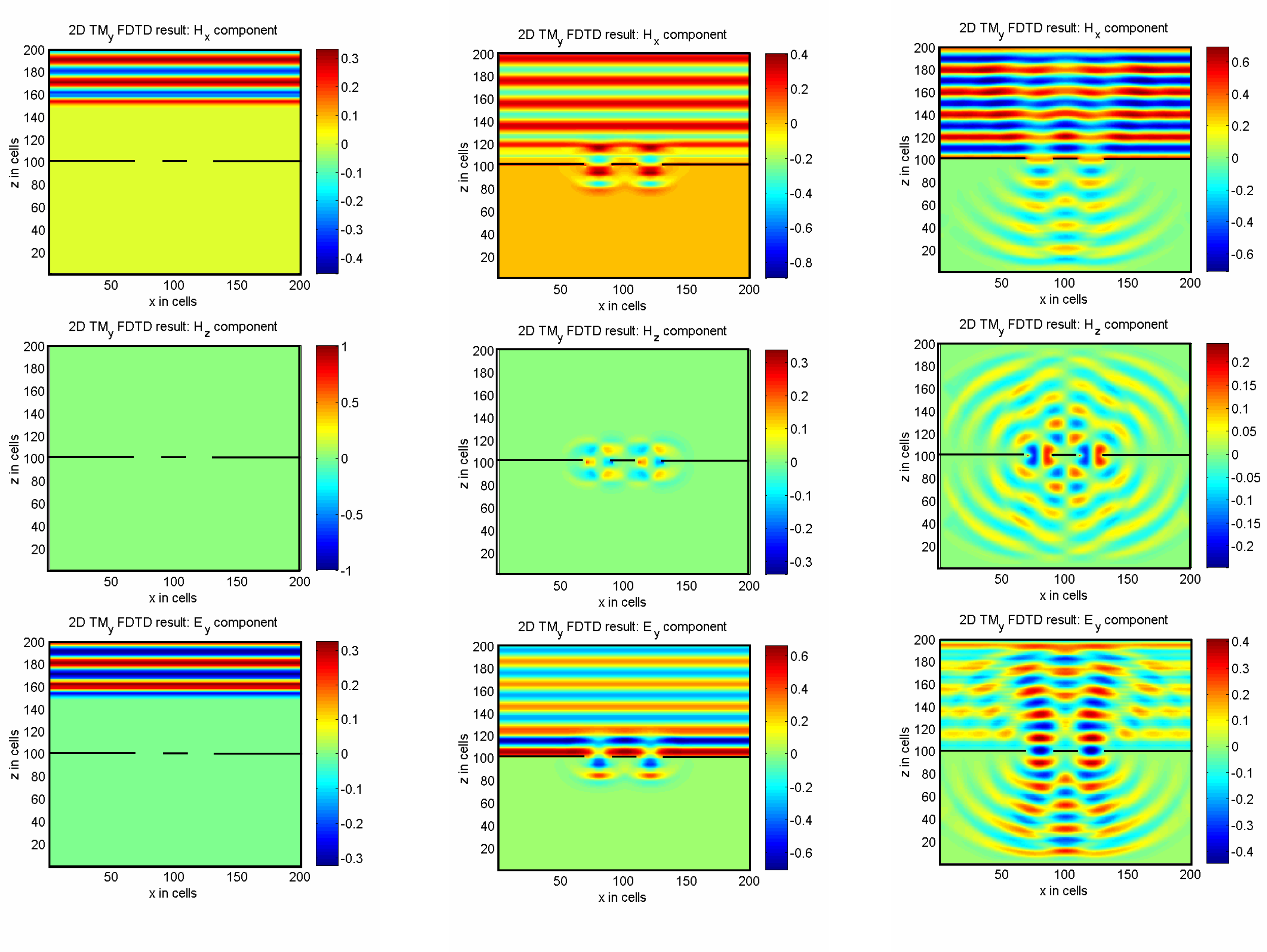
2–D TM FDTD – Diffraction on a Single Slit / 2D–TM–FDTD – Beugung am Spalt

Wave field movie of the H_x field component / Wellenfeldfilm der H_x–Feldkomponente

Wave field movie of the H_z field component / Wellenfeldfilm der H_z–Feldkomponente

