Numerical Methods of Electromagnetic Field Theory I (NFT I) Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

9th Lecture / 9. Vorlesung

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3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

Local grid equations in local notation /
Lokale Gittergleichungen in lokaler Notation

$$\frac{\mathrm{d}}{\mathrm{d}t}B_x^{(m)}(t)\Delta y\Delta z=-\left[E_y^{(u)}(t)\Delta y+E_z^{(f)}(t)\Delta z-E_y^{(d)}(t)\Delta y-E_z^{(b)}(t)\Delta z\right]-J_{\mathrm{mx}}^{(m)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_y^{(m)}(t)\Delta x\Delta z=-\left[-E_x^{(u)}(t)\Delta x+E_z^{(l)}(t)\Delta z+E_x^{(d)}(t)\Delta x-E_z^{(r)}(t)\Delta z\right]-J_{\mathrm{my}}^{(m)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_z^{(m)}(t)\Delta x\Delta y=-\left[E_x^{(b)}(t)\Delta x+E_y^{(r)}(t)\Delta y-E_x^{(f)}(t)\Delta x-E_y^{(l)}(t)\Delta y\right]-J_{\mathrm{mz}}^{(m)}(t)\Delta x\Delta y$$

Local grid equations in global grid node notation /
Lokale Gittergleichungen in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t}B_x^{(n)}(t)\Delta y\Delta z=-\left\{\left[E_y^{(n-M_z)}(t)-E_y^{(n)}(t)\right]\Delta y+\left[E_z^{(n)}(t)-E_z^{(n-M_y)}(t)\right]\Delta z\right\}-J_{\mathrm{mx}}^{(n)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_y^{(n)}(t)\Delta x\Delta z=-\left\{\left[E_x^{(n)}(t)-E_x^{(n-M_z)}(t)\right]\Delta x+\left[E_z^{(n-M_x)}(t)-E_z^{(n)}(t)\right]\Delta z\right\}-J_{\mathrm{my}}^{(n)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_z^{(n)}(t)\Delta x\Delta y=-\left\{\left[E_x^{(n-M_y)}(t)-E_x^{(n)}(t)\right]\Delta x+\left[E_y^{(n)}(t)-E_y^{(n-M_x)}(t)\right]\Delta y\right\}-J_{\mathrm{mz}}^{(n)}(t)\Delta x\Delta y$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

Local grid equations in global grid node notation / Lokale Gittergleichungen in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t} B_x^{(n)}(t)\Delta y \Delta z = -\left\{\left[E_y^{(n-M_z)}(t) - E_y^{(n)}(t)\right]\Delta y + \left[E_z^{(n)}(t) - E_z^{(n-M_y)}(t)\right]\Delta z\right\} - J_{\mathrm{mx}}^{(n)}(t)\Delta y \Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_y^{(n)}(t)\Delta x \Delta z = -\left\{\left[E_x^{(n)}(t) - E_x^{(n-M_z)}(t)\right]\Delta x + \left[E_z^{(n-M_x)}(t) - E_z^{(n)}(t)\right]\Delta z\right\} - J_{\mathrm{my}}^{(n)}(t)\Delta x \Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_z^{(n)}(t)\Delta x \Delta y = -\left\{\left[E_x^{(n-M_y)}(t) - E_x^{(n)}(t)\right]\Delta x + \left[E_y^{(n)}(t) - E_y^{(n-M_x)}(t)\right]\Delta y\right\} - J_{\mathrm{mz}}^{(n)}(t)\Delta x \Delta y$$

Local spatial shift operators / Lokale räumliche Schiebeoperatoren

$$S_{\pm M_i} f^{(n)} = f^{(n \pm M_i)}$$

$$S_0 f^{(n)} = f^{(n)}$$

$$S_0 = I$$

$$I\, f^{(n)} = f^{(n)}$$

Local grid equations with local spatial shift operators in global grid node notation / Lokale Gittergleichungen mit lokalen räumlichen Schiebeoperatoren in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t} B_x^{(n)}(t)\Delta y \Delta z = -\left\{\left[S_{-M_z} - I\right]E_y^{(n)}(t)\Delta y + \left[I - S_{M_y}\right]E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{mx}}^{(n)}(t)\Delta y \Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_y^{(n)}(t)\Delta x \Delta z = -\left\{\left[I - S_{-M_z}\right]E_x^{(n)}(t)\Delta x + \left[S_{-M_x} - I\right]E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{my}}^{(n)}(t)\Delta x \Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_z^{(n)}(t)\Delta x \Delta y = -\left\{\left[S_{-M_y} - I\right]E_x^{(n)}(t)\Delta x + \left[I - S_{-M_x}\right]E_y^{(n)}(t)\Delta y\right\} - J_{\mathrm{mz}}^{(n)}(t)\Delta x \Delta y$$

3-D FIT – Local Spatial Shift Operators /
3D-FIT – Lokale räumliche Schiebeoperatoren

1. Simple spatial shift operation / Einfache räumliche Schiebeoperation

$$S_{\pm M_i} f^{(n)} = f^{(n \pm M_i)}$$

2. Identity operation / Identitätsoperation

$$I\, f^{(n)} = f^{(n)}$$

3. Multiple shift operations / Zusammengesetzte Schiebeoperationen

$$S_{\pm M_i} S_{\pm M_j} f^{(n)} = S_{\pm M_j} S_{\pm M_i} f^{(n)} = f^{(n \pm M_i \pm M_j)}$$

Special case for $M_j = -M_i$ / Speziell folgt für $M_j = -M_i$

$$S_{\pm M_i} S_{\mp M_i} = I$$

4. Local difference operator / Lokaler Differenzoperator

$$P_{\pm M_i} = \mp I \pm S_{\pm M_i}$$

5. Local averaging operator / Lokaler Mittelungsoperator

$$A_{\pm M_i} = \frac{1}{2}\left(I + S_{\pm M_i} \right)$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

Local grid equations with local spatial shift operators in global grid node notation / Lokale Gittergleichungen mit lokalen räumlichen Schiebeoperatoren in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t} B_x^{(n)}(t)\Delta y\Delta z = -\left\{\left[S_{-M_z} - I\right]E_y^{(n)}(t)\Delta y + \left[I - S_{M_y}\right]E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{m}x}^{(n)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_y^{(n)}(t)\Delta x\Delta z = -\left\{\left[I - S_{-M_z}\right]E_x^{(n)}(t)\Delta x + \left[S_{-M_x} - I\right]E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{m}y}^{(n)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_z^{(n)}(t)\Delta x\Delta y = -\left\{\left[S_{-M_y} - I\right]E_x^{(n)}(t)\Delta x + \left[I - S_{-M_x}\right]E_y^{(n)}(t)\Delta y\right\} - J_{\mathrm{m}z}^{(n)}(t)\Delta x\Delta y$$

… in local matrix form / … in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{B\}^{(n)}(t)} = -\underbrace{\begin{bmatrix} 0 & S_{-M_z} - I & I - S_{M_y} \\ I - S_{-M_z} & 0 & S_{-M_x} - I \\ S_{-M_y} - I & I - S_{-M_x} & 0 \end{bmatrix}}_{=[\mathrm{curl}]} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=[R]} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{E\}^{(n)}(t)}$$

$$-\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \underbrace{\begin{Bmatrix} J_{\mathrm{m}x}^{(n)}(t) \\ J_{\mathrm{m}y}^{(n)}(t) \\ J_{\mathrm{m}z}^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{J_\mathrm{m}\}^{(n)}(t)}$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form / 3D-FIT – … diskreten Gittergleichungen in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \Delta y \Delta z & & \\ & \Delta x \Delta z & \\ & & \Delta x \Delta y \end{bmatrix}}_{=[S]} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{B\}^{(n)}(t)} = - \underbrace{\begin{bmatrix} 0 & S_{-M_z} - I & I - S_{M_y} \\ I - S_{-M_z} & 0 & S_{-M_x} - I \\ S_{-M_y} - I & I - S_{-M_x} & 0 \end{bmatrix}}_{=[\mathrm{curl}]} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=[R]} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{E\}^{(n)}(t)}$$

$$- \underbrace{\begin{bmatrix} \Delta y \Delta z & & \\ & \Delta x \Delta z & \\ & & \Delta x \Delta y \end{bmatrix}}_{=[S]} \underbrace{\begin{Bmatrix} J_{\mathrm{mx}}^{(n)}(t) \\ J_{\mathrm{mx}}^{(n)}(t) \\ J_{\mathrm{mx}}^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{J_{\mathrm{m}}\}^{(n)}(t)}$$

$$\begin{bmatrix} 0 & S_{-M_z} - I & I - S_{-M_y} \\ I - S_{-M_z} & 0 & S_{-M_x} - I \\ S_{-M_y} - I & I - S_{-M_x} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -P_{-M_z} & P_{-M_y} \\ P_{-M_z} & 0 & -P_{-M_x} \\ -P_{-M_y} & P_{-M_x} & 0 \end{bmatrix} = [\mathrm{curl}]$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form /
3D-FIT – … diskreten Gittergleichungen in lokaler Matrixform

Faraday's induction law in local matrix form / Faradaysches Induktionsgesetz in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{B\}^{(n)}(t)} = -\underbrace{\begin{bmatrix} 0 & -P_{-M_z} & P_{-M_y} \\ P_{-M_z} & 0 & -P_{-M_x} \\ -P_{-M_y} & P_{-M_x} & 0 \end{bmatrix}}_{=[\mathrm{curl}]} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=[R]} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{E\}^{(n)}(t)} - \underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \underbrace{\begin{Bmatrix} J_{\mathrm{mx}}^{(n)}(t) \\ J_{\mathrm{mx}}^{(n)}(t) \\ J_{\mathrm{mx}}^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{J_{\mathrm{m}}\}^{(n)}(t)}$$

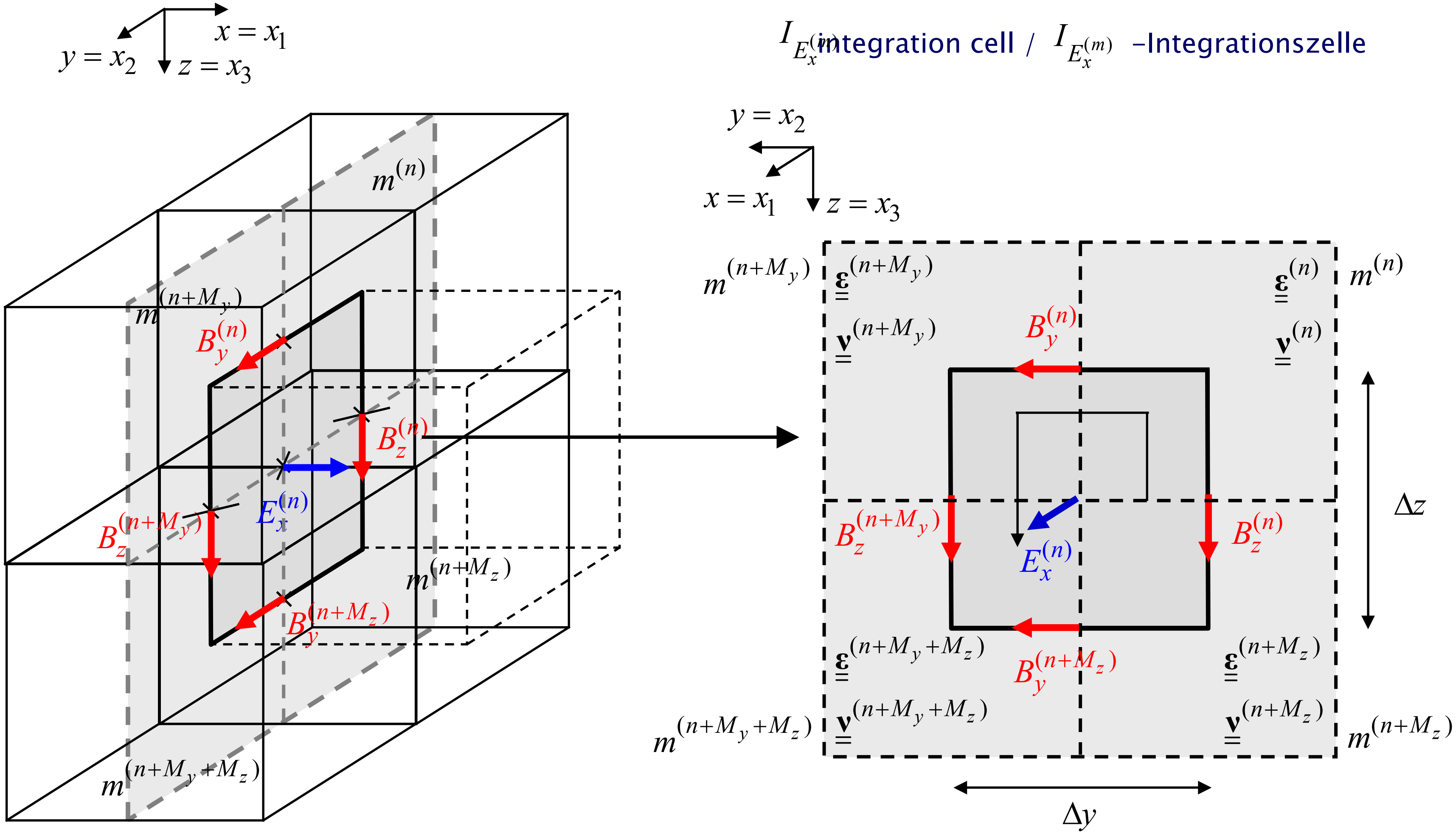
$$[S]\frac{\mathrm{d}}{\mathrm{d}t}\{B\}^{(n)}(t) = -[\mathrm{curl}][R]\{E\}^{(n)}(t) - [S]\{J_{\mathrm{m}}\}^{(n)}(t)$$

$[S]$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of elementary surfaces on the grid G / Diagonalmatrix der Elementarflächen auf dem Gitter G
$\{B\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$[\mathrm{curl}]$	$\in \mathbb{R}^{3\times 3}$	Topological curl operator in matrix form on the grid G / Topologischer Rotationsoperator in Matrixform auf dem Gitter G
$[R]$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of elementary lines on the grid G / Diagonalmatrix der Elementarstrecken auf dem Gitter G
$\{E\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic electric field strength vector / Algebraischer elektrische Feldstärkevektor
$\{J_{\mathrm{m}}\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic magnetic current density vector / Algebraischer magnetischer Stromdichtevektor

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$I_{E_x^{(n)}}$ integration cell / $I_{E_x^{(m)}}$ –Integrationszelle



3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}}}_{} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

