Numerical Methods of Electromagnetic Field Theory I (NFT I)
Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

6th Lecture / 6. Vorlesung

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EM Wave Propagation - Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung - Finite Differenzen im Zeitbereich (FDTD)

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

Equations of first order /
Gleichungen der ersten Ordnung

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underbrace{\begin{aligned}\frac{\partial}{\partial t}\mu\underline{\mathbf{H}}(\underline{\mathbf{R}},t) &= -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\\ \frac{\partial}{\partial t}\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\end{aligned}}_{f(\underline{\mathbf{H}},\underline{\mathbf{E}})}$$

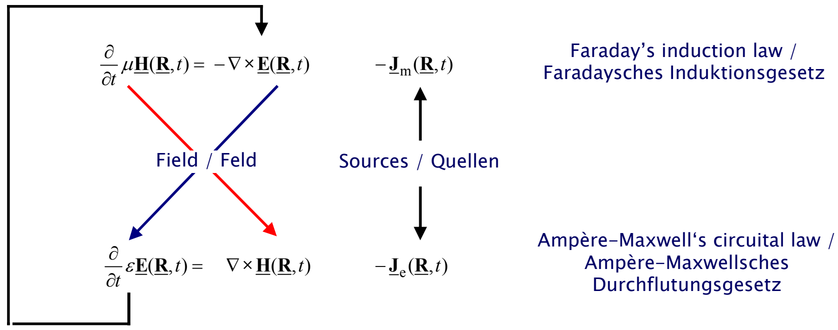
Constitutive Equations for Vacuum /
Konstituierende Gleichungen
(Materialgleichungen) für Vakuum

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \nu_0\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underbrace{\begin{aligned}\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) &= -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\\ \frac{\partial}{\partial t}\left[\varepsilon\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] &= \nabla\times\left[\nu\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right] - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\end{aligned}}_{f(\underline{\mathbf{B}},\underline{\mathbf{E}})}$$

EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
Idee: Entwurf eines Flussdiagramms



EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart /
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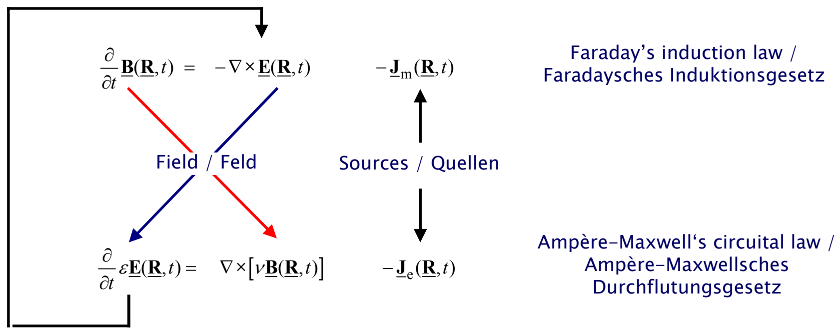


1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Constitutive Equations for Vacuum / Konstituierende Gleichungen (Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

Ansatz for the electric and magnetic field strength / Ansatz für die elektrische und magnetische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = E_x(z,t)\,\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = H_y(z,t)\,\underline{\mathbf{e}}_y$$

The first two Maxwell's Equations are: / Die ersten beiden Maxwellschen Gleichungen lauten:

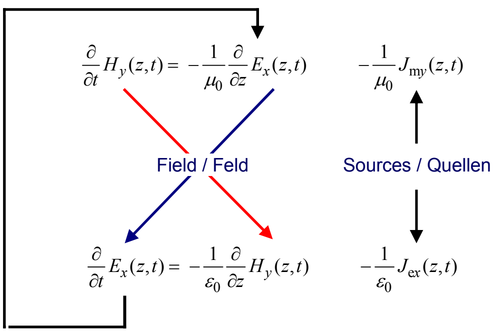
$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z}E_x(z,t) - \frac{1}{\mu_0}J_\mathrm{my}(z,t)$$
$$\frac{\partial}{\partial t}E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}H_y(z,t) - \frac{1}{\varepsilon_0}J_\mathrm{ex}(z,t)$$

$$\frac{d}{dt}f(t) = \frac{f\left(t+\frac{\Delta t}{2}\right) - f\left(t-\frac{\Delta t}{2}\right)}{\Delta t} + \mathrm{O}[(\Delta t)^2]$$
$$\frac{d}{dz}f(z) = \frac{f\left(z+\frac{\Delta z}{2}\right) - f\left(z-\frac{\Delta z}{2}\right)}{\Delta x} + \mathrm{O}[(\Delta x)^2]$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

Idea: Outline of a flow chart / Idee: Entwurf eines Flussdiagramms



1-D EM Wave Propagation – FDTD – Discretization of the 1st Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 1ten Gleichung

Spatial discretization of the 1st equation /
Räumliche Diskretisierung der 1ten Gleichung

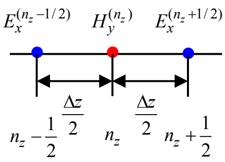
$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$

$$H_y : z \to n_z \Delta z, \qquad n_z = 1,\ldots,N_z$$

$$E_x : z \to \left(n_z + 1/2\right)\Delta z, \quad n_z = 1,\ldots,N_z$$

$$\frac{\partial}{\partial z} E_x(z,t) \;\to\; \frac{\partial}{\partial z} E_x(z,t)\Big|_z = \frac{1}{\Delta z}\left[E_x\left(z+\frac{\Delta z}{2}\right) - E_x\left(z-\frac{\Delta z}{2}\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\left[E_x^{(n_z)}\right]$$



$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

1-D EM Wave Propagation – FDTD – Discretization of the 2nd Equation / 1D EM Wellenausbreitung – FDTD – Diskretisierung der 2ten Gleichung

Spatial discretization of the 2nd equation /
Räumliche Diskretisierung der 2ten Gleichung

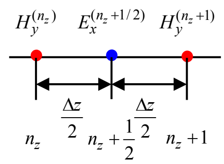
$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

$$H_y : z \to n_z \Delta z, \qquad n_z = 1,\ldots,N_z$$

$$E_x : z \to \left(n_z + 1/2\right)\Delta z, \quad n_z = 1,\ldots,N_z$$

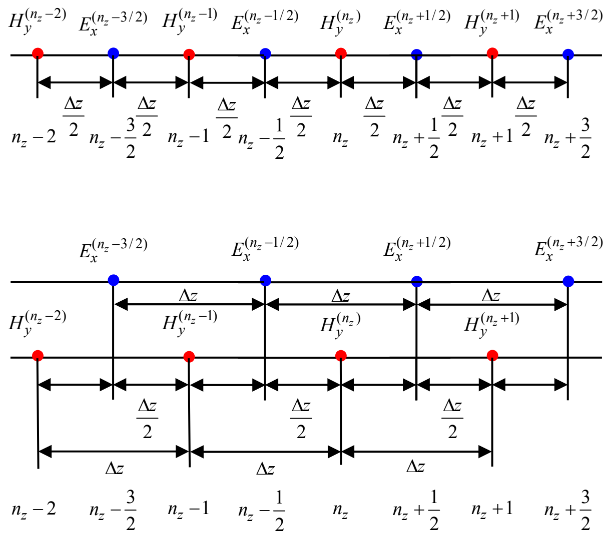
$$\frac{\partial}{\partial z} H_y(z,t) \;\to\; \frac{\partial}{\partial z} H_y(z,t)\Big|_{z+\frac{\Delta z}{2}} = \frac{1}{\Delta z}\left[H_y\left(z+\Delta z\right) - H_y\left(z\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\left[H_y^{(n_z+1/2)}\right]$$