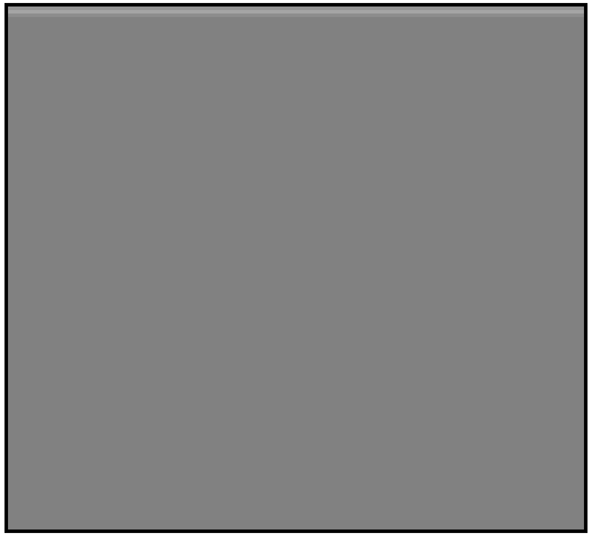
Wave field movie of the E_y field component / Wellenfeldfilm der E_y–Feldkomponente

2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

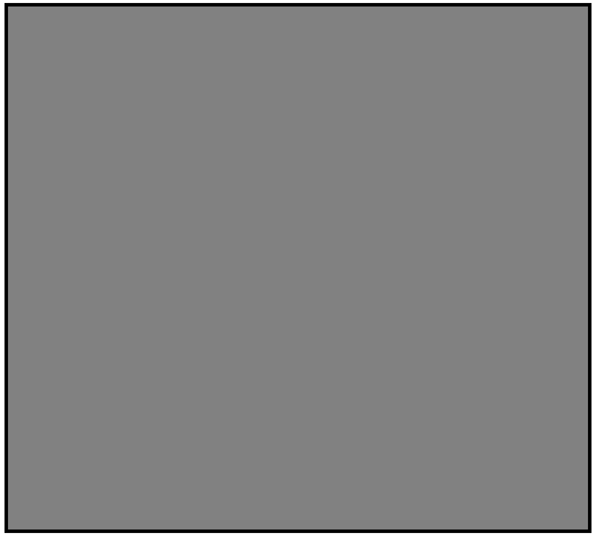


2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

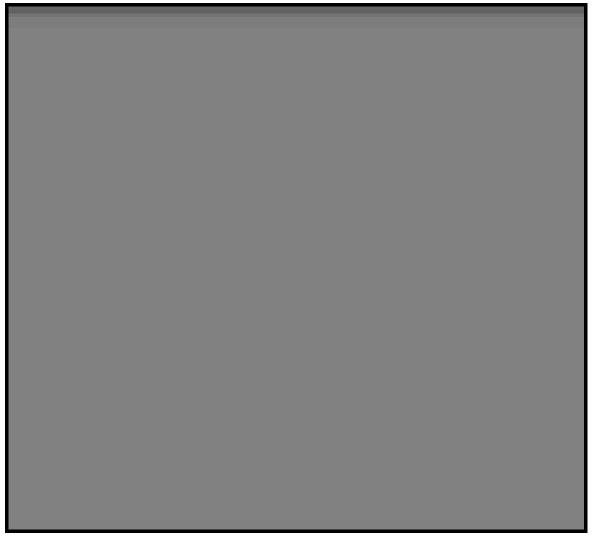
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



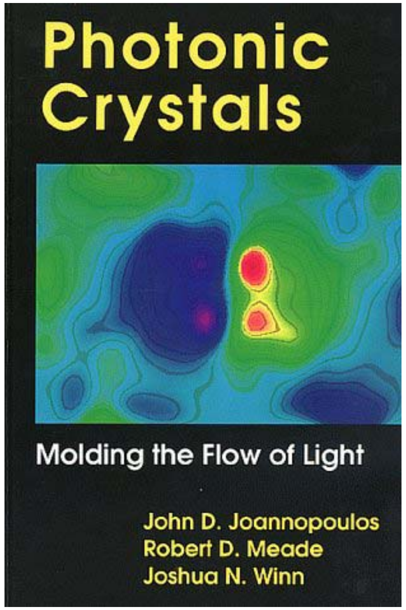
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