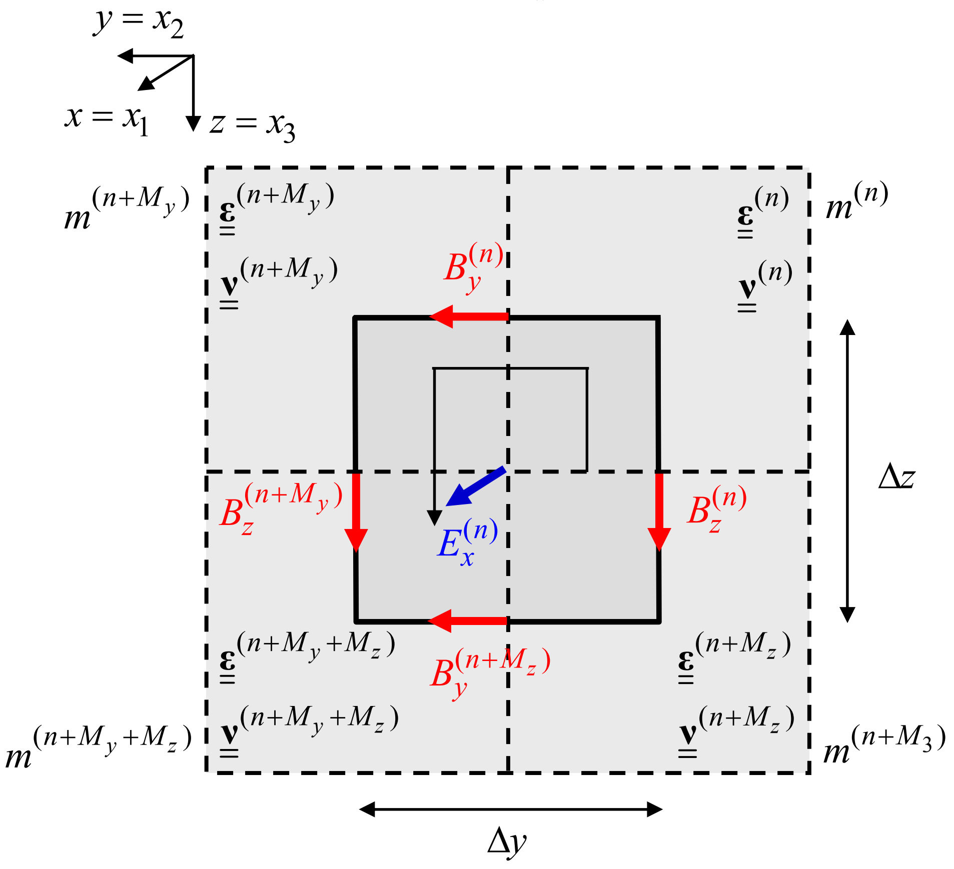
$$\longrightarrow \iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\underline{\mathbf{dS}} = \iint_S \underline{\mathbf{e}}_x\bullet\left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\mathrm{d}S$$

$$= \iint_S \varepsilon_{xx}(\underline{\mathbf{R}})\,E_x(\underline{\mathbf{R}},t)\,\mathrm{d}S$$

$$= E_x^{(n)}(t)\iint_S \varepsilon_{xx}(\underline{\mathbf{R}})\,\mathrm{d}S$$

$$+\mathrm{O}\left[(\Delta y)^3\,\Delta z + \Delta y(\Delta z)^3\right]$$

$I_{E_x^{(n)}}$ integration cell / $I_{E_x^{(m)}}$ –Integrationszelle



$$\iint_S \varepsilon_{xx}(\underline{\mathbf{R}})\,\mathrm{d}S$$

$$= \underbrace{\frac{1}{4}\left[\varepsilon_{xx}^{(n)} + \varepsilon_{xx}^{(n+M_y)} + \varepsilon_{xx}^{(n+M_z)} + \varepsilon_{xx}^{(n+M_y+M_z)}\right]}_{=\tilde{\varepsilon}_{xx}^{(n)}}\Delta y\Delta z$$

$$= \tilde{\varepsilon}_{xx}^{(n)}\,\Delta y\Delta z$$

$$\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\underline{\mathbf{dS}}$$

$$= E_x^{(n)}(t)\tilde{\varepsilon}_{xx}^{(n)}\,\Delta y\Delta z + \mathrm{O}\left[(\Delta y)^3\,\Delta z + \Delta y(\Delta z)^3\right]$$

$$\iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$= J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z + \mathrm{O}\left[(\Delta y)^3\,\Delta z + \Delta y(\Delta z)^3\right]$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

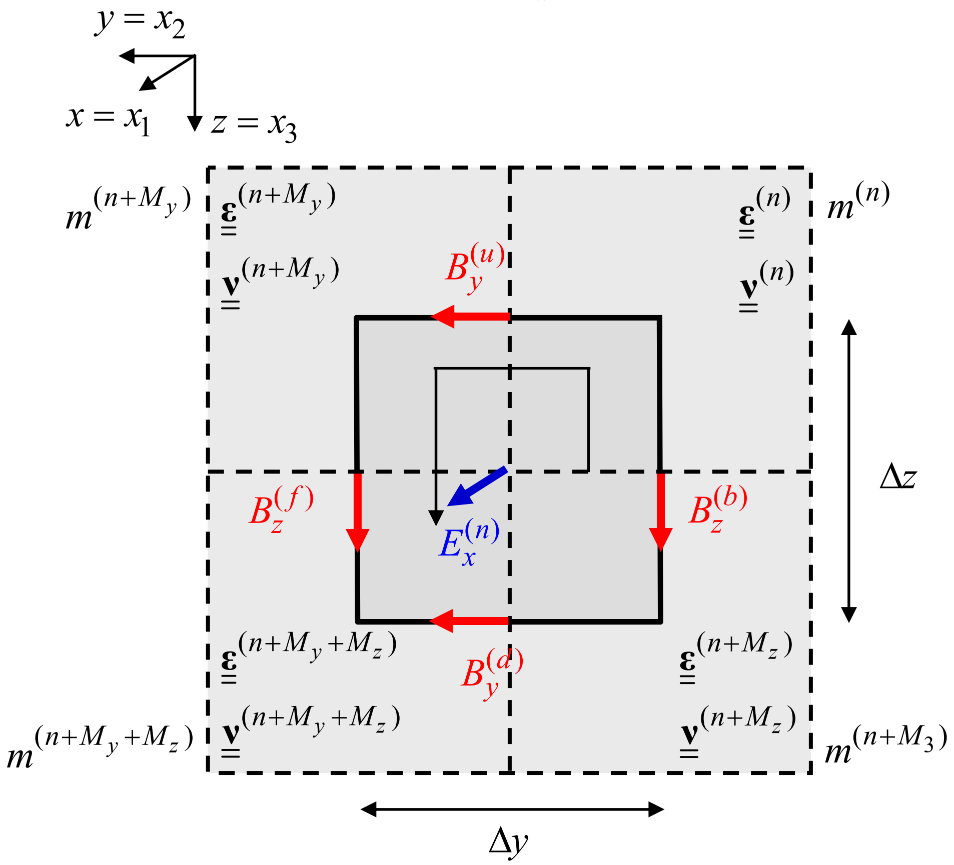
$$\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} = ?$$

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}\,\mathrm{d}S = \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z$$

$$\underline{\mathbf{dR}}_y = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_y\,\mathrm{d}y$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\,\mathrm{d}z$$

$I_{E_x^{(m)}}$ integration cell / $I_{E_x^{(m)}}$ –Integrationszelle



$$\begin{aligned}\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} &= \int_{C^{(u)}}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}\\ &+\int_{C^{(f)}}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}\\ &+\int_{C^{(d)}}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}\\ &+\int_{C^{(b)}}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}\end{aligned}$$

$$\begin{aligned}\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} &= \int_{C^{(u)}} \nu_{yy}(\underline{\mathbf{R}})B_y(\underline{\mathbf{R}},t)\,\mathrm{d}y\\ &+\int_{C^{(f)}} \nu_{zz}(\underline{\mathbf{R}})B_z(\underline{\mathbf{R}},t)\,\mathrm{d}z\\ &-\int_{C^{(d)}} \nu_{yy}(\underline{\mathbf{R}})B_y(\underline{\mathbf{R}},t)\,\mathrm{d}y\\ &-\int_{C^{(b)}} \nu_{zz}(\underline{\mathbf{R}})B_z(\underline{\mathbf{R}},t)\,\mathrm{d}z\end{aligned}$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}$$

$$=\int_{C^{(u)}}\nu_{yy}(\underline{\mathbf{R}})B_y(\underline{\mathbf{R}},t)\,\mathrm{d}y+\int_{C^{(f)}}\nu_{zz}(\underline{\mathbf{R}})B_z(\underline{\mathbf{R}},t)\,\mathrm{d}z-\int_{C^{(d)}}\nu_{yy}(\underline{\mathbf{R}})B_y(\underline{\mathbf{R}},t)\,\mathrm{d}y-\int_{C^{(b)}}\nu_{zz}(\underline{\mathbf{R}})B_z(\underline{\mathbf{R}},t)\,\mathrm{d}z$$

$$\int_{C^{(u)}}\nu_{yy}(\underline{\mathbf{R}})B_y(\underline{\mathbf{R}},t)\,\mathrm{d}y=B_y^{(u)}(t)\int_{C^{(u)}}\nu_{yy}(\underline{\mathbf{R}})\,\mathrm{d}y+\bigcirc\left[(\Delta y)^3\right]$$

$$\int_{C^{(u)}}\nu_{yy}(\underline{\mathbf{R}})\,\mathrm{d}y=\frac{1}{2}\left[\nu_{yy}^{(n)}+\nu_{yy}^{(n+M_y)}\right]\Delta y$$

$$\int_{C^{(f)}}\nu_{zz}(\underline{\mathbf{R}})B_z(\underline{\mathbf{R}},t)\,\mathrm{d}z=B_z^{(f)}(t)\int_{C^{(f)}}\nu_{zz}(\underline{\mathbf{R}})\,\mathrm{d}z+\bigcirc\left[(\Delta z)^3\right]$$

$$\int_{C^{(f)}}\nu_{zz}(\underline{\mathbf{R}})\,\mathrm{d}z=\frac{1}{2}\left[\nu_{yy}^{(n+M_z)}+\nu_{yy}^{(n+M_y+M_z)}\right]\Delta z$$

$$\int_{C^{(d)}}\nu_{yy}(\underline{\mathbf{R}})B_y(\underline{\mathbf{R}},t)\,\mathrm{d}y=B_y^{(d)}(t)\int_{C^{(d)}}\nu_{yy}(\underline{\mathbf{R}})\,\mathrm{d}y+\bigcirc\left[(\Delta y)^3\right]$$

$$\int_{C^{(d)}}\nu_{yy}(\underline{\mathbf{R}})\,\mathrm{d}y=\frac{1}{2}\left[\nu_{zz}^{(n+M_y)}+\nu_{zz}^{(n+M_y+M_z)}\right]\Delta y$$

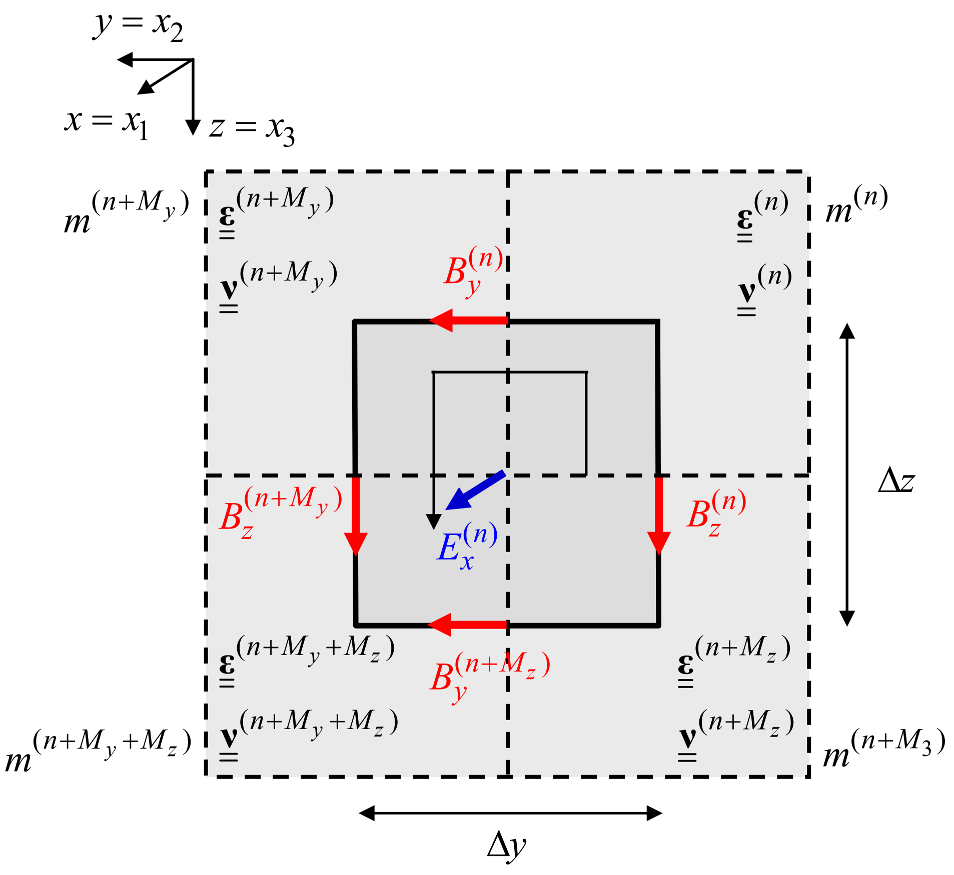
$$\int_{C^{(b)}}\nu_{zz}(\underline{\mathbf{R}})B_z(\underline{\mathbf{R}},t)\,\mathrm{d}z=B_z^{(b)}(t)\int_{C^{(b)}}\nu_{zz}(\underline{\mathbf{R}})\,\mathrm{d}z+\bigcirc\left[(\Delta z)^3\right]$$

$$\int_{C^{(b)}}\nu_{zz}(\underline{\mathbf{R}})\,\mathrm{d}z=\frac{1}{2}\left[\nu_{zz}^{(n)}+\nu_{zz}^{(n+M_z)}\right]\Delta z$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$I_{E_x^{(m)}}$-integration cell / $I_{E_x^{(m)}}$ -Integrationszelle



$$\oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}$$

$$= \underbrace{\frac{1}{2}\left[\nu_{yy}^{(n)} + \nu_{yy}^{(n+M_y)}\right]}_{=\tilde{\nu}_{yy}^{(n)}} B_y^{(n)}(t)\Delta y$$

$$-\underbrace{\frac{1}{2}\left[\nu_{yy}^{(n+M_z)} + \nu_{yy}^{(n+M_y+M_z)}\right]}_{=\tilde{\nu}_{yy}^{(n+M_y)}} B_y^{(n+M_z)}(t)\Delta y$$

$$+\underbrace{\frac{1}{2}\left[\nu_{zz}^{(n+M_y)} + \nu_{zz}^{(n+M_y+M_z)}\right]}_{=\tilde{\nu}_{zz}^{(n+M_z)}} B_z^{(n+M_y)}(t)\Delta z$$

$$-\underbrace{\frac{1}{2}\left[\nu_{zz}^{(n)} + \nu_{zz}^{(n+M_z)}\right]}_{=\tilde{\nu}_{zz}^{(n)}} B_z^{(n)}(t)\Delta z$$

$$= \tilde{\nu}_{yy}^{(n)} B_y^{(n)}(t)\Delta y - \tilde{\nu}_{yy}^{(n+M_y)} B_y^{(n+M_y)}(t)\Delta y$$

$$+\tilde{\nu}_{zz}^{(n+M_z)} B_z^{(n+M_y)}(t)\Delta z - \tilde{\nu}_{zz}^{(n)} B_z^{(n)}(t)\Delta z$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$I_{E_x^{(m)}}$ integration cell / $I_{E_x^{(m)}}$ –Integrationszelle

$$\tilde{\varepsilon}_{xx}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_x^{(n)}(t)\Delta y\Delta z = \tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - \tilde{\nu}_{yy}^{(n+M_z)}B_y^{(n+M_z)}(t)\Delta y$$

