Numerical Methods of Electromagnetic Field Theory II (NFT II)
Numerische Methoden der Elektromagnetischen Feldtheorie II (NFT II) /

2nd Lecture / 2. Vorlesung

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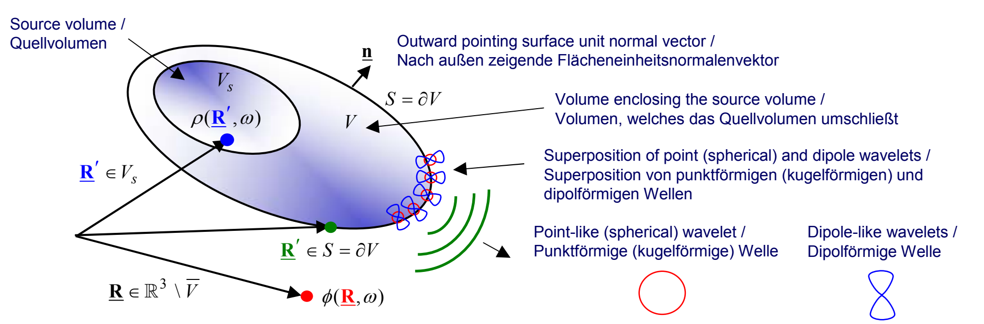
Scalar Huygens' Principle – Representation Theorem / Skalares Huygenssches Prinzip – Repräsentationstheorem



For / Für $\underline{\mathbf{R}} \in \mathbb{R}^3 \setminus \overline{V}$ we obtain the so-called representations theorem / erhalten wir das so genannte Repräsentationstheorem

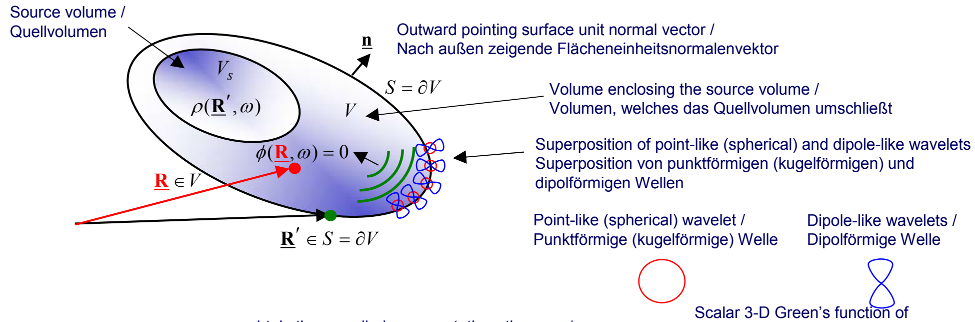
$$\phi(\underline{\mathbf{R}},\omega) = \begin{cases} \dfrac{1}{\varepsilon_0} \iiint_{\underline{\mathbf{R}}' \in V_s} G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \rho(\underline{\mathbf{R}}',\omega)\, \mathrm{d}^3\underline{\mathbf{R}}' \\ \oiint_{\underline{\mathbf{R}}' \in S = \partial V} \left[\phi(\underline{\mathbf{R}}',\omega) \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega) \nabla' \phi(\underline{\mathbf{R}}',\omega) \right] \bullet \underline{\mathbf{n}}'\, \mathrm{d}S' \end{cases}$$

Scalar 3-D Green's function of free-space / Skalare 3D-Greensche Funktion des Freiraumes

$$G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega) = \frac{\mathrm{e}^{\mathrm{j}k_0 |\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}}{4\pi |\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}$$

- ϕ: Density of double-layer potential / Dichte des Doppelschichtpotentials
- $\nabla' G$: Dipole wavelet / Dipolförmige Welle
- G: Spherical wavelet / Kugelförmige Welle
- $\nabla' \phi$: Density of single-layer potential / Dichte des Einfachschichtpotentials

Scalar Huygens' Principle – Extinction Theorem / Skalares Huygenssches Prinzip – (Aus)Löschungsstheorem