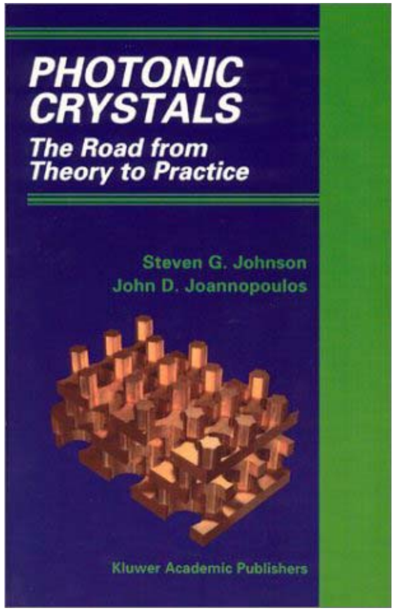
Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente



Photonic Crystals / Photonische Kristalle



Joannopoulos, J. D., R. D. Meade, J. N. Winn: *Photonic Crystals – Molding the Flow of Light.* *Princeton University Press, Princeton, 1995.*



Johnson, S. G.: *Photonic Crystals: The Road from Theory to Practice*. Kluwer Academic Press, 2001.

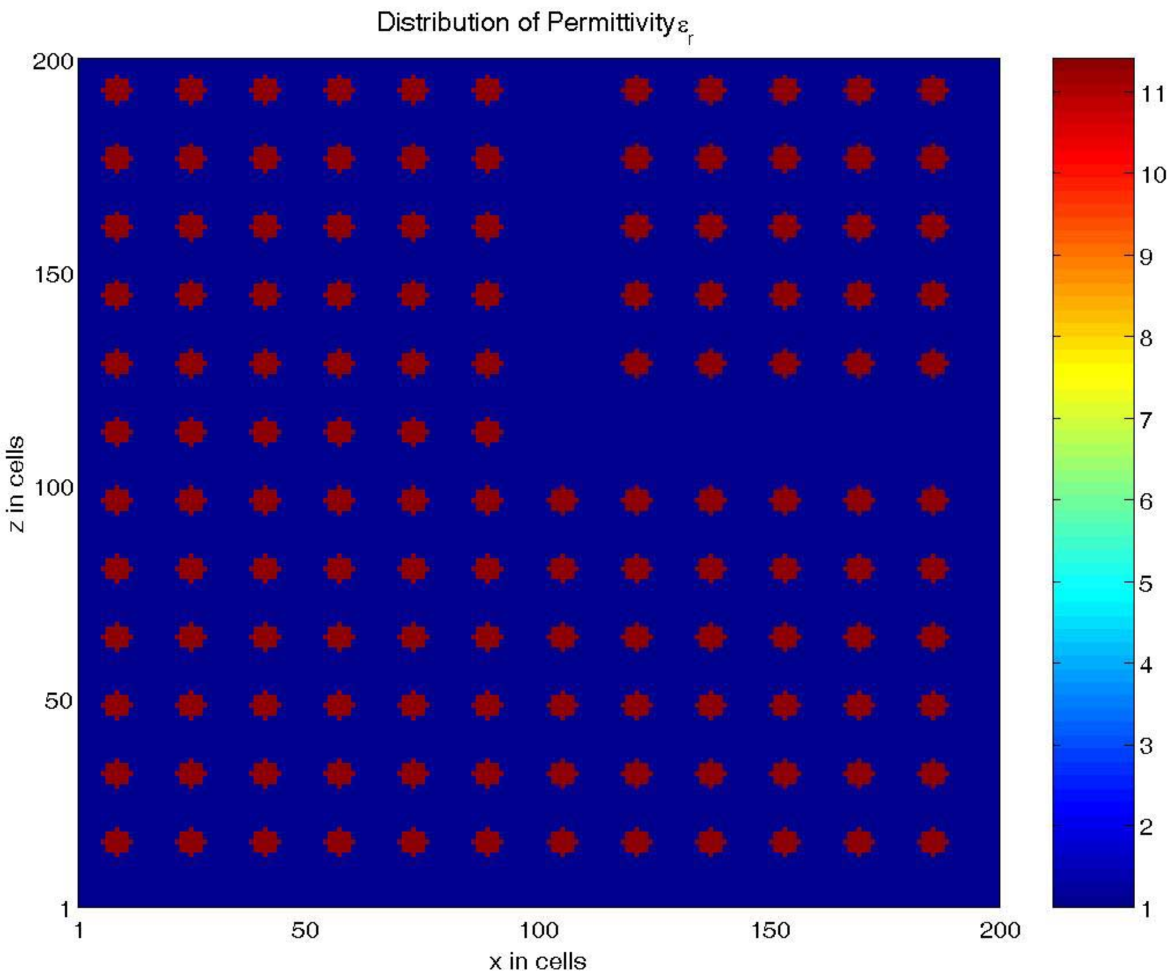
Links:

Photonic Crystals Research at MIT
Homepage of Prof. Sajeev John, University of Toronto, Canada

2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

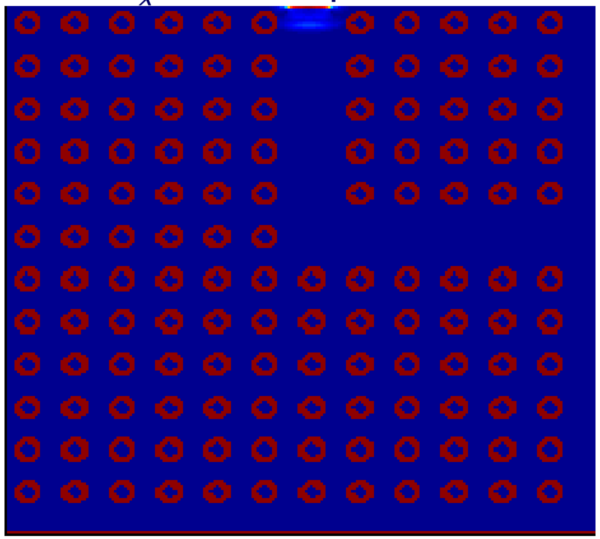
Relative permittivity of the background
Relative Permittivität des Hintergrundes $\varepsilon_{\mathrm{r}}^{(b)} = 1$

Relative permittivity of the rods
Relative Permittivität der Stäbe $\varepsilon_{\mathrm{r}}^{(r)} = 11.4$

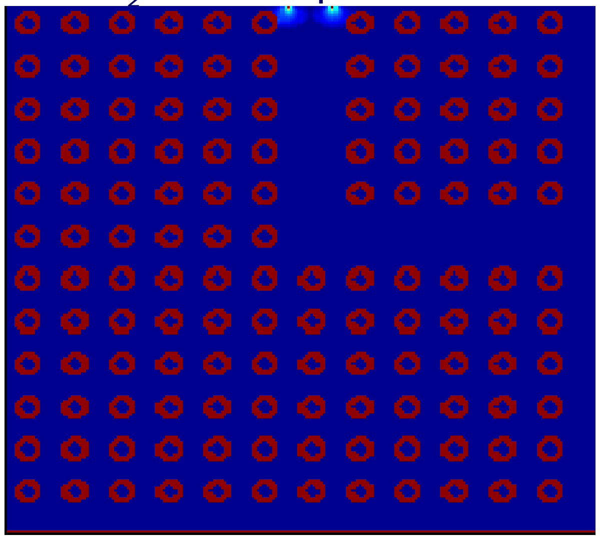


2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

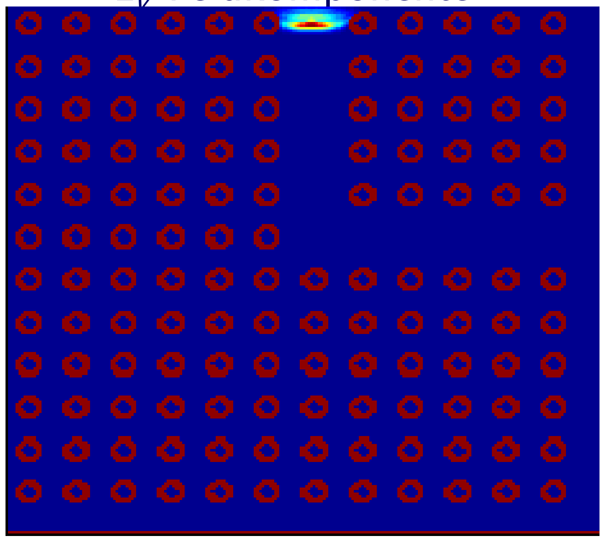
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente



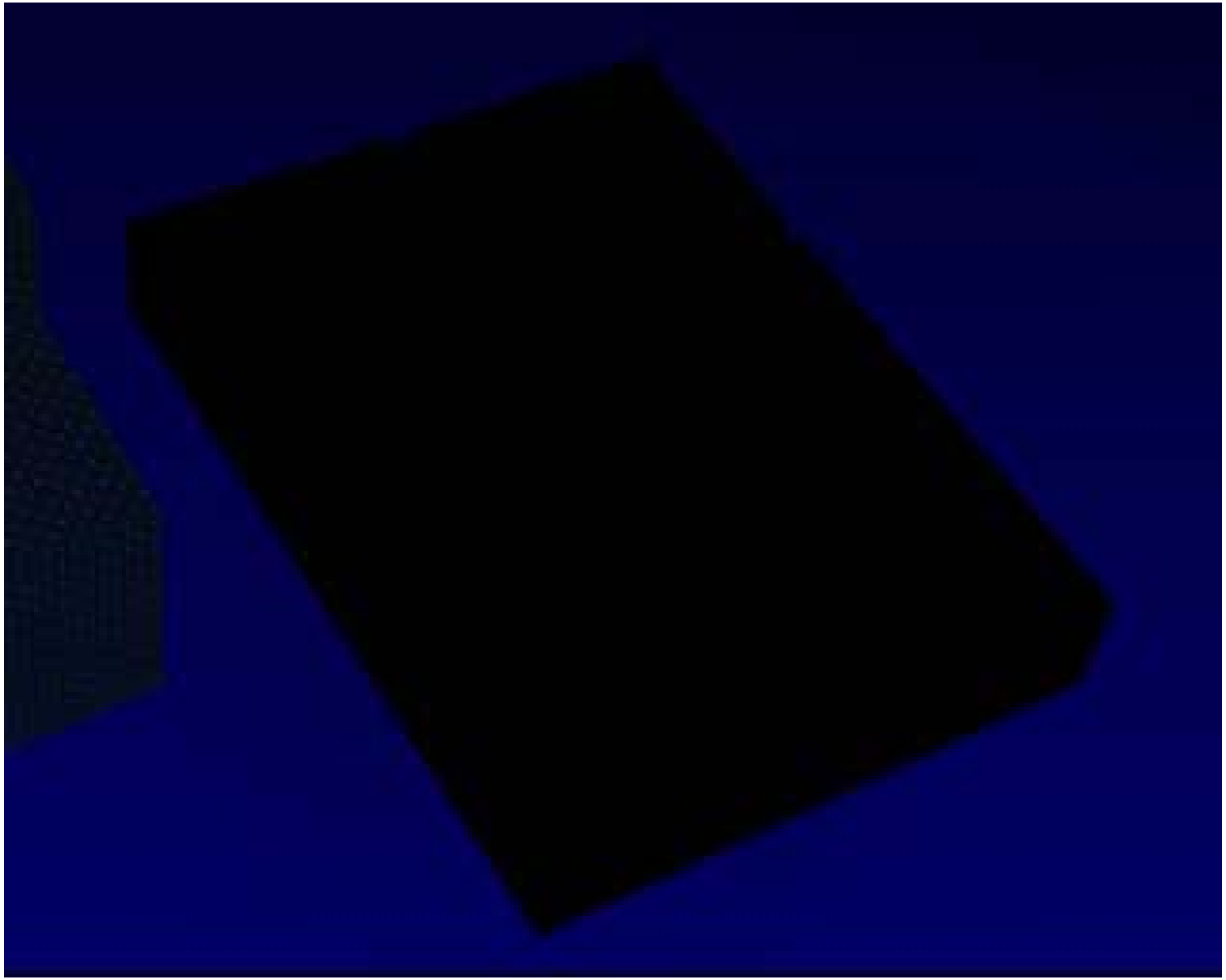
2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente

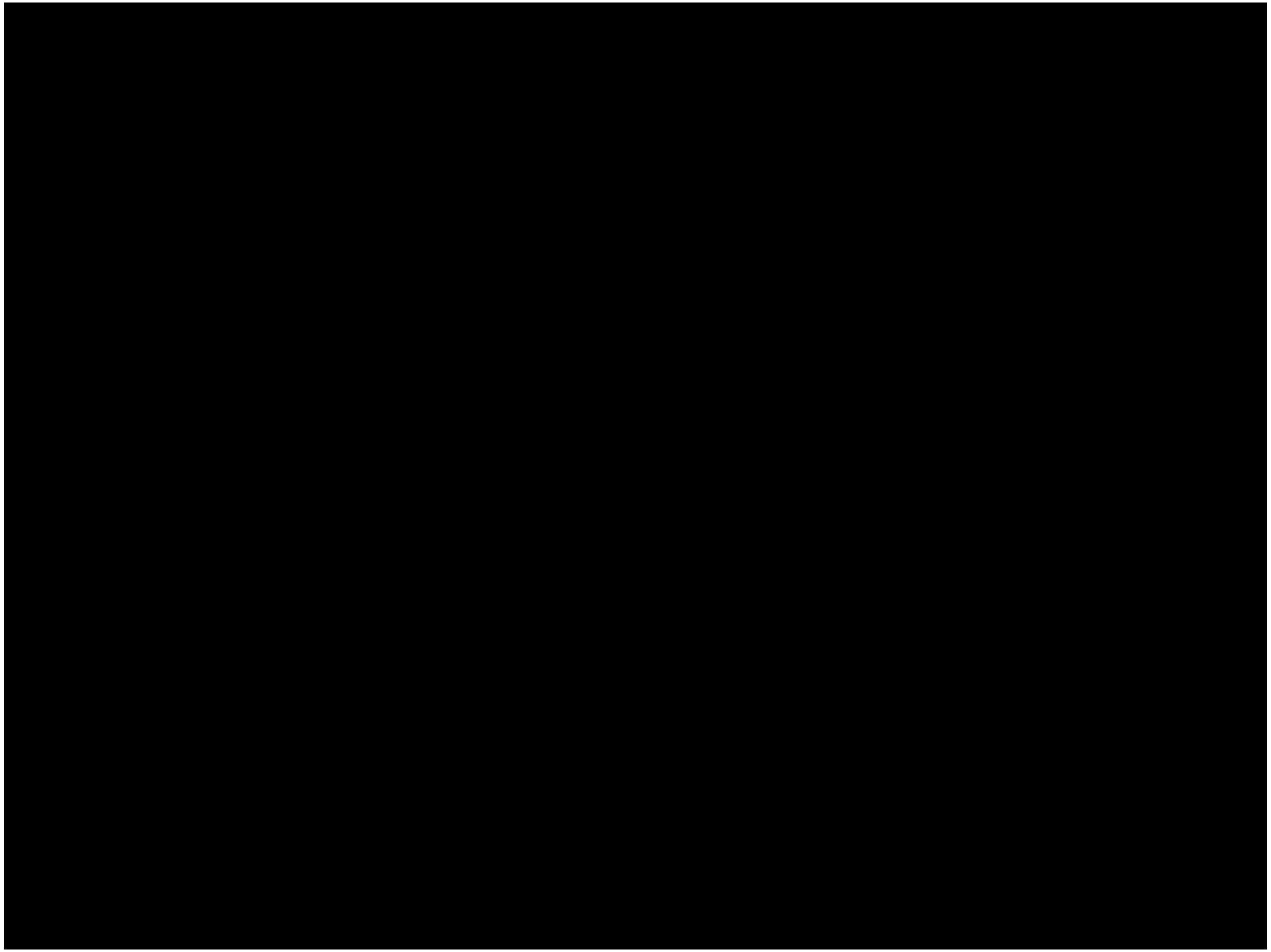
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

2-D TM FDTD - Photonic Crystals /
2D-TM-FDTD - Photonische Kristalle



2-D TM FDTD – Photonic Crystals /
2D-TM-FDTD – Photonische Kristalle



End of Lecture 6 /
Ende der 6. Vorlesung