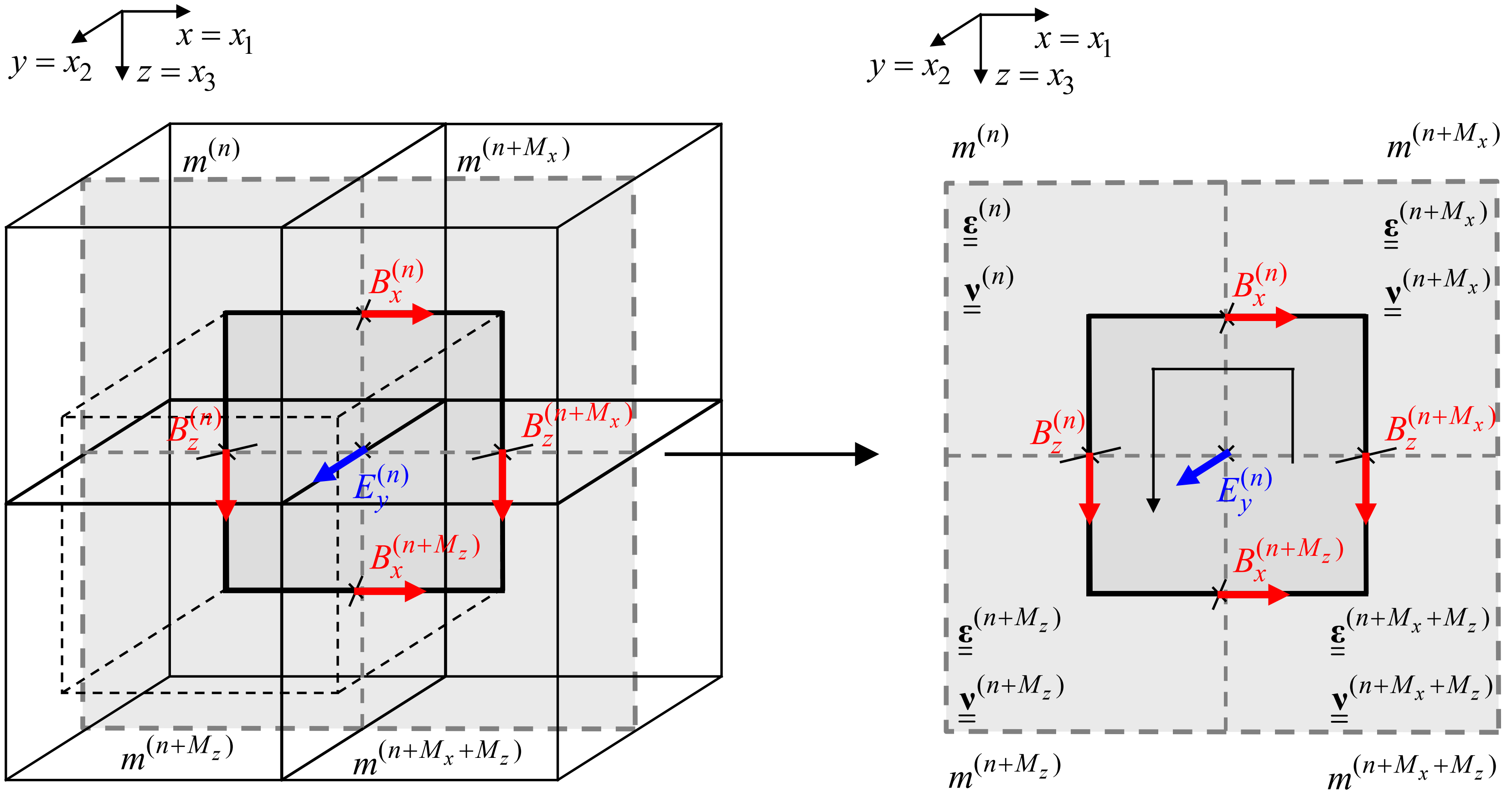
$$+\tilde{\nu}_{zz}^{(n+M_y)}B_z^{(n+M_y)}(t)\Delta z - \tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z$$

$$=\left(I - S_{M_z}\right)\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y + \left(S_{M_y} - I\right)\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

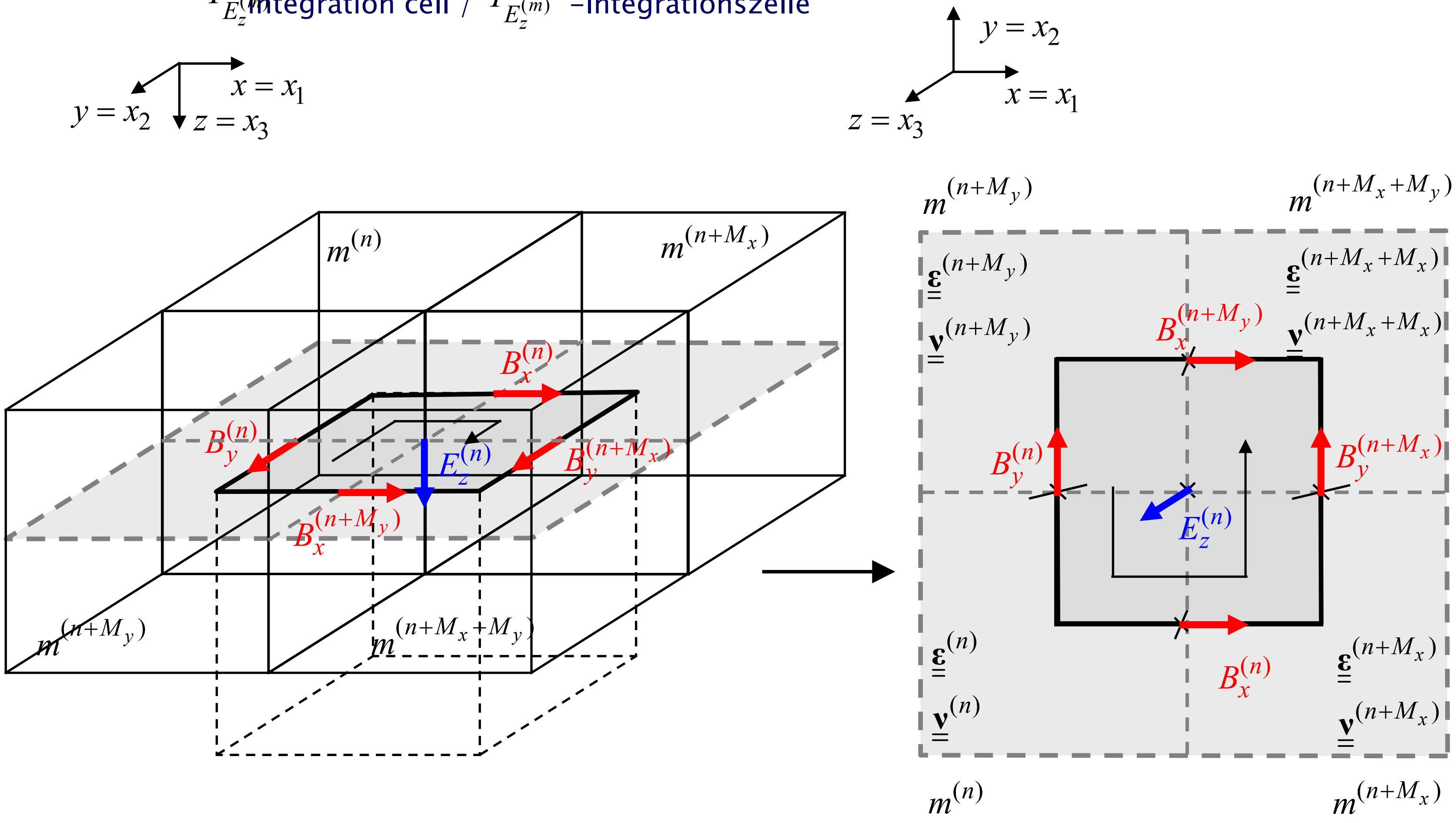
$I_{E_y^{(m)}}$ Integration cell / $I_{E_y^{(m)}}$ -Integrationszelle



3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S\left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}}=\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}}-\iint_S\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$I_{E_z^{(m)}}$ integration cell / $I_{E_z^{(m)}}$ –Integrationszelle



3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\Downarrow$$

$$\begin{aligned}
\tilde{\varepsilon}_{xx}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_x^{(n)}(t)\Delta y\Delta z &= \tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - \tilde{\nu}_{yy}^{(n+M_z)}B_y^{(n+M_z)}(t)\Delta y\\
&\quad +\tilde{\nu}_{zz}^{(n+M_y)}B_z^{(n+M_y)}(t)\Delta z - \tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z\\
&= \left(I - S_{M_z}\right)\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y + \left(S_{M_y} - I\right)\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z\\
\tilde{\varepsilon}_{yy}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_y^{(n)}(t)\Delta x\Delta z &= \tilde{\nu}_{xx}^{(n+M_3)}B_x^{(n+M_3)}(t)\Delta x - \tilde{\nu}_{xx}^{(n_z)}B_x^{(n)}(t)\Delta x\\
&\quad +\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - \tilde{\nu}_{zz}^{(n+M_x)}B_z^{(n+M_x)}(t)\Delta z - J_{\mathrm{ey}}^{(n)}(t)\Delta x\Delta z\\
&= \left(S_{M_z} - I\right)\tilde{\nu}_{xx}^{(n)}B_x^{(n)}(t)\Delta x + \left(I - S_{M_x}\right)\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ey}}^{(n)}(t)\Delta y\Delta z\\
\tilde{\varepsilon}_{zz}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_z^{(n)}(t)\Delta x\Delta y &= \tilde{\nu}_{xx}^{(n)}B_x^{(n)}(t)\Delta x - \tilde{\nu}_{xx}^{(n+M_y)}B_x^{(n+M_y)}(t)\Delta x\\
&\quad +\tilde{\nu}_{yy}^{(n+M_x)}B_y^{(n+M_x)}(t)\Delta y - \tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - J_{\mathrm{ez}}^{(n)}(t)\Delta x\Delta y\\
&= \left(I - S_{M_y}\right)\tilde{\nu}_{xx}^{(n)}B_x^{(n)}(t)\Delta x + \left(S_{M_x} - I\right)\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - J_{\mathrm{ex}}^{(n)}(t)\Delta x\Delta y
\end{aligned}$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form /
3D-FIT – ... diskreten Gittergleichungen in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \tilde{\varepsilon}_{xx}^{(n)} & & \\ & \tilde{\varepsilon}_{yy}^{(n)} & \\ & & \tilde{\varepsilon}_{zz}^{(n)} \end{bmatrix}}_{=\widetilde{[\varepsilon]}^{(n)}} \underbrace{\begin{bmatrix} \Delta y \Delta z & & \\ & \Delta x \Delta z & \\ & & \Delta x \Delta y \end{bmatrix}}_{=\widetilde{[S]}} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}}_{=\{E\}^{(n)}(t)}$$

$$= \underbrace{\begin{bmatrix} 0 & I - S_{M_z} & S_{M_y} - I \\ S_{M_z} - I & 0 & i - S_{M_x} \\ I - S_{M_y} & S_{M_x} - I & 0 \end{bmatrix}}_{=\widetilde{[\mathrm{curl}]}} \underbrace{\begin{bmatrix} \tilde{\nu}_{xx}^{(n)} & & \\ & \tilde{\nu}_{yy}^{(n)} & \\ & & \tilde{\nu}_{zz}^{(n)} \end{bmatrix}}_{=\widetilde{[\nu]}^{(n)}} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=\widetilde{[R]}} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}}_{=\{B\}^{(n)}(t)} - \underbrace{\begin{bmatrix} \Delta y \Delta z & & \\ & \Delta x \Delta z & \\ & & \Delta x \Delta y \end{bmatrix}}_{=\widetilde{[S]}} \underbrace{\begin{Bmatrix} J_{\mathrm{ex}}^{(n)}(t) \\ J_{\mathrm{ex}}^{(n)}(t) \\ J_{\mathrm{ex}}^{(n)}(t) \end{Bmatrix}}_{=\{J_{\mathrm{e}}\}^{(n)}(t)}$$

$$\begin{bmatrix} 0 & I - S_{M_z} & S_{M_y} - I \\ S_{M_z} - I & 0 & i - S_{M_x} \\ I - S_{M_y} & S_{M_x} - I & 0 \end{bmatrix} = \begin{bmatrix} 0 & -P_{M_z} & P_{M_y} \\ P_{M_z} & 0 & -P_{M_x} \\ -P_{M_y} & P_{M_x} & 0 \end{bmatrix} = \widetilde{[\mathrm{curl}]}$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form /
3D-FIT – … diskreten Gittergleichungen in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \tilde{\varepsilon}_{xx}^{(n)} & & \\ & \tilde{\varepsilon}_{yy}^{(n)} & \\ & & \tilde{\varepsilon}_{zz}^{(n)} \end{bmatrix}}_{=\widetilde{[\varepsilon]}^{(n)}} \underbrace{\begin{bmatrix} \Delta y \Delta z & & \\ & \Delta x \Delta z & \\ & & \Delta x \Delta y \end{bmatrix}}_{=\widetilde{[S]}} \frac{d}{dt} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}}_{=\{E\}^{(n)}(t)} = \underbrace{\begin{bmatrix} 0 & -P_{M_z} & P_{M_y} \\ P_{M_z} & 0 & -P_{M_x} \\ -P_{M_y} & P_{M_x} & 0 \end{bmatrix}}_{=\widetilde{[\mathrm{curl}]}} \underbrace{\begin{bmatrix} \tilde{\nu}_{xx}^{(n)} & & \\ & \tilde{\nu}_{yy}^{(n)} & \\ & & \tilde{\nu}_{zz}^{(n)} \end{bmatrix}}_{=\widetilde{[\nu]}^{(n)}} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=\widetilde{[R]}} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}}_{=\{B\}^{(n)}(t)} - \underbrace{\begin{bmatrix} \Delta y \Delta z & & \\ & \Delta x \Delta z & \\ & & \Delta x \Delta y \end{bmatrix}}_{=\widetilde{[S]}} \underbrace{\begin{Bmatrix} J_{\mathrm{ex}}^{(n)}(t) \\ J_{\mathrm{ex}}^{(n)}(t) \\ J_{\mathrm{ex}}^{(n)}(t) \end{Bmatrix}}_{=\{J_{\mathrm{e}}\}^{(n)}(t)}$$

$$\widetilde{[\varepsilon]}^{(n)} \widetilde{[S]} \frac{\mathrm{d}}{\mathrm{d}t} \{E\}^{(n)}(t) = \widetilde{[\mathrm{curl}]} \widetilde{[\nu]}^{(n)} \widetilde{[R]} \{B\}^{(n)}(t) - \widetilde{[S]} \{J_{\mathrm{e}}\}^{(n)}(t)$$