$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum



1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\text{my}}(z,t)$$
$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\text{ex}}(z,t)$$

$$\Downarrow$$

$$\frac{d}{dz} f(z) = \frac{1}{\Delta x}\left[f\left(z+\frac{\Delta z}{2}\right) - f\left(z-\frac{\Delta z}{2}\right)\right] + \mathrm{O}[(\Delta z)^2]$$

$$\Downarrow$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\text{my}}^{(n_z)}(t)$$
$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\text{ex}}^{(n_z+1/2)}(t)$$

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = ?$$
$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = ?$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

$$\frac{d}{dt} f(t) = \frac{1}{\Delta t}\left[f\left(t + \frac{\Delta t}{2}\right) - f\left(t - \frac{\Delta t}{2}\right)\right] + O[(\Delta t)^2]$$

Staggered grid in time / Versetztes Gitter in der Zeit

$$\frac{\partial}{\partial t} H_y^{(n_z)}(t) = \frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} + O[(\Delta t)^2]$$

$$\frac{\partial}{\partial t} E_x^{(n_z+1/2)}(t) = \frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t-1/2)}}{\Delta t} + O[(\Delta t)^2]$$

$$\frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t-1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

1-D EM Wave Propagation – Finite-Difference Time-Domain (FDTD) / 1D EM Wellenausbreitung – Finite Differenzen im Zeitbereich (FDTD)

$$\frac{H_y^{(n_z,n_t)} - H_y^{(n_z,n_t-1)}}{\Delta t} = -\frac{1}{\mu_0}\frac{1}{\Delta z}\left[E_x^{(n_z+1/2)}(t) - E_x^{(n_z-1/2)}(t)\right] - \frac{1}{\mu_0} J_{\mathrm{my}}^{(n_z)}(t)$$

$$\frac{E_x^{(n_z+1/2,n_t+1/2)} - E_x^{(n_z+1/2,n_t+1/2)}}{\Delta t} = -\frac{1}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1)}(t) - H_y^{(n_z)}(t)\right] - \frac{1}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2)}(t)$$

Explicit 1-D FDTD algorithm on a staggered grid in space and time / Expliziter 1D-FDTD-Algorithmus auf einem versetzten Gitter im Raum und Zeit

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t+1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ey}}^{(n_z+1/2,n_t)}$$

FDTD: Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

1-D EM Wave Propagation – 1-D FDTD / 1D EM Wellenausbreitung – 1D FDTD

The first two Maxwell's Equations are: /
Die ersten beiden Maxwellschen Gleichungen lauten:

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t)$$
$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

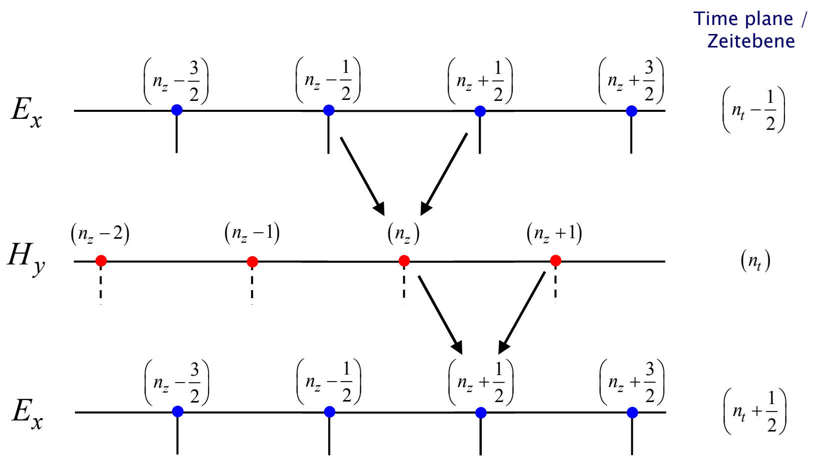
Explicit 1-D FDTD algorithm of leap-frog type on a staggered grid in space and time /
Expliziter 1D-FDTD-Algorithmus vom „Bocksprung“-Typ auf einem versetzten Gitter im Raum und Zeit

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$
$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

FDTD: Yee, K. S.: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas Propagation*, Vol. AP-14, pp. 302-307, 1966.

1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum

Interleaving of the E_x and H_y field components in space and time in the 1-D FDTD formulation /
Überlappung der E_x- und H_y-Feldkomponente in der 1D-FDTD-Formulierung im Raum und in der Zeit



1-D EM Wave Propagation – FDTD – Normalization / 1D EM Wellenausbreitung – FDTD – Normierung

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)} \right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)} \right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

$$\Delta t = \Delta t_{\mathrm{ref}}\,\widehat{\Delta t} \qquad \Delta t_{\mathrm{ref}} = \frac{\Delta x_{\mathrm{ref}}}{c_{\mathrm{ref}}} \qquad \Delta t = \frac{\Delta x_{\mathrm{ref}}}{c_{\mathrm{ref}}}\,\widehat{\Delta t}$$

$$\Delta z = \Delta x_{\mathrm{ref}}\,\widehat{\Delta z} \qquad c = c_{\mathrm{ref}}\,\hat{c} \qquad \varepsilon = \varepsilon_{\mathrm{ref}}\,\hat{\varepsilon} \qquad \mu = \mu_{\mathrm{ref}}\,\hat{\mu} \qquad \mu_{\mathrm{ref}} = \mu_0$$

$$E_x = E_{\mathrm{ref}}\,\hat{E}_x$$

$$H_y = H_{\mathrm{ref}}\,\hat{H}_y \qquad H_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{c_{\mathrm{ref}}\mu_{\mathrm{ref}}} = \frac{\sqrt{\varepsilon_{\mathrm{ref}}\mu_{\mathrm{ref}}}}{\mu_{\mathrm{ref}}} E_{\mathrm{ref}} = \sqrt{\frac{\varepsilon_{\mathrm{ref}}}{\mu_{\mathrm{ref}}}} E_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{Z_{\mathrm{ref}}}$$

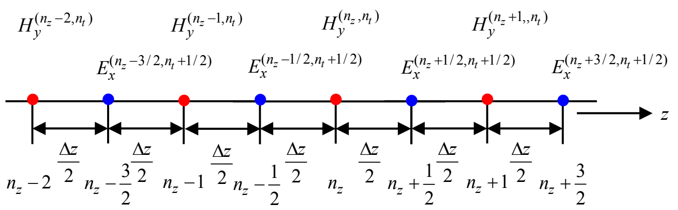
$$J_{\mathrm{ex}} = J_{\mathrm{e\,ref}}\,\hat{J}_{\mathrm{ex}} \qquad J_{\mathrm{e\,ref}} = \frac{\varepsilon_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}} E_{\mathrm{ref}}$$