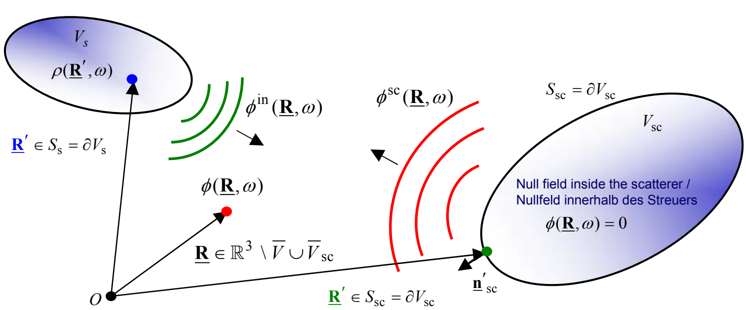
For / Für $\underline{\mathbf{R}} \in \mathbb{R}^3 \setminus \overline{V}$ we obtain the so-called representations theorem / erhalten wir das so genannte Repräsentationstheorem

Scalar 3-D Green's function of free-space / Skalare 3D-Greensche Funktion des Freiraumes

$$\oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S = \partial V} \left[\phi(\underline{\mathbf{R}}', \omega) \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) - G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) \nabla' \phi(\underline{\mathbf{R}}', \omega) \right] \bullet \underline{\mathbf{n}}' \, \mathrm{d}S' = \phi(\underline{\mathbf{R}}, \omega) = 0 \qquad G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) = \frac{\mathrm{e}^{\mathrm{j}k_0 |\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}}{4\pi |\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}$$

This means, that inside the volume V the Huygens wavelets interfere to zero. This zero wave field is called a null field (null field method) / Dies bedeutet, dass innerhalb des Volumens V die Huygens-Wellen (Wavelets) zu null interferieren. Dieses Null-Wellenfeld wird Nullfeld genannt (Nullfeld-Methode).

Scalar Huygens' Principle – Direct Scattering Problem / Skalares Huygenssches Prinzip – Direktes Streuproblem



$$\phi^{\mathrm{in}}(\underline{\mathbf{R}}, \omega) = \frac{1}{\varepsilon_0} \iiint_{\underline{\mathbf{R}}' \in V_s} G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) \, \rho(\underline{\mathbf{R}}', \omega) \, \mathrm{d}V(\underline{\mathbf{R}}')$$

$$\phi^{\mathrm{sc}}(\underline{\mathbf{R}}, \omega) = \oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S = \partial V} \left[\phi(\underline{\mathbf{R}}', \omega) \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) - G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) \nabla' \phi(\underline{\mathbf{R}}', \omega) \right] \bullet \underline{\mathbf{n}}' \, \mathrm{d}S'(\underline{\mathbf{R}}')$$

$$\phi(\underline{\mathbf{R}}, \omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}}, \omega) + \phi^{\mathrm{sc}}(\underline{\mathbf{R}}, \omega)$$

$$G(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) = \frac{\mathrm{e}^{\mathrm{j}k_0 |\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}}{4\pi |\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}$$

Scalar Integral Equations of the 1st and 2nd Kind / Skalare Integralgleichungen der 1. und 2. Art

Boundary Condition / Randbedingung

$$\phi(\underline{\mathbf{R}},\omega)=0,\quad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 1st Kind / Fredholmsche Integralgleichung der 1. Art

$$\oiint_{\underline{\mathbf{R}}'\in S=\partial V} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\underbrace{\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)}_{\text{Unknown / Unbekannt}}\,\mathrm{d}S'=\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega),\qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Boundary Condition / Randbedingung

$$\frac{\partial}{\partial n}\phi(\underline{\mathbf{R}},\omega)=0,\quad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 2nd Kind / Fredholmsche Integralgleichung der 2. Art

$$\frac{1}{2}\underbrace{\phi(\underline{\mathbf{R}},\omega)}_{\text{Unknown / Unbekannt}}-\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\underbrace{\phi(\underline{\mathbf{R}}',\omega)}_{\text{Unknown / Unbekannt}}\,\mathrm{d}S'=\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega),\qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

$$\phi(\underline{\mathbf{R}},\omega)=\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega)+\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)$$

Solution of the Scalar Integral Equations of the 1st and 2nd Kind / Lösung der skalaren Integralgleichungen der 1. und 2. Art