$\widetilde{[\varepsilon]}^{(n)}$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of permittivities on the grid $\tilde{G}$ / Diagonalmatrix der Permittivitäten auf dem Gitter $\tilde{G}$
$\widetilde{[S]}$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of elementary surfaces on the grid $\tilde{G}$ / Diagonalmatrix der Elementarflächen auf dem Gitter $\tilde{G}$
$\{E\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic electric field strength vector / Algebraischer elektrischer Feldstärkevektor
$\widetilde{[\mathrm{curl}]}$	$\in \mathbb{R}^{3\times 3}$	Topological curl operator in matrix form on the grid $\tilde{G}$ / Topologischer Rotationsoperator in Matrixform auf dem Gitter $\tilde{G}$
$\widetilde{[\nu]}^{(n)}$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of impermeabilities on the grid $\tilde{G}$ / Diagonalmatrix der Impermeabilitäten auf dem Gitter $\tilde{G}$
$\widetilde{[R]}$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of elementary lines on the grid $\tilde{G}$ / Diagonalmatrix der Elementarstrecken auf dem Gitter $\tilde{G}$
$\{B\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$\{J_{\mathrm{e}}\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic electric current density vector / Algebraischer elektrischer Stromdichtevektor

3-D FIT – … Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – ... diskreten Gittergleichungen in lokaler und globaler Matrixform

Maxwell's equations in integral form / Maxwellsche Gleichungen in Integralform

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S} \underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Discrete grid equations in local matrix form / Diskrete Gittergleichungen in lokaler Matrixform

$$[S]\frac{\mathrm{d}}{\mathrm{d}t}\{B\}^{(n)}(t) = -[\mathrm{curl}][R]\{E\}^{(n)}(t) \qquad -[S]\{J_{\mathrm{m}}\}^{(n)}(t)$$

$$\widetilde{[\varepsilon]}^{(n)}\widetilde{[S]}\frac{\mathrm{d}}{\mathrm{d}t}\{E\}^{(n)}(t) = \widetilde{[\mathrm{curl}]}\widetilde{[\nu]}^{(n)}\widetilde{[R]}\{B\}^{(n)}(t) - \widetilde{[S]}\{J_{\mathrm{e}}\}^{(n)}(t)$$

Discrete grid equations in global matrix form / Diskrete Gittergleichungen in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

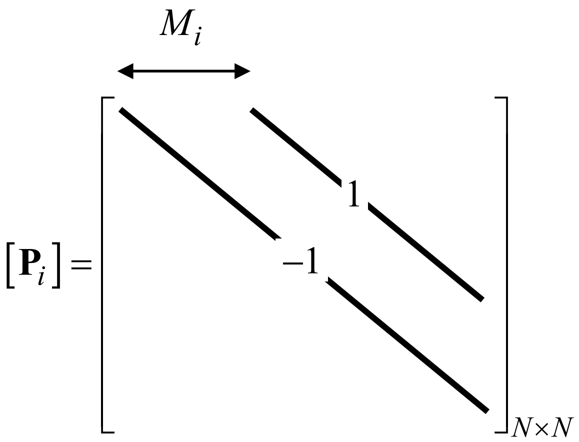
$$\widetilde{[\boldsymbol{\varepsilon}]}\widetilde{[\mathbf{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = \widetilde{[\mathbf{curl}]}\widetilde{[\boldsymbol{\nu}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t) - \widetilde{[\mathbf{S}]}\{\mathbf{J}_{\mathrm{e}}\}(t)$$

Elementary Difference Matrix [P$_i$] (P Matrix) / Elementare Differenzmatrix [P$_i$] (P–Matrix)

Elementary difference operator in global matrix form (P matrix) / Elementarer Differenzoperator in globaler Matrixform (P–Matrix)

$$\left[\mathbf{P}_{\pm i}\right] := \left(\left[\mathbf{P}_{\pm i}\right]\right)_{jk}, \quad j,k \in \{1,2,\ldots,N\}$$

$$\left(\left[\mathbf{P}_{\pm i}\right]\right)_{jk} = \begin{cases} \mp 1 & j = k \\ \pm 1 & j = k \mp M_i \text{ or / bzw. } k = j \pm M_i; \quad i = x,y,z; \;\; j,k \in \{1,2,\ldots,N\} \\ 0 & \text{else / sonst} \end{cases}$$



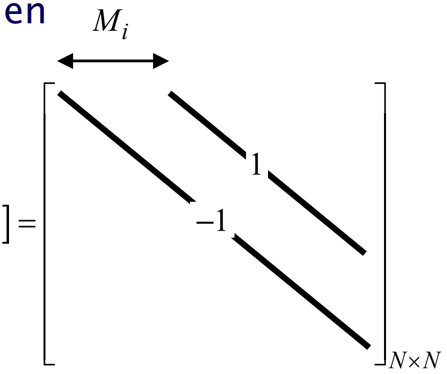
The P matrix has only two bands / Die P–Matrix hat nur zwei Bänder

Elementary Difference Matrix $[P_i]$ (P Matrix) (...) / Elementare Differenzmatrix $[P_i]$ (P–Matrix) (...)

The P matrix can be represented by a sum of an identity matrix [I] and a band matrix [B] /
Die P–Matrix kann als Summe aus einer Einheitsmatrix (Identitätsmatrix) [I] und Bandmatrix [B] dargestellt werden

$$\left[\mathbf{P}_{\pm i}\right] := \mp\left[\mathbf{I}\right] \pm \left[\mathbf{B}_{\pm i}\right], \qquad i = \{x, y, z\}$$

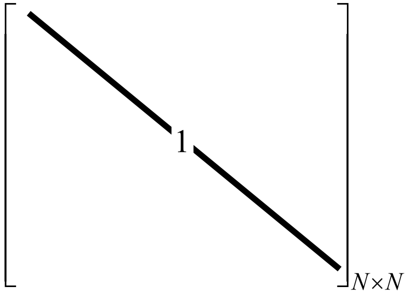
$[\mathbf{P}_i] =$



Identity matrix / Einheitsmatrix (Identitätsmatrix)

$$\left(\left[\mathbf{I}\right]\right)_{ij} = \delta_{ij} \qquad i, j \in \{1, 2, \ldots, N\}$$

$[\mathbf{I}] =$

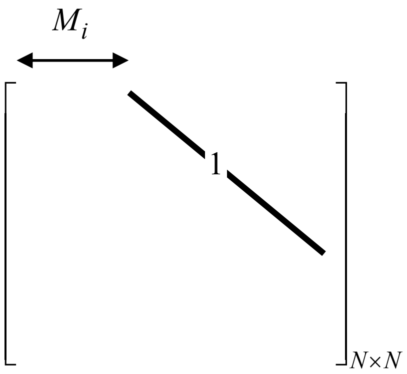


Band matrix / Bandmatrix

$$\left(\left[\mathbf{B}\right]_{\pm i}\right)_{jk} = \begin{cases} 1 & j = k \mp M_i \text{ or / bzw. } k = j \pm M_i \\ 0 & \text{else / sonst} \end{cases}$$

$$i = x, y, z; \;\; j, k \in \{1, 2, \ldots, N\}$$

$[\mathbf{B}_i] =$



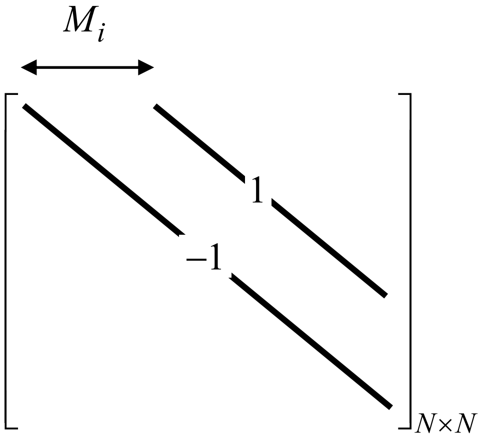
Properties of the Difference Matrix [P$_i$] (P Matrix) / Eigenschaften der Differenzmatrix [P$_i$] (P–Matrix)

$$\left[\mathbf{P}_{\pm i}\right] := \mp\left[\mathbf{I}\right] \pm \left[\mathbf{B}_{\pm i}\right], \quad i = \{x, y, z\}$$

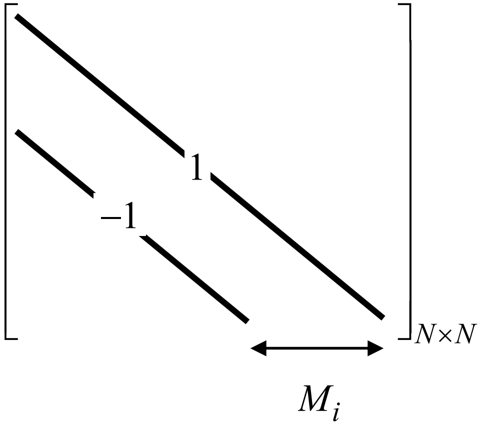
$$\left[\mathbf{P}_{i}\right] := -\left[\mathbf{I}\right] + \left[\mathbf{B}_{i}\right], \quad i = \{x, y, z\}$$

$$\left[\mathbf{P}_{-i}\right] := \left[\mathbf{I}\right] - \left[\mathbf{B}_{-i}\right], \quad i = \{x, y, z\}$$

$\left[\mathbf{P}_{i}\right] =$

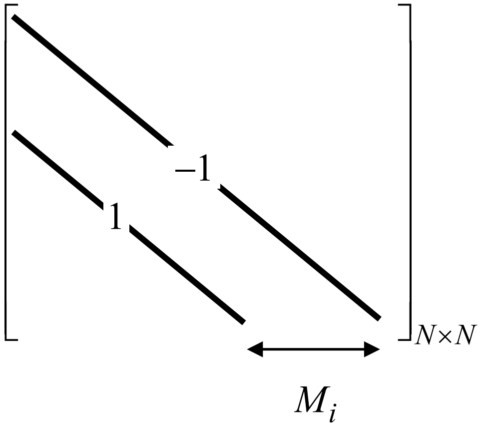


$\left[\mathbf{P}_{-i}\right] =$

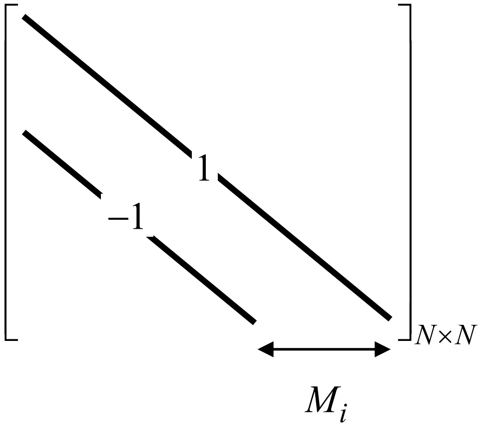


Property / Eigenschaft $\quad -\left[\mathbf{P}_{i}\right]^{\mathrm{T}} = \left[\mathbf{P}_{-i}\right]$

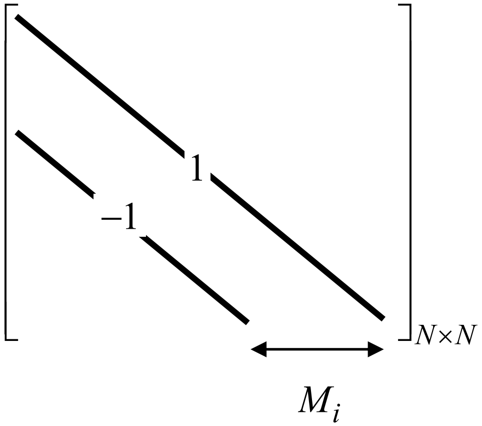
$\left[\mathbf{P}_{i}\right]^{\mathrm{T}} =$



$-\left[\mathbf{P}_{i}\right]^{\mathrm{T}} =$



$\left[\mathbf{P}_{-i}\right] =$



Discrete Global Gradient, Divergence, and Curl Operator / Diskreter globaler Gradienten-, Divergenz- und Rotationsoperator

Discrete gradient operator /
Diskreter Gradientenoperator

$$\left[\mathbf{grad}\right]=\begin{bmatrix}-\left[\mathbf{P}_x\right]^{\mathrm{T}}\\-\left[\mathbf{P}_y\right]^{\mathrm{T}}\\-\left[\mathbf{P}_z\right]^{\mathrm{T}}\end{bmatrix}_{3N\times N}$$

$$\widetilde{\left[\mathbf{grad}\right]}=\begin{bmatrix}\left[\mathbf{P}_x\right]\\\left[\mathbf{P}_y\right]\\\left[\mathbf{P}_z\right]\end{bmatrix}_{3N\times N}$$

Discrete divergence operator /
Diskreter Divergenzoperator

$$\left[\mathbf{div}\right]:=\left[-\left[\mathbf{P}_x\right]^{\mathrm{T}},-\left[\mathbf{P}_y\right]^{\mathrm{T}},-\left[\mathbf{P}_z\right]^{\mathrm{T}}\right]_{N\times 3N}$$

$$\widetilde{\left[\mathbf{div}\right]}:=\left[\left[\mathbf{P}_x\right],\left[\mathbf{P}_y\right],\left[\mathbf{P}_z\right]\right]_{N\times 3N}$$