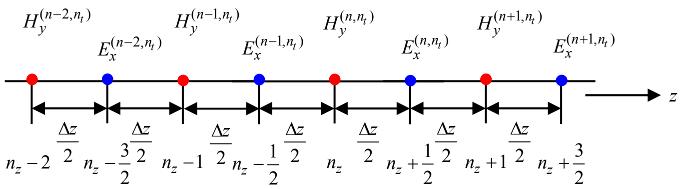
$$J_{\mathrm{mx}} = J_{\mathrm{m\,ref}}\,\hat{J}_{\mathrm{mx}} \qquad J_{\mathrm{m\,ref}} = \frac{\mu_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}} H_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}} c_{\mathrm{ref}}}$$

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)} \right] - \widehat{\Delta t}\,\widehat{J}_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

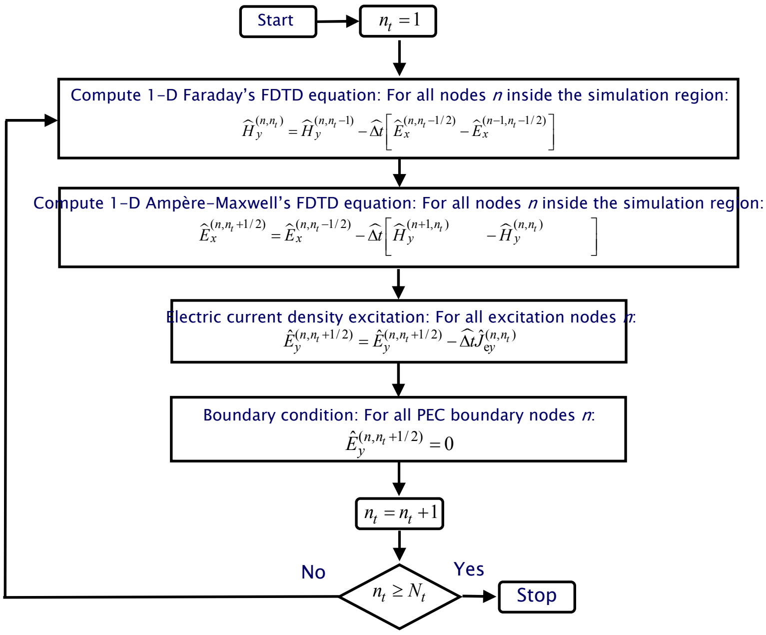
$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)} \right] - \widehat{\Delta t}\,\widehat{J}_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

1-D FDTD – Staggered Grid in Space – Global Node Numbering / 1D-FDTD – Versetztes Gitter im Raum – Globale Knotennummerierung

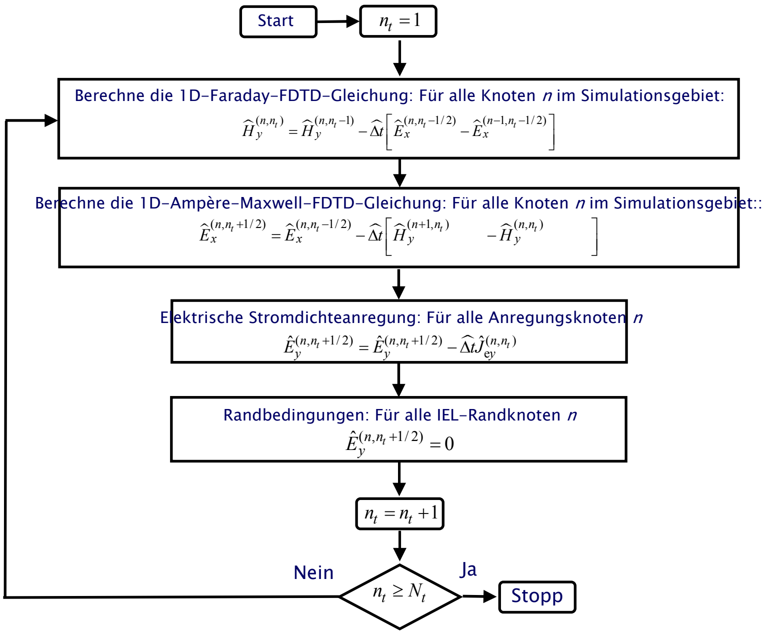




1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Maxwell's equations / Maxwellsche Gleichungen

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{my}(z,t) \quad \text{for / für} \quad \begin{cases} 0 \le z \le Z \\ 0 \le t \le T \end{cases}$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{ex}(z,t)$$

Initial condition / Anfangsbedingung

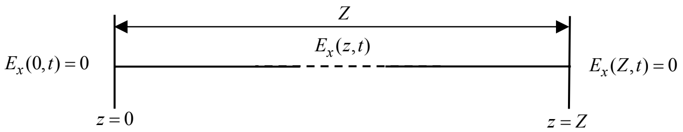
$$\begin{aligned} H_y(z,t) &= J_{my}(z,t) = 0 && t \le 0 \\ E_x(z,t) &= J_{ex}(z,t) = 0 && t \le 0 \\ J_{ex}(z,t) &= K_{e0}(z_0)\,\delta(z-z_0)\,f(t) && t > 0 \end{aligned}$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x(0,t) &= 0 \\ E_x(Z,t) &= 0 \end{aligned}\right\} \quad \forall t$$

Hyperbolic initial-boundary-value problem / Hyperbolisches Anfangs-Randwert-Problem



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Discrete 1-D FDTD equations / Diskrete 1D-FDTD-Gleichungen

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\widehat{J}_{my}^{(n_z,n_t-1/2)} \quad \text{for / für} \quad \begin{cases} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{cases}$$

$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\widehat{J}_{ex}^{(n_z+1/2,n_t)}$$

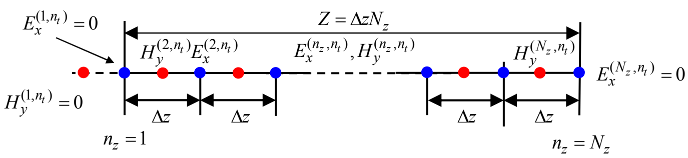
Initial condition / Anfangsbedingung

$$\begin{aligned} H_y^{(n_z,n_t)} &= J_{my}^{(n_z,n_t)} = 0 && n_t \le 1 \\ E_x^{(n_z,n_t)} &= J_{ex}^{(n_z,n_t)} = 0 && n_t \le 1 \\ J_{ex}^{(n_z,n_t)} &= K_{ex}^{(n_{z0})}\,\delta^{(n_z-n_{z0})}\,f^{(n_t)} && n_t > 1 \end{aligned}$$

