NFT II — Exercise Sheet 1 (NFT II — Übungsblatt 1)

RM/SS 03

Exercise 1 Consider the Helmholtz integral

$$\Phi(\underline{\mathbf{R}},\omega) = \oiint_{S=\partial V} \left[\Phi(\underline{\mathbf{R}}',\omega)\boldsymbol{\nabla}'\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\boldsymbol{\nabla}'\,\Phi(\underline{\mathbf{R}}',\omega)\right]\cdot \underline{\mathbf{n}}'\, dS' \,.$$

Identify the following quantities

1. elementary waves (wavelets) of the single–layer potential and double–layer potential
2. densities of the single–layer potential and double–layer potential.

Draw a picture of the elementary wavelets of the single–layer potential and double–layer potential.

Exercise 2 Derive the vector equation

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) - k_0^2\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \begin{cases} \mathrm{j}\,\omega\mu_0\,\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) - \boldsymbol{\nabla}\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega) & \text{case with sources} \\ \underline{\mathbf{0}} & \text{source–free case}\,. \end{cases}$$

Exercise 3 Derive the dyadic equation

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - k_0^2\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \underline{\underline{\mathbf{I}}}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}')\,.$$

Exercise 4 Calculate the magnetic dyadic Green's function of free space according to

$$\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \boldsymbol{\nabla}\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,,$$

where $\underline{\underline{\mathbf{G}}}$ is the electric dyadic Green's function of free space

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \left(\underline{\underline{\mathbf{I}}} - \frac{1}{k_0^2}\boldsymbol{\nabla}\boldsymbol{\nabla}\right) G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

with the scalar Green's function of free space

$$G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \frac{\mathrm{e}^{\mathrm{j}\,k_0|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}}{4\pi|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}\,.$$

Exercise 5 Derive the Franz integral, Stratton–Chu integral, and the Franz–Larmor integral for the $\underline{\mathbf{H}}$ field applying the duality principle

$$\left\{\begin{array}{c}\underline{\mathbf{E}}\\ \underline{\mathbf{H}}\\ \mu\end{array}\right\} \leftrightarrow \left\{\begin{array}{c}\underline{\mathbf{H}}\\ -\underline{\mathbf{E}}\\ \varepsilon\end{array}\right\}.$$