Discrete curl operator /
Diskreter Rotationsoperator

$$\left[\mathbf{curl}\right]=\begin{bmatrix}\left[\mathbf{0}\right] & \left[\mathbf{P}_z\right]^{\mathrm{T}} & -\left[\mathbf{P}_y\right]^{\mathrm{T}}\\-\left[\mathbf{P}_z\right]^{\mathrm{T}} & \left[\mathbf{0}\right] & \left[\mathbf{P}_x\right]^{\mathrm{T}}\\\left[\mathbf{P}_y\right]^{\mathrm{T}} & -\left[\mathbf{P}_x\right]^{\mathrm{T}} & \left[\mathbf{0}\right]\end{bmatrix}_{3N\times 3N}$$

$$\widetilde{\left[\mathbf{curl}\right]}=\begin{bmatrix}\left[\mathbf{0}\right] & -\left[\mathbf{P}_z\right] & \left[\mathbf{P}_y\right]\\\left[\mathbf{P}_z\right] & \left[\mathbf{0}\right] & -\left[\mathbf{P}_x\right]\\-\left[\mathbf{P}_y\right] & \left[\mathbf{P}_x\right] & \left[\mathbf{0}\right]\end{bmatrix}_{3N\times 3N}$$

The matrix operators /
Die Matrixoperatoren

$$\begin{matrix}\left[\mathbf{grad}\right] & \widetilde{\left[\mathbf{grad}\right]}\\\left[\mathbf{div}\right] & \widetilde{\left[\mathbf{div}\right]}\\\left[\mathbf{curl}\right] & \widetilde{\left[\mathbf{curl}\right]}\end{matrix}$$

are global matrix operators /
sind globale Matrixoperatoren

Properties of the Global Matrix Operators / Eigenschaften der globalen Matrixoperatoren

Some properties of the global matrix operators of the dual grid system /
Einige Eigenschaften der globalen Matrixoperatoren des dualen Gittersystems

$$-\widetilde{[\mathbf{div}]} = [\mathbf{grad}]^{\mathrm{T}}$$

$$\widetilde{[\mathbf{grad}]}^{\mathrm{T}} = [\mathbf{div}]$$

$$[\mathbf{curl}] = \widetilde{[\mathbf{curl}]}^{\mathrm{T}}$$

Conservation of important vector identities /
Erhaltung von wichtigen Vektoridentitäten

Vector identities /
Vektoridentitäten

$$\begin{aligned} \mathrm{curl\,grad} &= \nabla \times \nabla = \underline{\mathbf{0}} \\ \mathrm{div\,curl} &= \nabla \bullet \nabla = 0 \end{aligned}$$

$$\Updownarrow$$

are conserved in the dual grid system /
bleiben im dualen Gittersystem erhalten

$$[\mathbf{curl}][\mathbf{grad}] = [\mathbf{0}]$$

$$\widetilde{[\mathbf{curl}]}\widetilde{[\mathbf{grad}]} = [\mathbf{0}]$$

$$[\mathbf{div}][\mathbf{curl}] = [\mathbf{0}]$$

$$\widetilde{[\mathbf{div}]}\widetilde{[\mathbf{curl}]} = [\mathbf{0}]$$

Properties of the Global Matrix Operators / Eigenschaften der globalen Matrixoperatoren

Consistency test / Konsistenztest

$$\left[\widetilde{\mathbf{curl}}\right]\left[\widetilde{\mathbf{grad}}\right]=\begin{bmatrix}[\mathbf{0}] & -[\mathbf{P}_z] & [\mathbf{P}_y]\\ [\mathbf{P}_z] & [\mathbf{0}] & -[\mathbf{P}_x]\\ -[\mathbf{P}_y] & [\mathbf{P}_x] & [\mathbf{0}]\end{bmatrix}\begin{bmatrix}[\mathbf{P}_x]\\ [\mathbf{P}_y]\\ [\mathbf{P}_z]\end{bmatrix}$$

$$=\begin{bmatrix}[\mathbf{P}_y][\mathbf{P}_z]-[\mathbf{P}_z][\mathbf{P}_y]\\ [\mathbf{P}_z][\mathbf{P}_x]-[\mathbf{P}_x][\mathbf{P}_z]\\ [\mathbf{P}_x][\mathbf{P}_y]-[\mathbf{P}_y][\mathbf{P}_x]\end{bmatrix}$$

$$\begin{aligned}[\mathbf{P}_i][\mathbf{P}_j]-[\mathbf{P}_j][\mathbf{P}_i]&=\left(-[\mathbf{I}]+[\mathbf{B}_i]\right)\left(-[\mathbf{I}]+[\mathbf{B}_j]\right)-\left(-[\mathbf{I}]+[\mathbf{B}_j]\right)\left(-[\mathbf{I}]+[\mathbf{B}_i]\right)\\
&=\left(-[\mathbf{I}][\mathbf{I}]-[\mathbf{I}][\mathbf{B}_j]-[\mathbf{B}_i][\mathbf{I}]+[\mathbf{B}_i][\mathbf{B}_j]\right)\\
&\quad-\left(-[\mathbf{I}][\mathbf{I}]-[\mathbf{I}][\mathbf{B}_i]-[\mathbf{B}_j][\mathbf{I}]+[\mathbf{B}_j][\mathbf{B}_i]\right)\\
&=\left(-[\mathbf{I}]-[\mathbf{B}_j]-[\mathbf{B}_i]+[\mathbf{B}_i][\mathbf{B}_j]\right)\\
&\quad-\left(-[\mathbf{I}]-[\mathbf{B}_i]-[\mathbf{B}_j]+[\mathbf{B}_j][\mathbf{B}_i]\right)\\
&=-[\mathbf{I}]-[\mathbf{B}_j]-[\mathbf{B}_i]+[\mathbf{B}_i][\mathbf{B}_j]+[\mathbf{I}]+[\mathbf{B}_i]+[\mathbf{B}_j]-[\mathbf{B}_j][\mathbf{B}_i]\\
&=[\mathbf{B}_i][\mathbf{B}_j]-[\mathbf{B}_j][\mathbf{B}_i]\end{aligned}$$

Properties of the Global Matrix Operators / Eigenschaften der globalen Matrixoperatoren

With the property /
Mit der Eigenschaft

$$\left(\left[\mathbf{B}_{\pm i}\right]\left[\mathbf{B}_{\pm j}\right]\right)_{kl} = \begin{cases} 1 & k = l \mp M_i \mp M_j \\ 0 & \text{else / sonst} \end{cases}$$

⇨ *i* and *j* can be arbitrarily interchanged /
i und *j* können beliebig vertauscht werden

⇨ This means that the matrices $\left[\mathbf{B}_{\pm i}\right]$ and $\left[\mathbf{B}_{\pm j}\right]$ as well as $\left[\mathbf{P}_{\pm i}\right]$ and $\left[\mathbf{P}_{\pm j}\right]$ are commutative!
Das bedeutet, dass die Matrizen $\left[\mathbf{B}_{\pm i}\right]$ und $\left[\mathbf{B}_{\pm j}\right]$ als auch $\left[\mathbf{P}_{\pm i}\right]$ und $\left[\mathbf{P}_{\pm j}\right]$ kommutativ sind!

⇨

$$\left[\mathbf{B}_{\pm i}\right]\left[\mathbf{B}_{\pm j}\right] = \left[\mathbf{B}_{\pm j}\right]\left[\mathbf{B}_{\pm i}\right]$$

$$\left[\mathbf{P}_{\pm i}\right]\left[\mathbf{P}_{\pm j}\right] = \left[\mathbf{P}_{\pm j}\right]\left[\mathbf{P}_{\pm i}\right]$$

⇨

$$\begin{aligned} \widetilde{\left[\mathbf{curl}\right]}\widetilde{\left[\mathbf{grad}\right]} &= \left[\mathbf{P}_i\right]\left[\mathbf{P}_j\right] - \left[\mathbf{P}_j\right]\left[\mathbf{P}_i\right] \\ &= \left[\mathbf{B}_i\right]\left[\mathbf{B}_j\right] - \underbrace{\left[\mathbf{B}_j\right]\left[\mathbf{B}_i\right]}_{=\left[\mathbf{B}_i\right]\left[\mathbf{B}_j\right]} \\ &= \left[\mathbf{B}_i\right]\left[\mathbf{B}_j\right] - \left[\mathbf{B}_i\right]\left[\mathbf{B}_j\right] \\ &= \left[\mathbf{0}\right] \end{aligned}$$

3-D FIT – … Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in lokaler und globaler Matrixform

Maxwell's equations in integral form / Maxwellsche Gleichungen in Integralform

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S} \underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Discrete grid equations in local matrix form / Diskrete Gittergleichungen in lokaler Matrixform

$$[S]\frac{\mathrm{d}}{\mathrm{d}t}\{B\}^{(n)}(t) = -[\mathrm{curl}][R]\{E\}^{(n)}(t) \quad -[S]\{J_{\mathrm{m}}\}^{(n)}(t)$$

$$\widetilde{[\varepsilon]}^{(n)}\widetilde{[S]}\frac{\mathrm{d}}{\mathrm{d}t}\{E\}^{(n)}(t) = \widetilde{[\mathrm{curl}]}\widetilde{[\nu]}^{(n)}\widetilde{[R]}\{B\}^{(n)}(t) - \widetilde{[S]}\{J_{\mathrm{e}}\}^{(n)}(t)$$

$$n = 1, 2, \ldots, N$$

Discrete grid equations in global matrix form / Diskrete Gittergleichungen in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$\widetilde{[\boldsymbol{\varepsilon}]}\widetilde{[\mathbf{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = \widetilde{[\mathbf{curl}]}\widetilde{[\mathbf{v}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t) - \widetilde{[\mathbf{S}]}\{\mathbf{J}_{\mathrm{e}}\}(t)$$

3-D FIT – … Discrete Grid Equations in Global Matrix Form / 3D-FIT – … diskrete Gittergleichungen in globaler Matrixform

Faraday's induction law in global matrix form / Faradaysches Induktionsgesetz in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$[\mathbf{S}]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary surfaces on the grid G / Diagonalmatrix der Elementarflächen auf dem Gitter G
$\{\mathbf{B}\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$[\mathbf{curl}]$	$\in \mathbb{R}^{3N\times 3N}$	Topological curl operator in matrix form on the grid G / Topologischer Rotationsoperator in Matrixform auf dem Gitter G
$[\mathbf{R}]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary lines on the grid G / Diagonalmatrix der Elementarstrecken auf dem Gitter G
$\{\mathbf{E}\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic electric field strength vector / Algebraischer elektrische Feldstärkevektor
$\{\mathbf{J}_\mathrm{m}\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic magnetic current density vector / Algebraischer magnetischer Stromdichtevektor

3-D FIT – … Discrete Grid Equations in Global Matrix Form / 3D-FIT – … diskrete Gittergleichungen in globaler Matrixform

Faraday’s induction law in global matrix form / Faradaysches Induktionsgesetz in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$[\mathbf{S}] = \begin{bmatrix} \Delta y\Delta z & & & & & & & & \\ & \ddots & & & & & & & \\ & & \Delta y\Delta z & & & & & & \\ & & & \Delta x\Delta y & & & & & \\ & & & & \ddots & & & & \\ & & & & & \Delta x\Delta y & & & \\ & & & & & & \Delta x\Delta y & & \\ & & & & & & & \ddots & \\ & & & & & & & & \Delta x\Delta y \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} [\mathrm{diag}\{\Delta y\Delta z\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\Delta x\Delta z\}]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\Delta x\Delta y\}]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

$$\{\mathbf{B}\}(t) = \begin{Bmatrix} \{B_x\}(t) \\ \{B_y\}(t) \\ \{B_z\}(t) \end{Bmatrix}_{3N} \qquad \{B_i\}(t) = \begin{Bmatrix} B_i^{(1)}(t) \\ B_i^{(2)}(t) \\ \vdots \\ B_i^{(N)}(t) \end{Bmatrix}_N \qquad i = x, y, z$$

$$[\mathbf{curl}] = \begin{bmatrix} [\mathbf{0}] & [\mathbf{P}_z]^\mathrm{T} & -[\mathbf{P}_y]^\mathrm{T} \\ -[\mathbf{P}_z]^\mathrm{T} & [\mathbf{0}] & [\mathbf{P}_x]^\mathrm{T} \\ [\mathbf{P}_y]^\mathrm{T} & -[\mathbf{P}_x]^\mathrm{T} & [\mathbf{0}] \end{bmatrix}_{3N\times 3N}$$

$$[\mathbf{R}] = \begin{bmatrix} \Delta x & & & & & & & & \\ & \ddots & & & & & & & \\ & & \Delta x & & & & & & \\ & & & \Delta y & & & & & \\ & & & & \ddots & & & & \\ & & & & & \Delta y & & & \\ & & & & & & \Delta z & & \\ & & & & & & & \ddots & \\ & & & & & & & & \Delta z \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} [\mathrm{diag}\{\Delta x\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\Delta y\}]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\Delta z\}]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

3-D FIT – … Discrete Grid Equations in Global Matrix Form / 3D-FIT – … diskrete Gittergleichungen in globaler Matrixform

Faraday’s induction law in global matrix form /
Faradaysches Induktionsgesetz in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$\{\mathbf{E}\}(t) = \begin{Bmatrix} \{E_x\}(t) \\ \{E_y\}(t) \\ \{E_z\}(t) \end{Bmatrix}_{3N} \qquad \{E_i\}(t) = \begin{Bmatrix} E_i^{(1)}(t) \\ E_i^{(2)}(t) \\ \vdots \\ E_i^{(N)}(t) \end{Bmatrix}_N \qquad i = x, y, z$$

$$\{\mathbf{J}_\mathrm{m}\}(t) = \begin{Bmatrix} \{J_{\mathrm{m}x}\}(t) \\ \{J_{\mathrm{m}y}\}(t) \\ \{J_{\mathrm{m}z}\}(t) \end{Bmatrix}_{3N} \qquad \{J_{\mathrm{m}i}\}(t) = \begin{Bmatrix} J_{\mathrm{m}i}^{(1)}(t) \\ J_{\mathrm{m}i}^{(2)}(t) \\ \vdots \\ J_{\mathrm{m}i}^{(N)}(t) \end{Bmatrix}_N \qquad i = x, y, z$$