Causality / Kausalität

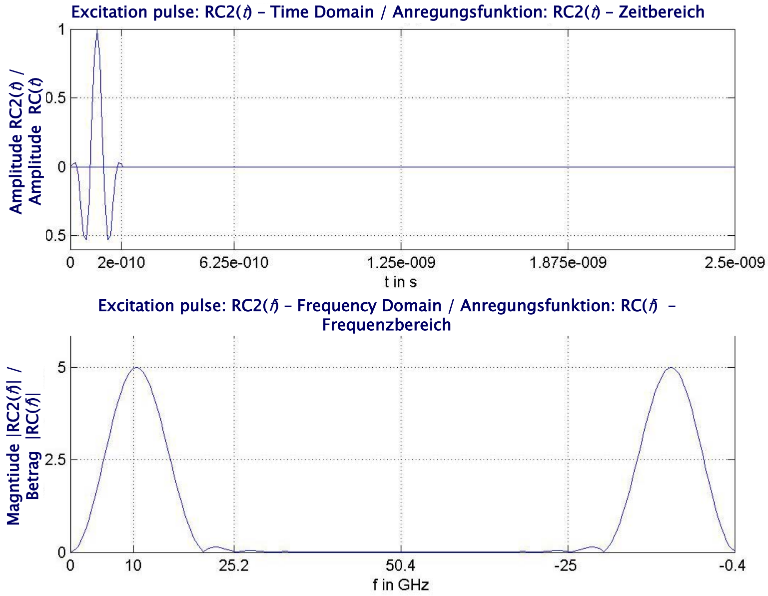
Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

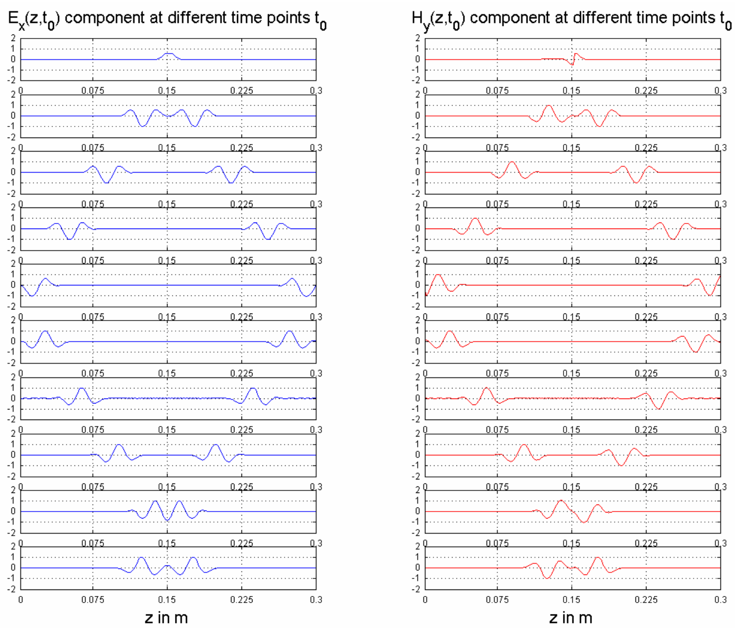


Discrete hyperbolic initial-boundary-value problem / Diskretes hyperbolisches Anfangs-Randwert-Problem

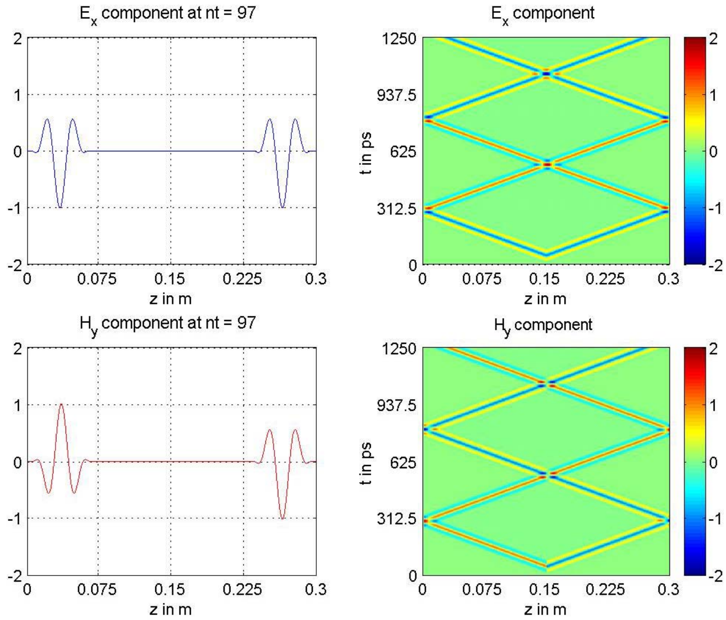
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



Implementation of Boundary Conditions / Implementierung von Randbedingungen

Boundary condition for a perfectly electrically conducting (PEC) material
/
Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

Absorbing/open boundary condition /
Absorbierende/offene Randbedingung

Space-time-extrapolation of the first order /
Raum-Zeit-Extrapolation der ersten Ordnung

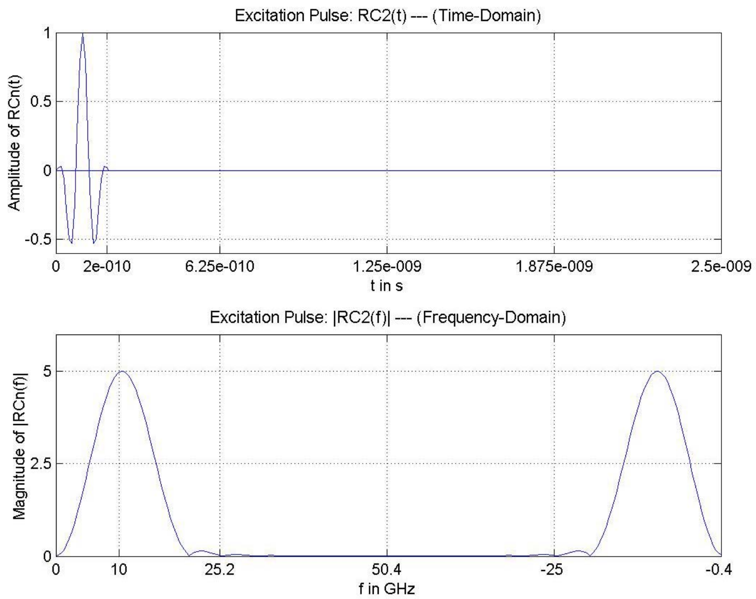
For / Für $\widehat{\Delta t} = 0.5$

a plane wave needs two time steps, 2 n_t, to travel over one grid cell with the size Δz /
braucht eine ebene Welle zwei Zeitschritte, 2 n_t, um sich über eine Gitterzelle der Größe Δz auszubreiten

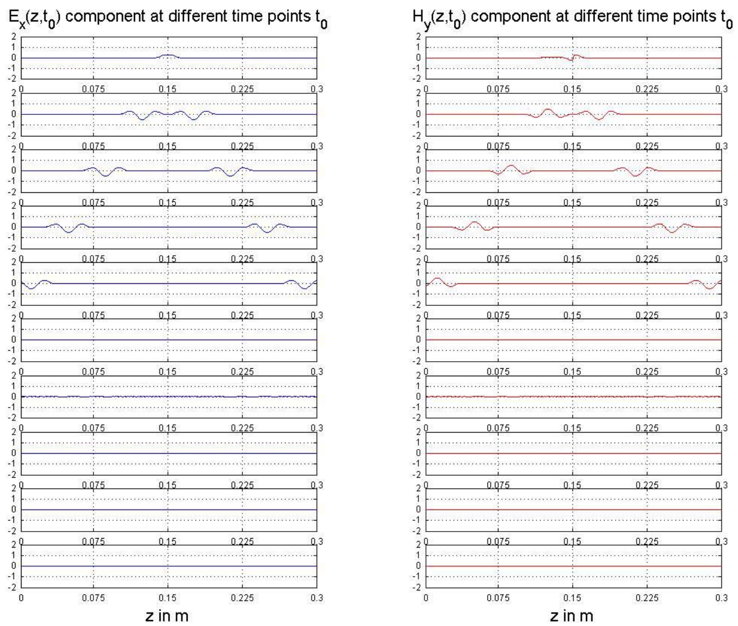
$$\left.\begin{aligned} E_x^{(1,n_t)} &= E_x^{(2,n_t-2)} \\ E_x^{(N_z,n_t)} &= E_x^{(N_z-1,n_t-2)} \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

Space-time-extrapolation of the first order /
Raum-Zeit-Extrapolation der ersten Ordnung

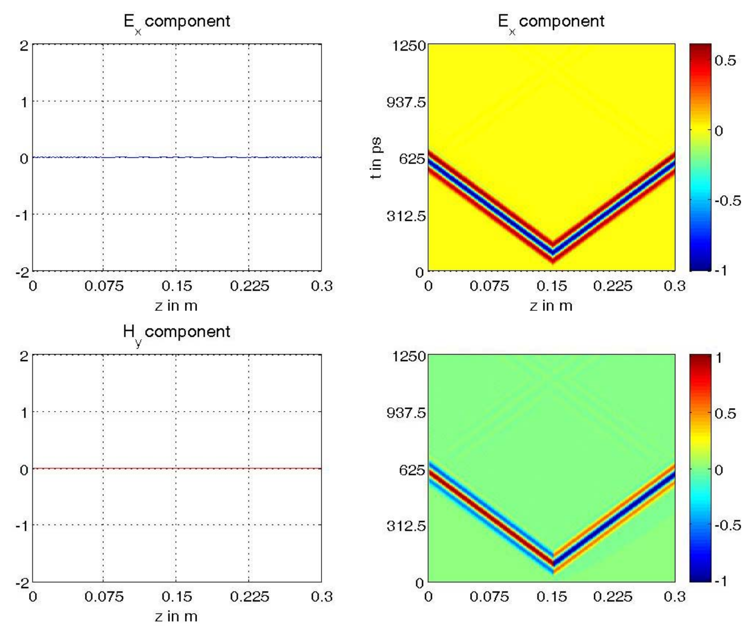
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



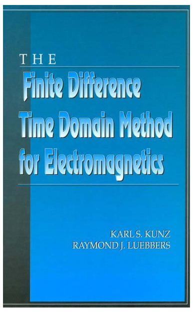
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



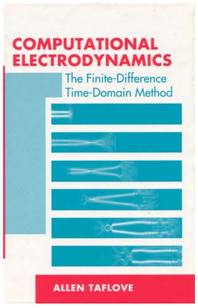
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



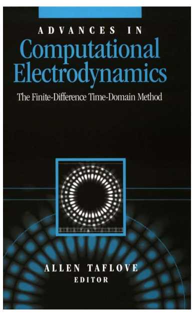
FDTD Books / FDTD-Bücher



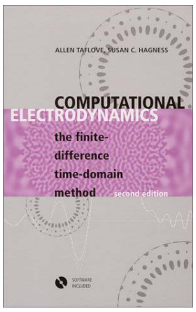
Kunz, K. S., Luebbers, R. J.: ***The Finite Difference Time Domain Method for Electromagnetics.*** **1993**



Taflove, A. (Editor): ***Computational Electrodynamics: The Finite-Difference Time-Domain Method.*** **Artech House, Boston, 1995.**

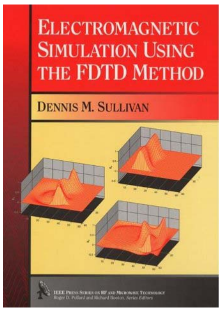


Taflove, A. (Editor): ***Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method.*** **Artech House, 1998.**



Taflove, A. (Editor): ***Computational Electrodynamics: The Finite-Difference Time-Domain Method.*** **2nd Editon, Artech House, Boston, 2000.**

FDTD Books / FDTD-Bücher



Sullivan, D. M.: *Electromagnetic Simulation Using the FDTD Method.* IEEE Press, New York, 2000.

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

The first two Maxwell's Equations are in differential form / Die ersten beiden Maxwellschen Gleichungen lauten in Differentialform:

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

In Cartesian Coordinates we find for the Curl operator applied to E and H / Im Kartesischen Koordinatensystem finden wir für den Rotationsoperator angewendet auf E und H:

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(\underline{\mathbf{R}},t) & E_y(\underline{\mathbf{R}},t) & E_z(\underline{\mathbf{R}},t) \end{vmatrix}$$

$$= \left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(\underline{\mathbf{R}},t) & H_y(\underline{\mathbf{R}},t) & H_z(\underline{\mathbf{R}},t) \end{vmatrix}$$

$$= \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z$$

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

If we insert the last expressions into the first two Maxwell's equations are in differential form read / Wenn wir die letzten Ausdrücke in the ersten beiden Maxwellschen Gleichungen in Differentialform einsetzen, erhalten wir:

$$\frac{\partial}{\partial t}\mathbf{B}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\left[B_x(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + B_y(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + B_z(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$
$$= -\left\{\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z\right\}$$
$$-\left[J_{\mathrm{m}x}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + J_{\mathrm{m}y}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + J_{\mathrm{m}z}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$

$$\frac{\partial}{\partial t}\mathbf{D}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\left[D_x(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + D_y(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + D_z(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$
$$= \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right]\underline{\mathbf{e}}_z$$
$$-\left[J_{\mathrm{e}x}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + J_{\mathrm{e}y}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + J_{\mathrm{e}z}(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]$$

Six decoupled scalar equations! /
Sechs entkoppelte skalare Gleichungen!

3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

If we insert the last expressions into the first two Maxwell's equations are in differential form we read / Wenn wir die letzten Ausdrücke in die ersten beiden Maxwellschen Gleichungen in Differentialform einsetzen, erhalten wir:

$$\frac{\partial}{\partial t}B_x(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{m}x}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}B_y(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{m}y}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}B_z(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right] - J_{\mathrm{m}z}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}D_x(\underline{\mathbf{R}},t) = \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{e}x}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}D_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{e}y}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}D_z(\underline{\mathbf{R}},t) = \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right] - J_{\mathrm{e}z}(\underline{\mathbf{R}},t)$$

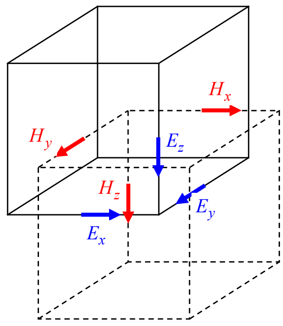
3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