3-D FIT – … Discrete Grid Equations in Global Matrix Form / 3D-FIT – … diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form / Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$\left[\widetilde{\boldsymbol{\varepsilon}}\right]\left[\widetilde{\mathbf{S}}\right]\frac{\mathrm{d}}{\mathrm{d}t}\left\{\mathbf{E}\right\}(t) = \left[\widetilde{\mathbf{curl}}\right]\left[\widetilde{\boldsymbol{\nu}}\right]\left[\widetilde{\mathbf{R}}\right]\left\{\mathbf{B}\right\}(t) - \left[\widetilde{\mathbf{S}}\right]\left\{\mathbf{J}_{\mathrm{e}}\right\}(t)$$

$\left[\widetilde{\boldsymbol{\varepsilon}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of permittivities on the grid $\widetilde{G}$ / Diagonalmatrix der Permittivitäten auf dem Gitter $\widetilde{G}$
$\left[\widetilde{\mathbf{S}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary surfaces on the grid $\widetilde{G}$ / Diagonalmatrix der Elementarflächen auf dem Gitter $\widetilde{G}$
$\left\{\mathbf{E}\right\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic electric field strength vector / Algebraischer elektrischer Feldstärkevektor
$\left[\widetilde{\mathrm{curl}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Topological curl operator in matrix form on the grid $\widetilde{G}$ / Topologischer Rotationsoperator in Matrixform auf dem Gitter $\widetilde{G}$
$\left[\widetilde{\boldsymbol{\nu}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of impermeabilities on the grid $\widetilde{G}$ / Diagonalmatrix der Impermeabilitäten auf dem Gitter $\widetilde{G}$
$\left[\widetilde{\mathbf{R}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary lines on the grid $\widetilde{G}$ / Diagonalmatrix der Elementarstrecken auf dem Gitter $\widetilde{G}$
$\left\{\mathbf{B}\right\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$\left\{\mathbf{J}_{\mathrm{e}}\right\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic electric current density vector / Algebraischer elektrischer Stromdichtevektor

3–D FIT – … Discrete Grid Equations in Global Matrix Form / 3D–FIT – … diskrete Gittergleichungen in globaler Matrixform

Ampère–Maxwell's circuital law in global matrix form /
Ampère–Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$[\underset{\sim}{\boldsymbol{\varepsilon}}][\underset{\sim}{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\underset{\sim}{\mathbf{curl}}][\underset{\sim}{\mathbf{v}}][\underset{\sim}{\mathbf{R}}]\{\mathbf{B}\}(t) - [\underset{\sim}{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} \overset{\sim(1)}{\varepsilon}_{xx} & & & & & & & & \\ & \ddots & & & & & & & \\ & & \overset{\sim(N)}{\varepsilon}_{xx} & & & & & & \\ & & & \overset{\sim(1)}{\varepsilon}_{yy} & & & & & \\ & & & & \ddots & & & & \\ & & & & & \overset{\sim(N)}{\varepsilon}_{yy} & & & \\ & & & & & & \overset{\sim(1)}{\varepsilon}_{zz} & & \\ & & & & & & & \ddots & \\ & & & & & & & & \overset{\sim(N)}{\varepsilon}_{zz} \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} \left[\mathrm{diag}\{\overset{\sim(1)}{\varepsilon}_{xx}, \overset{\sim(2)}{\varepsilon}_{xx}, \ldots, \overset{\sim(N)}{\varepsilon}_{xx}\}\right]_{N\times N} & [0] & [0] \\ [0] & \left[\mathrm{diag}\{\overset{\sim(1)}{\varepsilon}_{yy}, \overset{\sim(2)}{\varepsilon}_{yy}, \ldots, \overset{\sim(N)}{\varepsilon}_{yy}\}_N\right]_{N\times N} & [0] \\ [0] & [0] & \left[\mathrm{diag}\{\overset{\sim(1)}{\varepsilon}_{zz}, \overset{\sim(2)}{\varepsilon}_{zz}, \ldots, \overset{\sim(N)}{\varepsilon}_{zz}\}_N\right]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

3-D FIT – … Discrete Grid Equations in Global Matrix Form /
3D-FIT – … diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form /
Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$\widetilde{[\boldsymbol{\varepsilon}]}\widetilde{[\mathbf{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = \widetilde{[\mathbf{curl}]}\widetilde{[\boldsymbol{\nu}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t) - \widetilde{[\mathbf{S}]}\{\mathbf{J}_\mathrm{e}\}(t)$$

$$\widetilde{[\mathbf{S}]} = \begin{bmatrix} \Delta y\Delta z &&&&&&&& \\ & \ddots &&&&&&& \\ && \Delta y\Delta z &&&&&& \\ &&& \Delta x\Delta z &&&&& \\ &&&& \ddots &&&& \\ &&&&& \Delta x\Delta z &&& \\ &&&&&& \Delta x\Delta y && \\ &&&&&&& \ddots & \\ &&&&&&&& \Delta y\Delta z \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} [\mathrm{diag}\{\Delta y\Delta z\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\Delta x\Delta z\}]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\Delta x\Delta y\}]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

$$\widetilde{[\mathbf{R}]} = \begin{bmatrix} \Delta x &&&&&&&& \\ & \ddots &&&&&&& \\ && \Delta x &&&&&& \\ &&& \Delta y &&&&& \\ &&&& \ddots &&&& \\ &&&&& \Delta y &&& \\ &&&&&& \Delta z && \\ &&&&&&& \ddots & \\ &&&&&&&& \Delta z \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} [\mathrm{diag}\{\Delta x\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\Delta y\}]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\Delta z\}]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

$$\widetilde{[\mathbf{curl}]} = \begin{bmatrix} [\mathbf{0}] & -[\mathbf{P}_z] & [\mathbf{P}_y] \\ [\mathbf{P}_z] & [\mathbf{0}] & -[\mathbf{P}_x] \\ -[\mathbf{P}_y] & [\mathbf{P}_x] & [\mathbf{0}] \end{bmatrix}_{3N\times 3N}$$

$$\{\mathbf{J}_\mathrm{e}\}(t) = \begin{Bmatrix} \{J_\mathrm{ex}\}(t) \\ \{J_\mathrm{ey}\}(t) \\ \{J_\mathrm{ez}\}(t) \end{Bmatrix}_{3N} \qquad \{J_{\mathrm{e}i}\}(t) = \begin{Bmatrix} J_{\mathrm{e}i}^{(1)}(t) \\ J_{\mathrm{e}i}^{(2)}(t) \\ \vdots \\ J_{\mathrm{e}i}^{(N)}(t) \end{Bmatrix}_N \qquad i = x, y, z$$

3-D FIT – … Discrete Grid Equations in Global Matrix Form / 3D-FIT – … diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form /
Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$\left[\widetilde{\boldsymbol{\varepsilon}}\right]\left[\widetilde{\mathbf{S}}\right]\frac{\mathrm{d}}{\mathrm{d}t}\left\{\mathbf{E}\right\}(t)=\left[\widetilde{\mathbf{curl}}\right]\left[\widetilde{\boldsymbol{\nu}}\right]\left[\widetilde{\mathbf{R}}\right]\left\{\mathbf{B}\right\}(t)-\left[\widetilde{\mathbf{S}}\right]\left\{\mathbf{J}_{\mathrm{e}}\right\}(t)$$

$$\left[\widetilde{\nu}\right]=\begin{bmatrix} \tilde{\nu}_{xx}^{(1)} &&&&&&&& \\ & \ddots &&&&&&& \\ && \tilde{\nu}_{xx}^{(N)} &&&&&& \\ &&& \tilde{\nu}_{yy}^{(1)} &&&&& \\ &&&& \ddots &&&& \\ &&&&& \tilde{\nu}_{yy}^{(N)} &&& \\ &&&&&& \tilde{\nu}_{zz}^{(1)} && \\ &&&&&&& \ddots & \\ &&&&&&&& \tilde{\nu}_{zz}^{(N)} \end{bmatrix}_{3N\times 3N}$$

$$=\begin{bmatrix} \left[\operatorname{diag}\{\tilde{\nu}_{xx}^{(1)},\tilde{\nu}_{xx}^{(2)},\ldots,\tilde{\nu}_{xx}^{(N)}\}\right]_{N\times N} & [0] & [0] \\ [0] & \left[\operatorname{diag}\{\tilde{\nu}_{yy}^{(1)},\tilde{\nu}_{yy}^{(2)},\ldots,\tilde{\nu}_{yy}^{(N)}\}_{N}\right]_{N\times N} & [0] \\ [0] & [0] & \left[\operatorname{diag}\{\tilde{\nu}_{zz}^{(1)},\tilde{\nu}_{zz}^{(2)},\ldots,\tilde{\nu}_{zz}^{(N)}\}_{N}\right]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

3-D FIT – … Discrete Grid Equations in Local and Global Matrix Form /
3D-FIT – ... diskrete Gittergleichungen in lokaler und globaler Matrixform

The two discrete grid equations in global matrix form read /
Die beiden diskreten Gittergleichungen in globaler Matrixform lauten

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t)\quad-[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$\widetilde{[\boldsymbol{\varepsilon}]}\widetilde{[\mathbf{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t)=\quad\widetilde{[\mathbf{curl}]}\widetilde{[\boldsymbol{\nu}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t)-\widetilde{[\mathbf{S}]}\{\mathbf{J}_{\mathrm{e}}\}(t)$$

We arrange the last equations in the form /
Wir bringen die letzten beiden Gleichungen in die Form

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{S}]^{-1}\qquad[\mathbf{curl}]\quad[\mathbf{R}]\{\mathbf{E}\}(t)-[\mathbf{S}]^{-1}\qquad[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t)=\quad\widetilde{[\mathbf{S}]}^{-1}\widetilde{[\boldsymbol{\varepsilon}]}^{-1}\,\widetilde{[\mathbf{curl}]}\widetilde{[\boldsymbol{\nu}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t)-\widetilde{[\mathbf{S}]}^{-1}\widetilde{[\boldsymbol{\varepsilon}]}^{-1}\widetilde{[\mathbf{S}]}\{\mathbf{J}_{\mathrm{e}}\}(t)$$

$$[\mathbf{S}]^{-1}[\mathbf{S}]=[\mathbf{I}]$$

$$\widetilde{[\mathbf{S}]}^{-1}\widetilde{[\boldsymbol{\varepsilon}]}^{-1}\widetilde{[\mathbf{S}]}=\underbrace{\widetilde{[\mathbf{S}]}^{-1}\widetilde{[\mathbf{S}]}}_{=[\mathbf{I}]}\widetilde{[\boldsymbol{\varepsilon}]}^{-1}=\widetilde{[\boldsymbol{\varepsilon}]}^{-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{S}]^{-1}\qquad[\mathbf{curl}]\quad[\mathbf{R}]\{\mathbf{E}\}(t)-\qquad\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t)=\quad\widetilde{[\mathbf{S}]}^{-1}\widetilde{[\boldsymbol{\varepsilon}]}^{-1}\,\widetilde{[\mathbf{curl}]}\widetilde{[\boldsymbol{\nu}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t)-\widetilde{[\boldsymbol{\varepsilon}]}^{-1}\{\mathbf{J}_{\mathrm{e}}\}(t)$$

3-D FIT – … Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in lokaler und globaler Matrixform

The two discrete grid equations in global matrix form read /
Die beiden diskreten Gittergleichungen in globaler Matrixform lauten

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t)\quad-[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$[\widetilde{\boldsymbol{\varepsilon}}][\widetilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t)=\quad[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t)-[\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

We arrange the last equations in the form /
Wir bringen die letzten beiden Gleichungen in die Form

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{S}]^{-1}\qquad[\mathbf{curl}]\quad[\mathbf{R}]\{\mathbf{E}\}(t)-[\mathbf{S}]^{-1}\qquad[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t)=\quad[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\,[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t)-[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

$$[\mathbf{S}]^{-1}[\mathbf{S}]=[\mathbf{I}]$$

$$[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{S}}]=\underbrace{[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\mathbf{S}}]}_{=[\mathbf{I}]}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}=[\widetilde{\boldsymbol{\varepsilon}}]^{-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{S}]^{-1}\qquad[\mathbf{curl}]\quad[\mathbf{R}]\{\mathbf{E}\}(t)-\qquad\{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t)=\quad[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\,[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t)-[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\,\{\mathbf{J}_\mathrm{e}\}(t)$$

3-D FIT – … Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – … diskrete Gittergleichungen in lokaler und globaler Matrixform

The two discrete grid equations in global matrix form read /
Die beiden diskreten Gittergleichungen in globaler Matrixform lauten

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{S}]^{-1} \quad [\mathbf{curl}] \quad [\mathbf{R}]\{\mathbf{E}\}(t) - \quad \{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = \quad [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1} \quad [\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}(t)$$

Now we write these two matrix equations in matrix form and find a first-order system of differential equations / Nun schreiben wir die beiden Matrixgleichungen in Matrixform und finden das folgende System von Differentialgleichungen erster Ordnung

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{y}\}(t) = [\mathbf{A}]\{\mathbf{y}\}(t) + \{\mathbf{q}\}(t)$$

with / mit

Solution vector / Lösungsvektor

$$\{\mathbf{y}\}(t) = \begin{Bmatrix} \{\mathbf{B}\}(t) \\ \{\mathbf{E}\}(t) \end{Bmatrix}$$

System matrix / Systemmatrix

$$[\mathbf{A}] = \begin{bmatrix} [0] & [\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}] \\ [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1} \; [\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}] & [0] \end{bmatrix}$$

Source vector / Quellvektor

$$\{\mathbf{q}\}(t) = \begin{Bmatrix} - \quad \{\mathbf{J}_\mathrm{m}\}(t) \\ -[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}(t) \end{Bmatrix}$$

3-D FIT – … Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

A general solution of the initial value problem (IVP) with the initial value {y}(t_0) is /
Eine allgemeine Lösung des Anfangswertproblems (AWP) mit dem Anfangswert {y}(t_0) ist

$$\{\mathbf{y}\}(t) = \{\mathbf{y}\}(t_0) + \underbrace{\int\limits_{t'=t_0}^{t} \underbrace{\{[\mathbf{A}]\{\mathbf{y}\}(t) + \{\mathbf{q}\}(t)\}}_{=\{\dot{\mathbf{y}}\}(t)}\,\mathrm{d}t}_{\substack{\text{time integration /}\\ \text{zeitliche Integration}}}$$

- implicit time integration / implizierte Zeitintegration
- explicit time integration / explizite Zeitintegration

Explicit time integration / Explizite Zeitintegration

$$\{\mathbf{B}\}(t) = \boxed{\{\mathbf{B}\}(t_0)} + \int\limits_{t'=t_0}^{t} \{\dot{\mathbf{B}}\}(t')\,\mathrm{d}t'$$

$$\{\mathbf{E}\}(t) = \boxed{\{\mathbf{E}\}(t_0)} + \int\limits_{t'=t_0}^{t} \{\dot{\mathbf{E}}\}(t')\,\mathrm{d}t'$$