Constitutive equation for homogeneous isotropic materials / Konstituierende Gleichungen für homogene isotrope Materialien:

$$B_x(\underline{\mathbf{R}},t) = \mu H_x(\underline{\mathbf{R}},t) \qquad D_x(\underline{\mathbf{R}},t) = \mu E_x(\underline{\mathbf{R}},t)$$
$$B_y(\underline{\mathbf{R}},t) = \mu H_y(\underline{\mathbf{R}},t) \qquad D_y(\underline{\mathbf{R}},t) = \mu E_y(\underline{\mathbf{R}},t)$$
$$B_z(\underline{\mathbf{R}},t) = \mu H_z(\underline{\mathbf{R}},t) \qquad D_z(\underline{\mathbf{R}},t) = \mu E_z(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu H_x(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{m}x}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\mu H_y(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{m}y}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\mu H_z(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial y}\right] - J_{\mathrm{m}z}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_x(\underline{\mathbf{R}},t) = \left[\frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{e}x}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\varepsilon E_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{\mathrm{e}y}(\underline{\mathbf{R}},t)$$
$$\frac{\partial}{\partial t}\varepsilon E_z(\underline{\mathbf{R}},t) = \left[\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - \frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial y}\right] - J_{\mathrm{e}z}(\underline{\mathbf{R}},t)$$



$$H_{x_i} = J_{\mathrm{m}x_i}, i = 1,2,3$$
$$E_{x_i} = J_{\mathrm{e}x_i}, i = 1,2,3$$

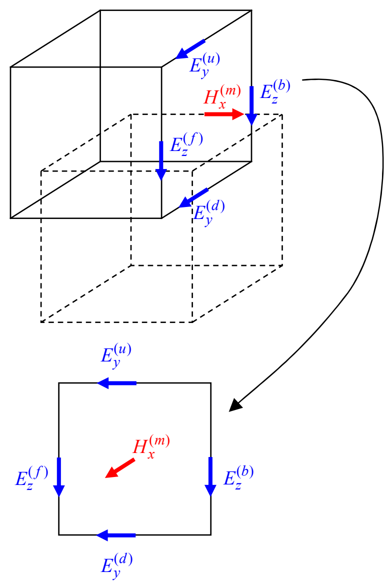
3-D FDTD – Derivation of the Discrete Equations / 3D-FDTD – Ableitung der diskreten Gleichungen

$$\frac{\partial}{\partial t}H_x(\underline{\mathbf{R}},t) = \dot{H}_x(\underline{\mathbf{R}},t)$$
$$\mu\dot{H}_x(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} - \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z}\right] - J_{\mathrm{m}x}(\underline{\mathbf{R}},t)$$

$$\mu\dot{H}_x(\underline{\mathbf{R}},t) = \dot{H}_x^{(m)}(t)$$
$$J_{\mathrm{m}x}(\underline{\mathbf{R}},t) = J_{\mathrm{m}x}^{(m)}(t)$$
$$\frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial y} = \frac{E_z^{(f)}(t) - E_z^{(b)}(t)}{\Delta y} + O\left[(\Delta y)^2\right]$$
$$\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z} = \frac{E_y^{(d)}(t) - E_y^{(u)}(t)}{\Delta z} + O\left[(\Delta z)^2\right]$$

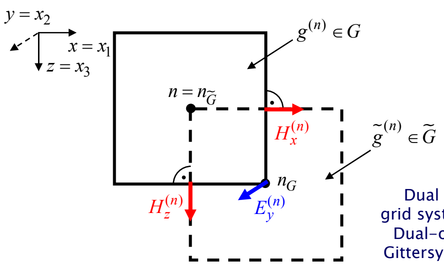
$$\mu\dot{H}_x^{(m)}(t) = \underbrace{-\frac{E_z^{(f)}(t) - E_z^{(b)}(t)}{\Delta y} + \frac{E_y^{(d)}(t) - E_y^{(u)}(t)}{\Delta z}}_{} - J_{\mathrm{m}x}^{(m)}(t)$$

A part of the discrete curl operator / Ein Teil des diskreten Rotationsoperators

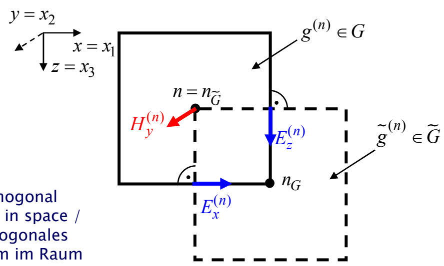


2-D EM Wave Propagation – 2-D FDTD – TM and TE Case / 2D EM Wellenausbreitung – 2D-FDTD – TM- und TE-Fall

2-D TM Case / 2D-TM-Fall



2-D TE Case / 2D-TE-Fall



Dual orthogonal grid system in space / Dual-orthogonales Gittersystem im Raum

$G \perp \tilde{G}$

$$\frac{\partial}{\partial t}\mu H_x(\underline{\mathbf{R}},t) = \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z} - J_{mx}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu H_z(\underline{\mathbf{R}},t) = -\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - J_{mz}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{ey}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + z\underline{\mathbf{e}}_z$$

$$\frac{\partial}{\partial t}\mu H_y(\underline{\mathbf{R}},t) = -\left[\frac{\partial E_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial E_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{my}(\underline{\mathbf{R}},t)$$

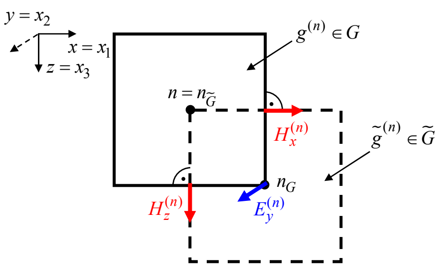
$$\frac{\partial}{\partial t}\varepsilon E_x(\underline{\mathbf{R}},t) = -\frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial z} - J_{ex}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_z(\underline{\mathbf{R}},t) = \frac{\partial H_y(\underline{\mathbf{R}},t)}{\partial x} - J_{ez}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + z\underline{\mathbf{e}}_z$$

2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall

2-D TM Case / 2D-TM-Fall



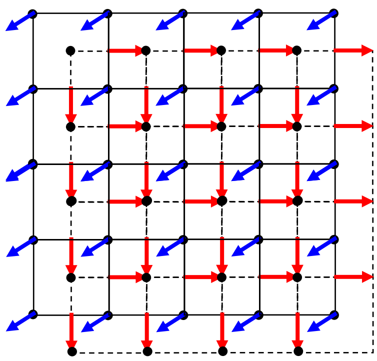
$$\frac{\partial}{\partial t}\mu H_x(\underline{\mathbf{R}},t) = \frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial z} - J_{mx}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\mu H_z(\underline{\mathbf{R}},t) = -\frac{\partial E_y(\underline{\mathbf{R}},t)}{\partial x} - J_{mz}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\varepsilon E_y(\underline{\mathbf{R}},t) = \left[\frac{\partial H_x(\underline{\mathbf{R}},t)}{\partial z} - \frac{\partial H_z(\underline{\mathbf{R}},t)}{\partial x}\right] - J_{ey}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + z\underline{\mathbf{e}}_z$$

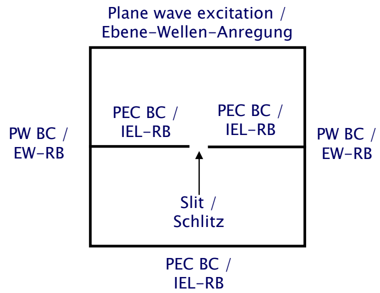
Two-dimensional staggered grid system in the 2-D TM case / Zweidimensionales versetztes Gittersystem im 2D-TM-Fall



Implementation of Boundary Conditions / Implementierung von Randbedingungen

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

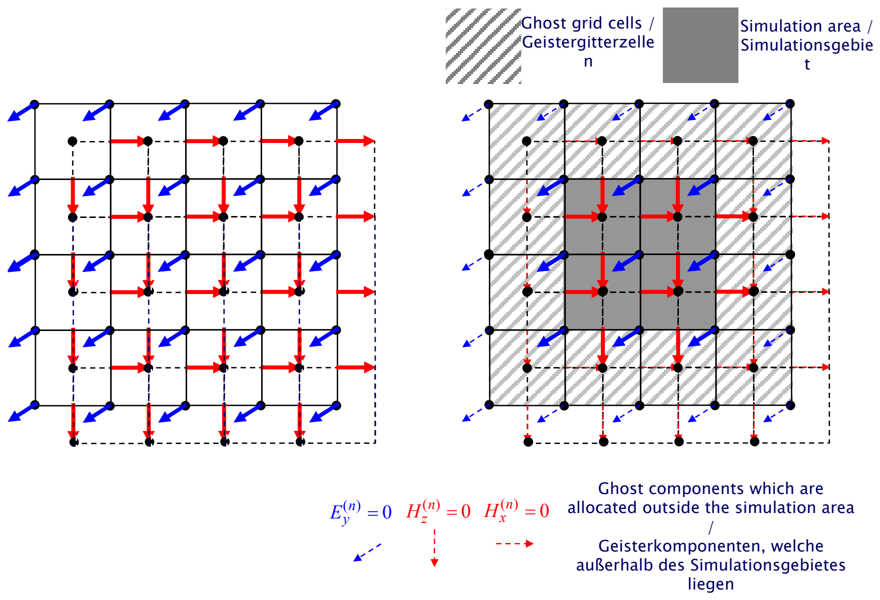
$$\left.\begin{aligned} E_y^{(\bullet,\bullet,n_t)} &= 0 \\ E_y^{(\bullet,\bullet,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$



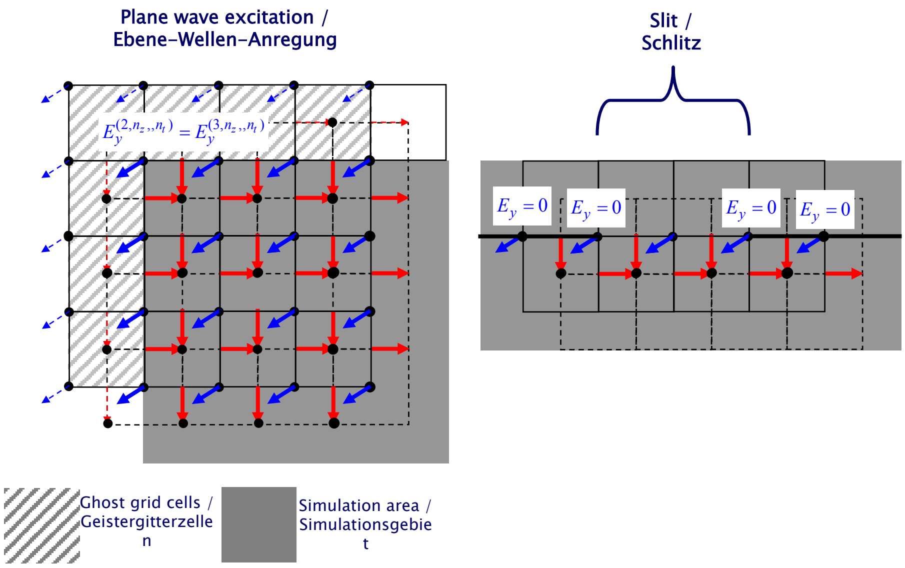
Plane wave boundary condition for a vertical incident plane wave / Ebene-Wellen-Randbedingung für eine vertikal einfallende ebene Welle

$$\left.\begin{aligned} E_y^{(2,n_z,n_t)} &= E_y^{(3,n_z,n_t)} \\ E_y^{(N_x-1,n_z,n_t)} &= E_y^{(N_x-2,n_z,n_t-2)} \end{aligned}\right\} \quad \begin{aligned} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{aligned}$$

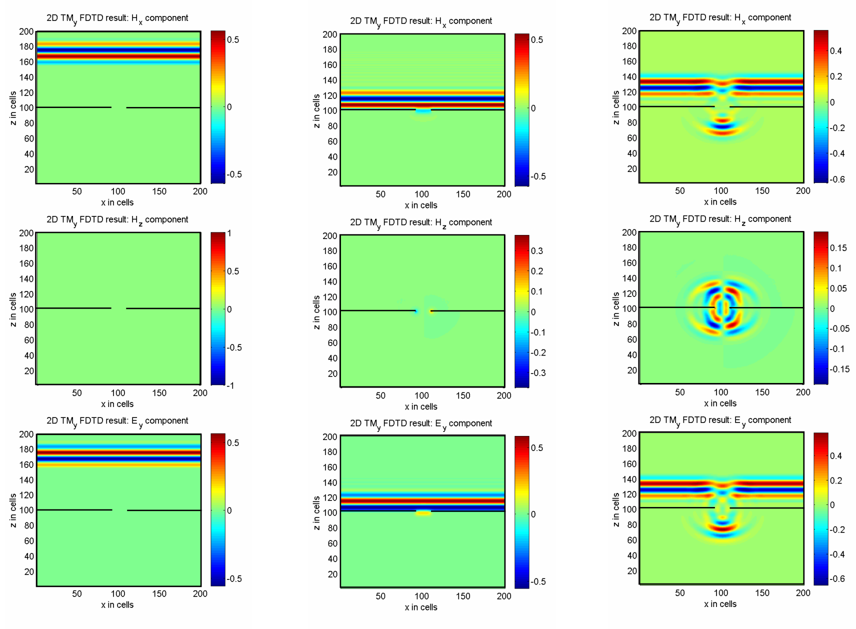
2-D EM Wave Propagation – 2-D FDTD – TM Case / 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall



2-D EM Wave Propagation – 2-D FDTD – TM Case/ 2D EM Wellenausbreitung – 2D-FDTD – TM-Fall