Initial value / Anfangswert

$$t = [0, T]; \qquad T: \begin{matrix}\text{time interval to be simulated}\\ \text{zu simulierendes Zeitinterval}\end{matrix}$$

3-D FIT – … Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

Discretization in time on a staggered grid in time /
Diskretisierung in der Zeit auf einem versetzten Gitter in der Zeit

$$\{\mathbf{B}\}(t) \rightarrow \{\mathbf{B}\}(n_t \Delta t) \qquad \rightarrow \{\mathbf{B}\}^{(n_t)}$$

$$\{\mathbf{E}\}(t) \rightarrow \{\mathbf{E}\}\left[\left(n_t + \frac{1}{2}\right)\Delta t\right] \rightarrow \{\mathbf{E}\}^{(n_t + 1/2)}$$

$$\{\mathbf{B}\}(t) = \{\mathbf{B}\}(t_0) + \int\limits_{t'=t_0}^{t} \{\dot{\mathbf{B}}\}(t')\, \mathrm{d}t'$$

$$\{\mathbf{E}\}(t) = \{\mathbf{E}\}(t_0) + \int\limits_{t'=t_0}^{t} \{\dot{\mathbf{E}}\}(t')\, \mathrm{d}t'$$

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t - 1)} + \int\limits_{t'=(n_t-1)\Delta t}^{n_t \Delta t} \{\dot{\mathbf{B}}\}(t')\, \mathrm{d}t'$$

$$\{\mathbf{E}\}^{(n_t + 1/2)} = \{\mathbf{E}\}^{(n_t - 1/2)} + \int\limits_{t'=(n_t-1/2)\Delta t}^{(n_t+1/2)\Delta t} \{\dot{\mathbf{E}}\}(t')\, \mathrm{d}t'$$

Mid point rule /
Mittelpunktsregel

$$\int\limits_{t'=(n_t-1)\Delta t}^{n_t \Delta t} \{\dot{\mathbf{B}}\}(t')\, \mathrm{d}t' = \{\dot{\mathbf{B}}\}\left[(n_t - 1/2)\Delta t\right]\Delta t = \{\dot{\mathbf{B}}\}^{(n_t - 1/2)} \Delta t$$

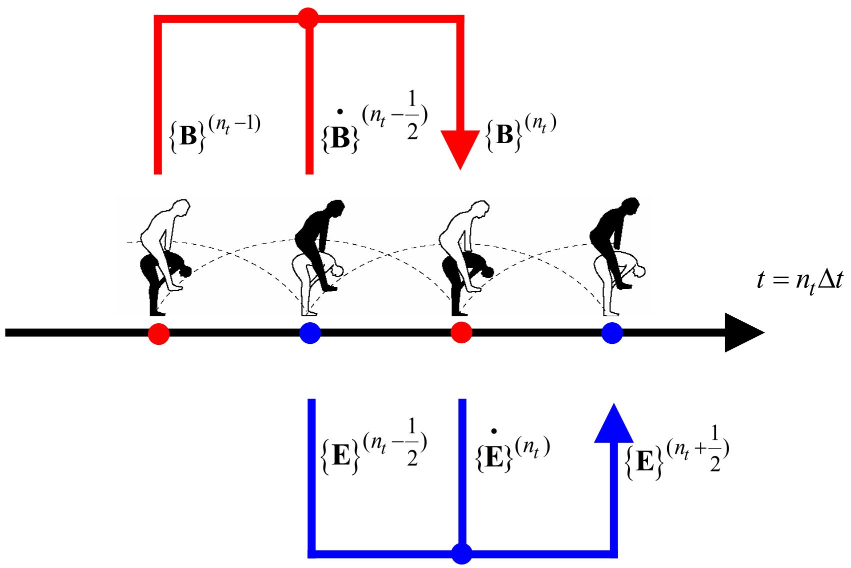
$$\int\limits_{t'=(n_t-1/2)\Delta t}^{(n_t+1/2)\Delta t} \{\dot{\mathbf{E}}\}(t')\, \mathrm{d}t' = \{\dot{\mathbf{E}}\}\left(n_t \Delta t\right)\Delta t = \{\dot{\mathbf{E}}\}^{(n_t)} \Delta t$$

3-D FIT – … Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

The leapfrog structure of the algorithm in time / Die Bocksprung-Struktur des Algorithmus in der Zeit

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} + \Delta t \{\dot{\mathbf{B}}\}^{(n_t-1/2)}$$

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t \{\dot{\mathbf{E}}\}^{(n_t)}$$



3-D FIT – … Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

Electromagnetic grid equations (EMGE) of the so-called
Electromagnetic Finite Integration Technique (EMFIT) algorithm /
Elektromagnetische Gittergleichungen (EMGG) des so genannten
Elektromagnetischen Finite Integrationstechnik (EMFIT) Algorithmus

Faraday’s induction grid equation / Faradaysche Induktionsgittergleichung

$$\{\dot{\mathbf{B}}\}^{(n_t-1/2)} = -[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}^{(n_t-1/2)} - \{\mathbf{J}_\mathrm{m}\}^{(n_t-1/2)}$$

Time integration / Zeitintegration

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} \quad + \Delta t\{\dot{\mathbf{B}}\}^{(n_t-1/2)}$$

Ampère-Maxwell’s circuital grid equation / Ampère-Maxwellsche Durchflutungsgittergleichung

$$\{\dot{\mathbf{E}}\}^{(n_t} = [\widetilde{\mathbf{S}}]^{-1}\,[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\,\widetilde{[\mathbf{curl}]}[\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}^{(n_t)} - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}^{(n_t)}$$

Time integration / Zeitintegration

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t\{\dot{\mathbf{E}}\}^{(n_t)}$$

3-D FIT – … Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

Electromagnetic grid equations (EMGE) of the so-called EMFIT algorithm /
Elektromagnetische Gittergleichungen (EMGG) des so genannten EMFIT-Algorithmus

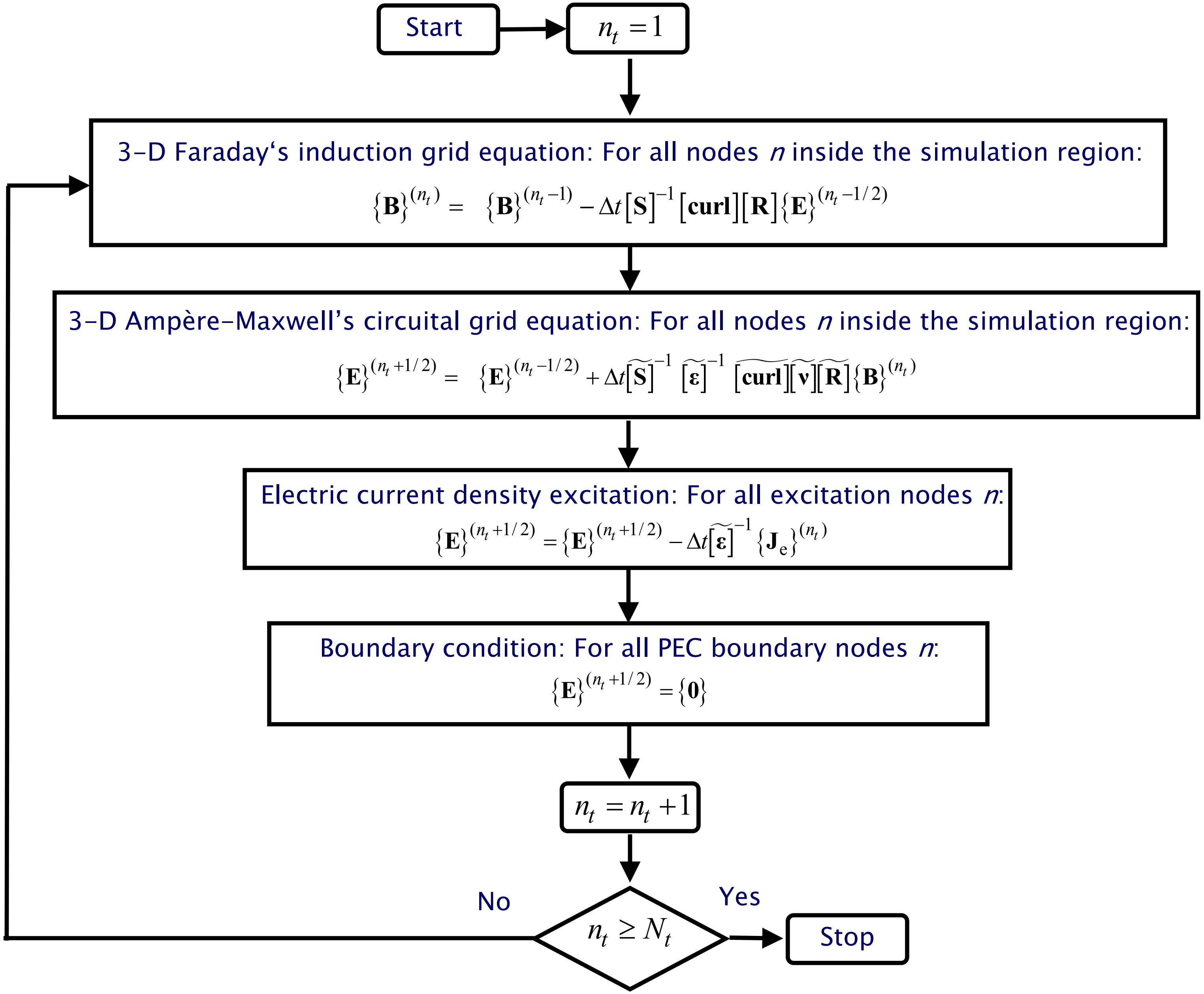
Time-integrated Faraday's induction grid equation /
Zeitlich integrierte Faradaysche Induktionsgittergleichung

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} + \Delta t\left[-[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}^{(n_t-1/2)} - \{\mathbf{J}_{\mathrm{m}}\}^{(n_t-1/2)}\right]$$

Time-integrated Ampère-Maxwell's circuital grid equation /
Zeitlich integrierte Ampère-Maxwellsche Durchflutungsgittergleichung

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t\left[\widetilde{[\mathbf{S}]}^{-1}\,\widetilde{[\boldsymbol{\varepsilon}]}^{-1}\,\widetilde{[\mathbf{curl}]}\widetilde{[\boldsymbol{\nu}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}^{(n_t)} - \widetilde{[\boldsymbol{\varepsilon}]}^{-1}\{\mathbf{J}_{\mathrm{e}}\}^{(n_t)}\right]$$

3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



Start

$n_t = 1$

3-D Faraday‘s induction grid equation: For all nodes *n* inside the simulation region:

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} - \Delta t[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}^{(n_t-1/2)}$$

3-D Ampère-Maxwell’s circuital grid equation: For all nodes *n* inside the simulation region:

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\mathbf{v}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}^{(n_t)}$$

Electric current density excitation: For all excitation nodes *n*:

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t+1/2)} - \Delta t[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}^{(n_t)}$$

Boundary condition: For all PEC boundary nodes *n*:

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{0}\}$$

$n_t = n_t + 1$

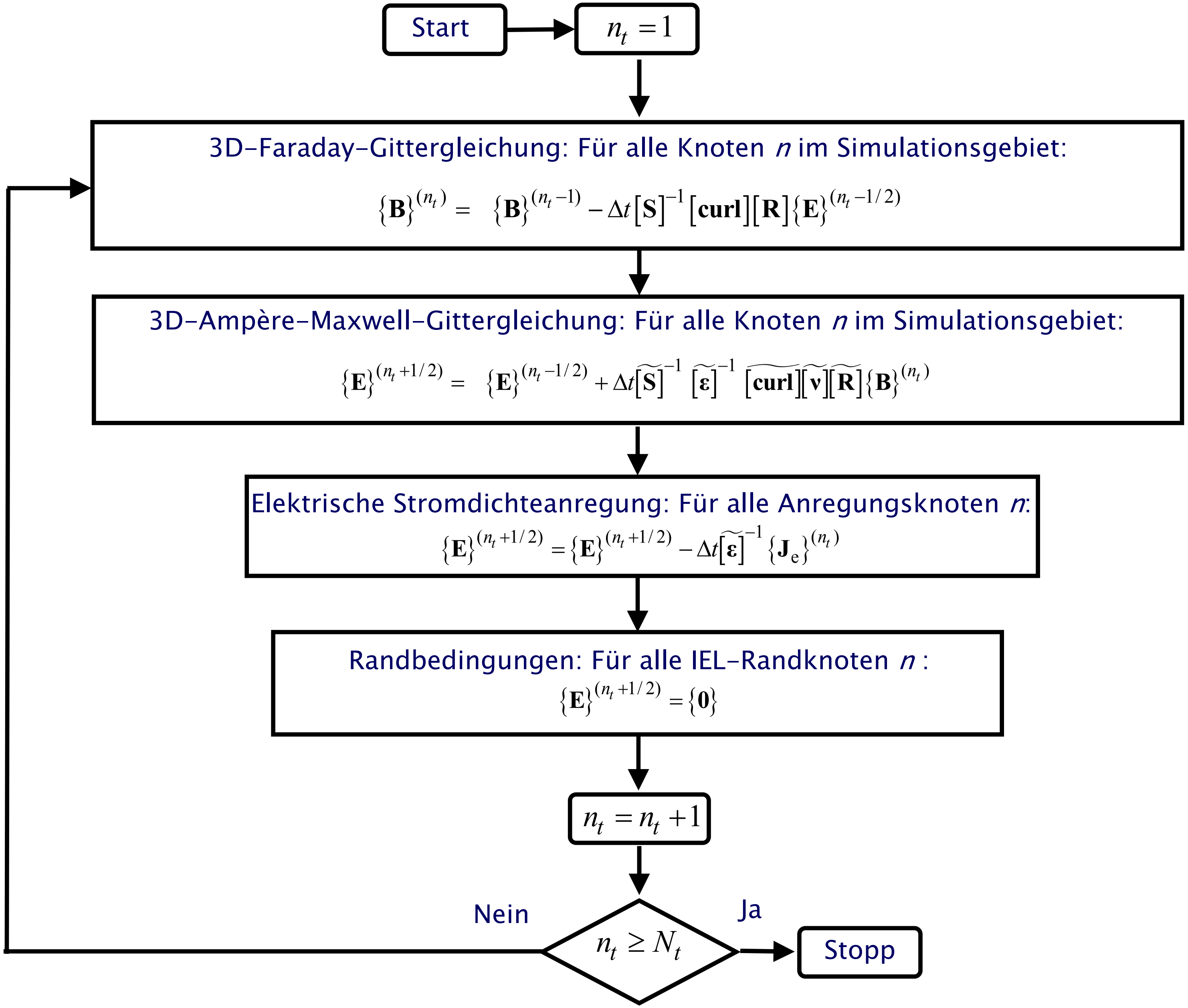
$n_t \geq N_t$

No

Yes

Stop

3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



Start

$n_t = 1$

3D-Faraday-Gittergleichung: Für alle Knoten *n* im Simulationsgebiet:

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} - \Delta t[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}^{(n_t-1/2)}$$

3D-Ampère-Maxwell-Gittergleichung: Für alle Knoten *n* im Simulationsgebiet:

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\mathbf{v}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}^{(n_t)}$$

Elektrische Stromdichteanregung: Für alle Anregungsknoten *n*:

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t+1/2)} - \Delta t[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}^{(n_t)}$$

Randbedingungen: Für alle IEL-Randknoten *n* :

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{0}\}$$

$n_t = n_t + 1$

$n_t \geq N_t$

Stopp

3-D FIT – … Normalized … Grid Equations / 3D-FIT – … normierte … Gittergleichungen

Normalized electromagnetic grid equations (EMGE) of the so-called EMFIT algorithm /
Normierte elektromagnetische Gittergleichungen (EMGG) des so genannten EMFIT-Algorithmus

Normalized time-integrated Faraday’s induction grid equation /
Normierte zeitlich integrierte Faradaysche Induktionsgittergleichung

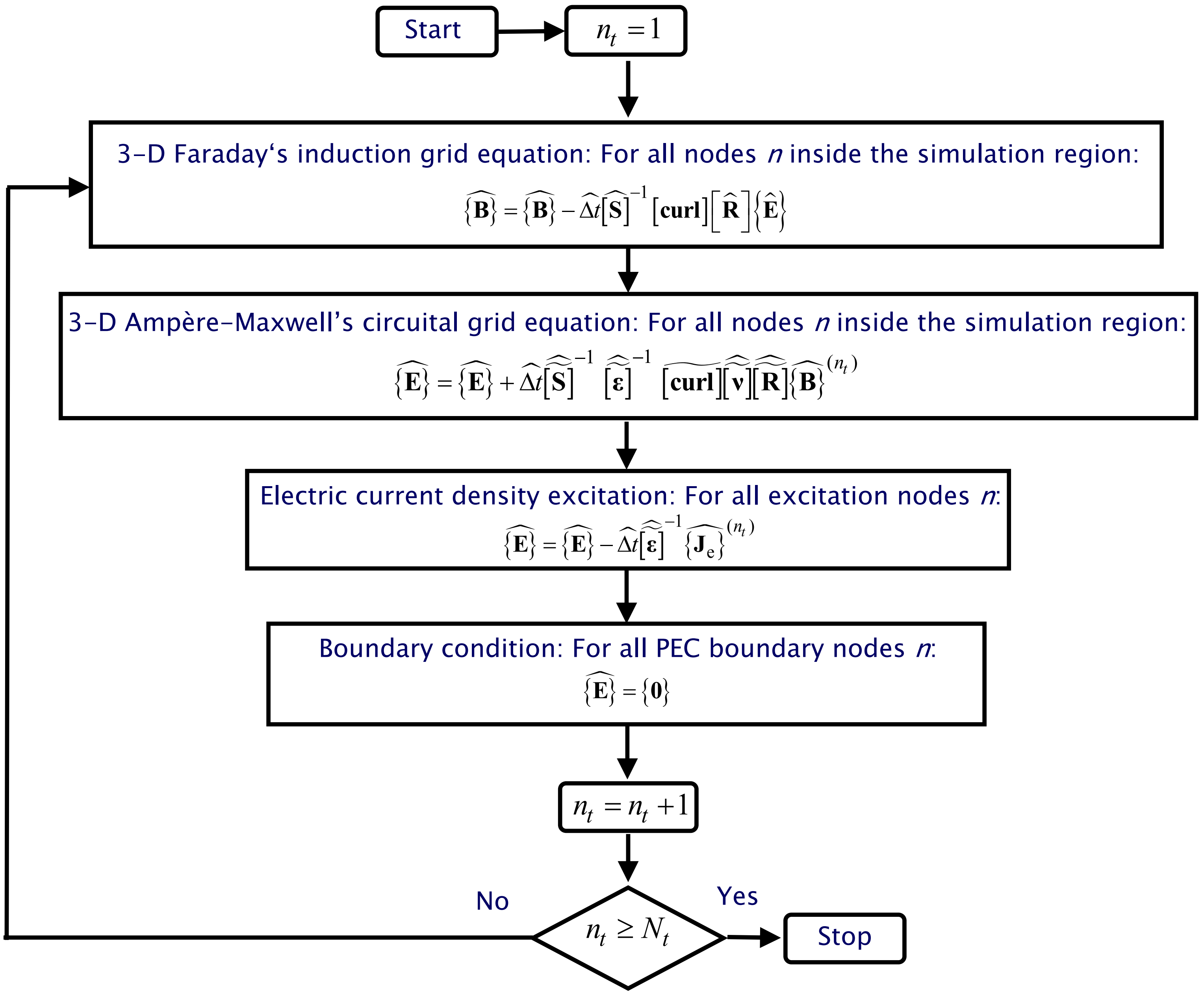
$$\widehat{\{\mathbf{B}\}}^{(n_t)} = \widehat{\{\mathbf{B}\}}^{(n_t-1)} + \widehat{\Delta t}\left[-\widehat{[\mathbf{S}]}^{-1}[\mathbf{curl}]\left[\widehat{\mathbf{R}}\right]\left\{\widehat{\mathbf{E}}\right\}^{(n_t-1/2)} - \widehat{\{\mathbf{J}_\mathrm{m}\}}^{(n_t-1/2)}\right]$$

Normalized time-integrated Ampère-Maxwell’s circuital grid equation /
Normierte zeitlich integrierte Ampère-Maxwellsche Durchflutungsgittergleichung

$$\widehat{\{\mathbf{E}\}}^{(n_t+1/2)} = \widehat{\{\mathbf{E}\}}^{(n_t-1/2)} + \widehat{\Delta t}\left[\widehat{\widehat{[\mathbf{S}]}}^{-1}\,\widehat{\widehat{[\boldsymbol{\varepsilon}]}}^{-1}\,\widetilde{[\mathbf{curl}]}\widehat{\widehat{[\boldsymbol{\nu}]}}\widehat{\widehat{[\mathbf{R}]}}\widehat{\{\mathbf{B}\}}^{(n_t)} - \widehat{\widehat{[\boldsymbol{\varepsilon}]}}^{-1}\,\widehat{\{\mathbf{J}_\mathrm{e}\}}^{(n_t)}\right]$$

In a computer implementation we can neglect the integer time step counter n_t. /
In der Rechnerimplementierung kann der ganzzahlige Zeitschrittzähler n_t unterdrückt werden.

3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



Start

$n_t = 1$

3-D Faraday‘s induction grid equation: For all nodes *n* inside the simulation region:

$$\widehat{\{\mathbf{B}\}} = \widehat{\{\mathbf{B}\}} - \widehat{\Delta t}\widehat{[\mathbf{S}]}^{-1}[\mathbf{curl}]\left[\widehat{\mathbf{R}}\right]\left\{\widehat{\mathbf{E}}\right\}$$

3-D Ampère-Maxwell’s circuital grid equation: For all nodes *n* inside the simulation region:

$$\widehat{\{\mathbf{E}\}} = \widehat{\{\mathbf{E}\}} + \widehat{\Delta t}\widehat{\widetilde{[\mathbf{S}]}}^{-1}\ \widehat{\widetilde{[\boldsymbol{\varepsilon}]}}^{-1}\ \widetilde{[\mathbf{curl}]}\widehat{\widetilde{[\mathbf{v}]}}\widehat{\widetilde{[\mathbf{R}]}}\widehat{\{\mathbf{B}\}}^{(n_t)}$$

Electric current density excitation: For all excitation nodes *n*:

$$\widehat{\{\mathbf{E}\}} = \widehat{\{\mathbf{E}\}} - \widehat{\Delta t}\widehat{\widetilde{[\boldsymbol{\varepsilon}]}}^{-1}\ \widehat{\{\mathbf{J}_\mathrm{e}\}}^{(n_t)}$$

Boundary condition: For all PEC boundary nodes *n*:

$$\widehat{\{\mathbf{E}\}} = \{\mathbf{0}\}$$

$n_t = n_t + 1$

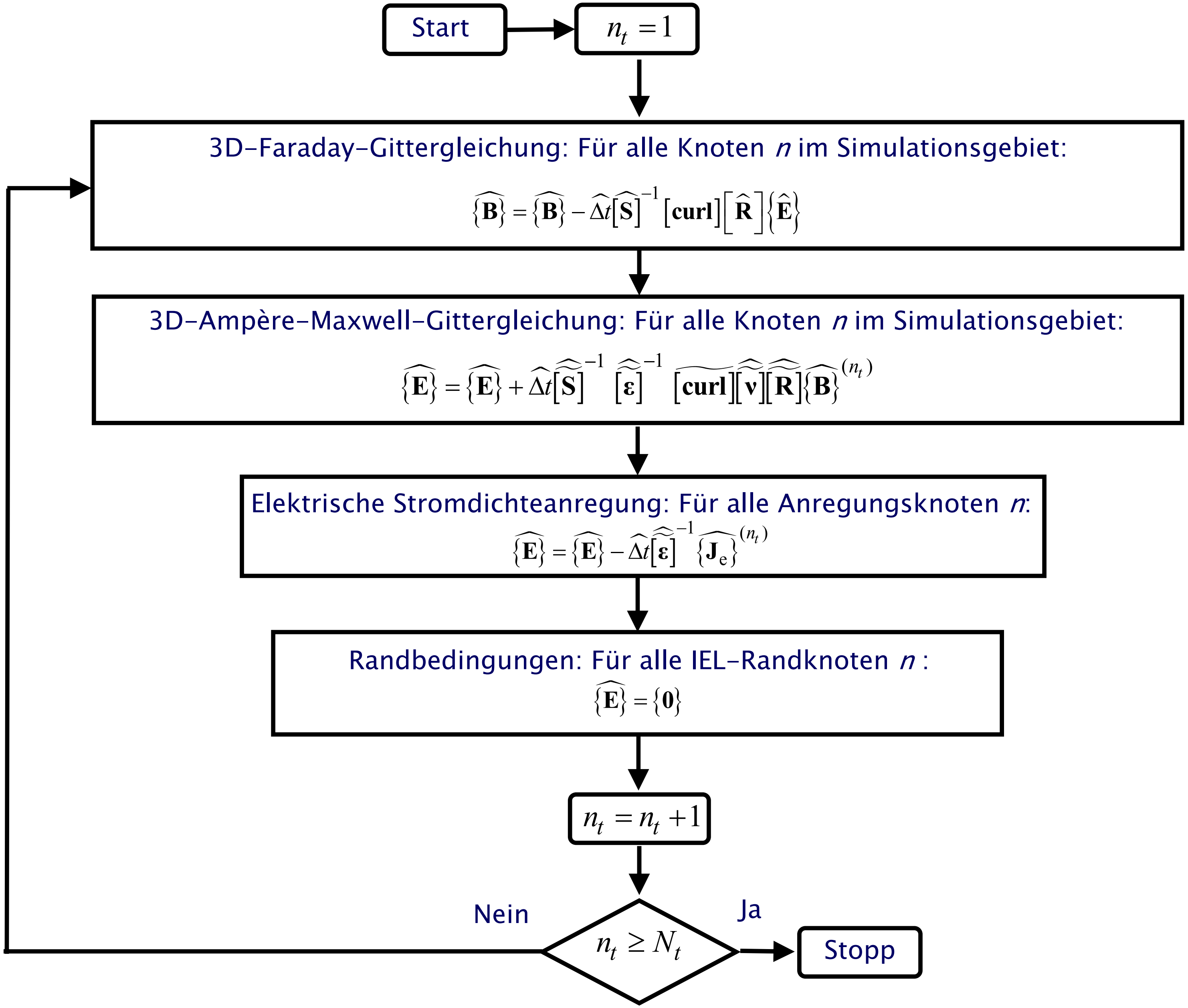
$n_t \geq N_t$

No

Yes

Stop

3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



Start

$n_t = 1$

3D-Faraday-Gittergleichung: Für alle Knoten *n* im Simulationsgebiet:

$$\widehat{\{\mathbf{B}\}} = \widehat{\{\mathbf{B}\}} - \widehat{\Delta t}\widehat{[\mathbf{S}]}^{-1}[\mathbf{curl}]\left[\widehat{\mathbf{R}}\right]\left\{\widehat{\mathbf{E}}\right\}$$

3D-Ampère-Maxwell-Gittergleichung: Für alle Knoten *n* im Simulationsgebiet:

$$\widehat{\{\mathbf{E}\}} = \widehat{\{\mathbf{E}\}} + \widehat{\Delta t}\widehat{\widetilde{[\mathbf{S}]}}^{-1}\widehat{\widetilde{[\boldsymbol{\varepsilon}]}}^{-1}\widetilde{[\mathbf{curl}]}\widehat{\widetilde{[\boldsymbol{\nu}]}}\widehat{\widetilde{[\mathbf{R}]}}\widehat{\{\mathbf{B}\}}^{(n_t)}$$

Elektrische Stromdichteanregung: Für alle Anregungsknoten *n*:

$$\widehat{\{\mathbf{E}\}} = \widehat{\{\mathbf{E}\}} - \widehat{\Delta t}\widehat{\widetilde{[\boldsymbol{\varepsilon}]}}^{-1}\widehat{\{\mathbf{J}_{\mathrm{e}}\}}^{(n_t)}$$

Randbedingungen: Für alle IEL-Randknoten *n* :

$$\widehat{\{\mathbf{E}\}} = \{\mathbf{0}\}$$

$n_t = n_t + 1$

$n_t \geq N_t$

Nein

Ja

Stopp

FIT Discretization of the 3rd and 4th Maxwell's Equation / FIT–Diskretisierung der 3. und 4. Maxwellschen Gleichung

Maxwell's equations in integral form /
Maxwellsche Gleichungen in Integralform

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\left.\begin{aligned} &\oint\!\!\!\oint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,dV \\ &\oint\!\!\!\oint_{S=\partial V}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,dV \end{aligned}\right\}\ ?$$

FIT

Maxwell's grid equations /
Maxwellsche Gittergleichungen

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$[\widetilde{\boldsymbol{\varepsilon}}][\widetilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{curl}}][\widetilde{\mathbf{v}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\mathbf{S}}]\{\mathbf{J}_{\mathrm{e}}\}(t)$$

End of Lecture 9 /
Ende der 9. Vorlesung