2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung an einem Spalt



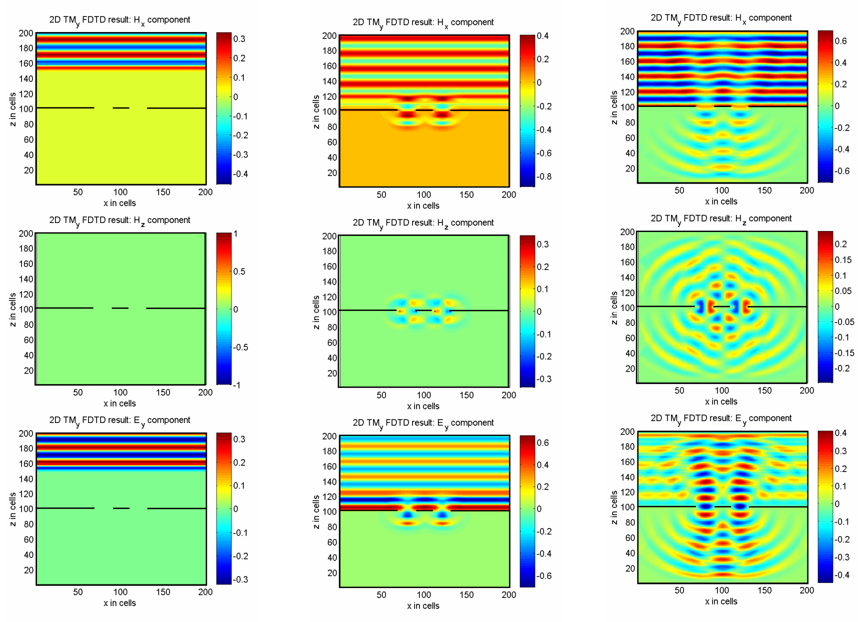
2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung am Spalt

Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente

Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt



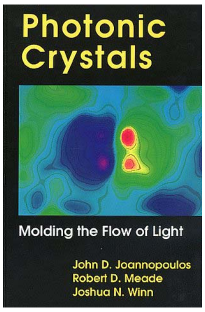
2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente

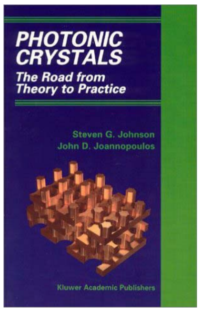
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

Photonic Crystals / Photonische Kristalle



Joannopoulos, J. D., R. D. Meade, J. N. Winn: *Photonic Crystals – Molding the Flow of Light. Princeton University Press, Princeton, 1995.*



Johnson, S. G.: *Photonic Crystals: The Road from Theory to Practice.* Kluwer Academic Press, 2001.

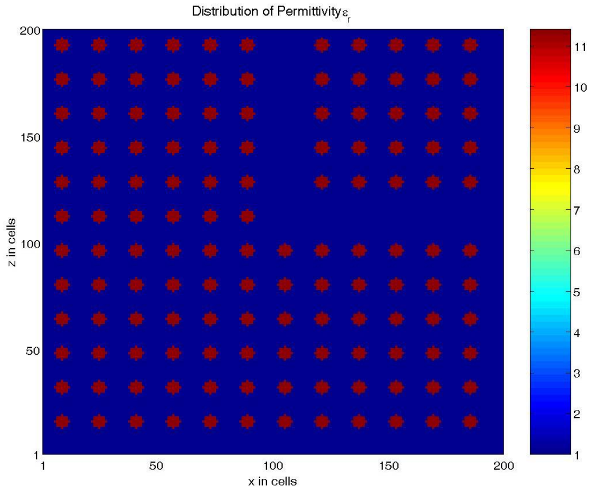
Links:

Photonic Crystals Research at MIT
Homepage of Prof. Sajeev John, University of Toronto, Canada

2-D TM FDTD - Photonic Crystals /
2D-TM-FDTD - Photonische Kristalle

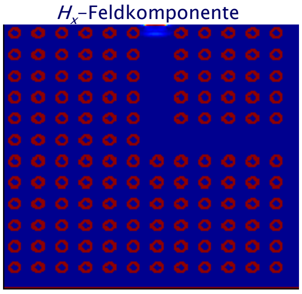
Relative permittivity of the background
Relative Permittivität des Hintergrundes $\varepsilon_{\mathrm{r}}^{(b)} = 1$

Relative permittivity of the rods
Relative Permittivität der Stäbe $\varepsilon_{\mathrm{r}}^{(r)} = 11.4$

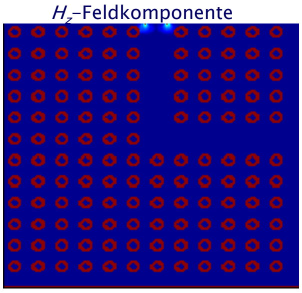


2-D TM FDTD - Photonic Crystals /
2D-TM-FDTD - Photonische Kristalle

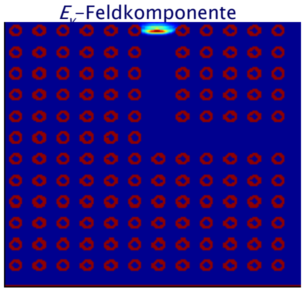
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

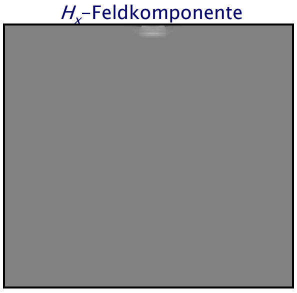


Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

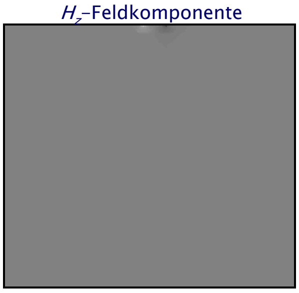


2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

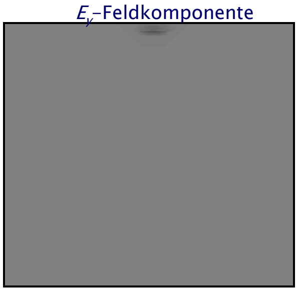
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



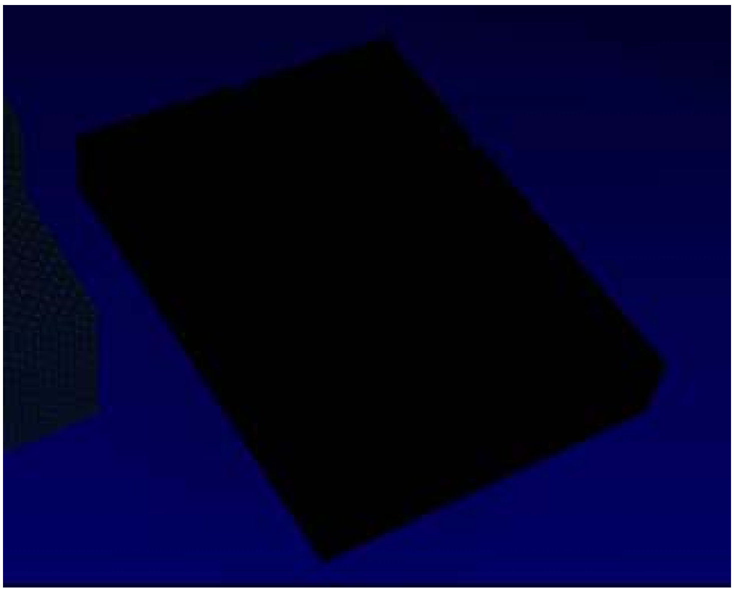
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente



2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle



2-D TM FDTD – Photonic Crystals /
2D-TM-FDTD – Photonische Kristalle

End of Lecture 6 /
Ende der 6. Vorlesung