Boundary Condition / Randbedingung

$$\phi(\underline{\mathbf{R}},\omega)=0,\quad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 1st Kind / Fredholmsche Integralgleichung der 1. Art

$$\oiint_{\underline{\mathbf{R}}'\in S=\partial V} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\,\mathrm{d}S'=\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega)\qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

⇩ Discretization (Method of Moments) / Diskretisierung (Momenten-Methode)

$$[G_{\mathrm{F1}}]\{\phi\}(\omega)=\left\{\phi^{\mathrm{in}}\right\}(\omega)\quad\Rightarrow\quad \{\phi\}(\omega)=[G_{\mathrm{F1}}]^{-1}\left\{\phi^{\mathrm{in}}\right\}(\omega)$$

Boundary Condition / Randbedingung

$$\frac{\partial}{\partial n}\phi(\underline{\mathbf{R}},\omega)=0,\quad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 2nd Kind / Fredholmsche Integralgleichung der 2. Art

$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega)-\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\phi(\underline{\mathbf{R}}',\omega)\,\mathrm{d}S'=\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega)\qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

⇩ Discretization (Method of Moments) / Diskretisierung (Momenten-Methode)

$$[G_{\mathrm{F2}}]\{\phi\}(\omega)=\left\{\phi^{\mathrm{in}}\right\}(\omega)\quad\Rightarrow\quad \{\phi\}(\omega)=[G_{\mathrm{F2}}]^{-1}\left\{\phi^{\mathrm{in}}\right\}(\omega)$$

Electromagnetic Huygens’ Principle – Representation Theorem / Elektromagnetisches Huygenssches Prinzip – Repräsentationstheorem

Three Different Versions of EFIE and MFIE / Drei unterschiedliche Versionen von EFIE und MFIE:

1. Stratton-Chu Version [1939] / Stratton-Chu-Version [1939]
2. Franz Version [1948]; Mathematical Formulation / Franz-Version [1948]; mathematische Formulierung
3. Franz-Larmor Version [Franz, 1948; Larmor, 1903] / Franz-Larmor-Version [Franz, 1948; Larmor, 1903]

Larmor, J.: On the mathematical expression of the principle of Huygens. *London Math. Soc. Proc.*, Vol. 1, pp. 1, 1903.
Franz, W.: Zur Formulierung des Huygensschen Prinzips. *Z. Naturforschung*, Vol. 3a, pp. 500, 1948.
Stratton, J. A., L.. J. Chu: Diffraction theory of electromagnetic wave. *Phys. Rev.*, Vol. 56, pp. 99, 1939.

Christian Huygens
1629-1695

Joseph Larmor
1857-1942

http://www.uni-saarland.de/fak7/hartmann/scientists/gallery.html

Electromagnetic Huygens’ Principle – Representation Theorem / Elektromagnetisches Huygenssches Prinzip – Repräsentationstheorem

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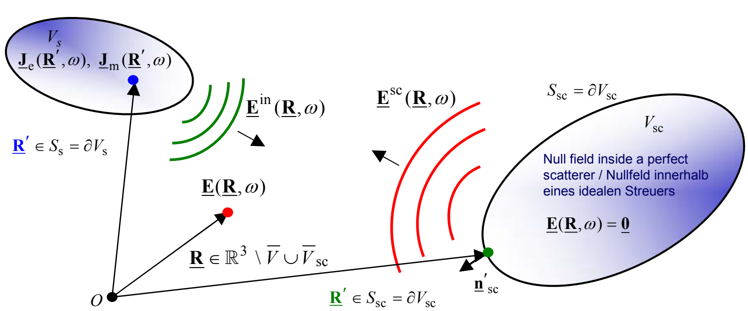

Electromagnetic Huygens' Principle – Representation Theorem / Elektromagnetisches Huygenssches Prinzip – Repräsentationstheorem

Three Different Versions of EFIE and MFIE / Drei unterschiedliche Versionen von EFIE und MFIE:

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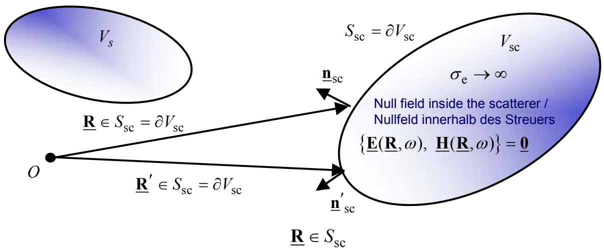
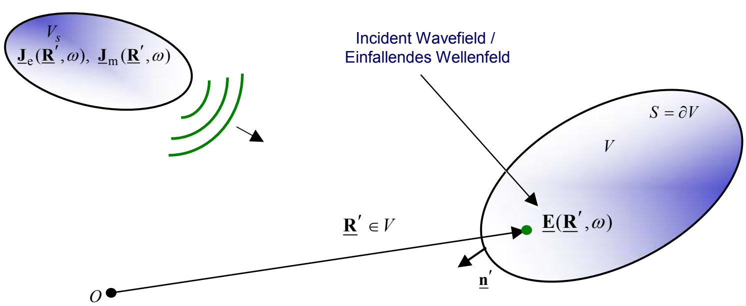


Boundary Conditions / Randbedingungen

		PEC / IEL	PMC / IML
$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$	=	$\{\ \underline{\mathbf{0}}$	$-\underline{\mathbf{K}}_m(\underline{\mathbf{R}},\omega)\}$
$\underline{\mathbf{n}}_{sc} \cdot \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$	=	$\{\frac{1}{\varepsilon_0}\eta_e(\underline{\mathbf{R}},\omega),$	$0\ \}$
$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)$	=	$\{\ \underline{\mathbf{K}}_e(\underline{\mathbf{R}},\omega),$	$\underline{\mathbf{0}}\ \}$
$\underline{\mathbf{n}}_{sc} \cdot \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)$	=	0	$\frac{1}{\mu_0}\eta_m(\underline{\mathbf{R}},\omega)\}$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version



We start with the Governing Equations for the Electric Field Strength and the Electric Dyadic Green's Function / Wir beginnen mit den Grundgleichung für die elektrische Feldstärke und die elektrische dyadische Greensche Funktion

$$\nabla' \times \nabla' \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) - k_0^2 \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) = \underline{\mathbf{0}} \qquad (1)$$

$$\nabla' \times \nabla' \times \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - k_0^2 \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \underline{\underline{\mathbf{I}}}\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}}) \qquad (2)$$

With the Dyadic Electric Green's Function of Free-Space / Mit der dyadischen elektrische Greenschen Funktion des Freiraumes

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \left(\underline{\underline{\mathbf{I}}} - \frac{1}{k_0^2}\nabla\nabla\right) G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

Scalar Green's Function of Free-Space / Skalare Greensche Funktion des Freiraumes

$$G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \frac{e^{\mathrm{j}k_0|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}}{4\pi|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}$$

Dyadic Electric Green’s Function of Free-Space / Dyadische Greensche Funktion des Freiraumes

Scalar Green’s Function of Free-Space / Skalare Greensche Funktion des Freiraumes

$$G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\frac{\mathrm{e}^{\mathrm{j}k_0\left|\underline{\mathbf{R}}-\underline{\mathbf{R}}'\right|}}{4\pi\left|\underline{\mathbf{R}}-\underline{\mathbf{R}}'\right|}$$

Solution of the Scalar PDE / Lösung der skalaren PDGL

$$\left(\Delta+k_0^2\right)G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=-\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})$$

Dyadic Electric Green’s Function of Free-Space / Dyadische elektrische Greenschen Funktion des Freiraumes

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\left(\underline{\underline{\mathbf{I}}}-\frac{1}{k_0^2}\nabla\nabla\right)G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

Solution of the dyadic PDE / Lösung der dyadischen PDGL

$$-\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)+k_0^2\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=-\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})\underline{\underline{\mathbf{I}}}$$

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\left(\underline{\underline{\mathbf{I}}}-\frac{1}{k_0^2}\nabla\nabla\right)\frac{\mathrm{e}^{\mathrm{j}k_0\left|\underline{\mathbf{R}}-\underline{\mathbf{R}}'\right|}}{4\pi\left|\underline{\mathbf{R}}-\underline{\mathbf{R}}'\right|}$$

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}},\omega)=\left(\underline{\underline{\mathbf{I}}}-\frac{1}{k_0^2}\nabla\nabla\right)\frac{\mathrm{e}^{\mathrm{j}k_0\left|\underline{\mathbf{R}}\right|}}{4\pi\left|\underline{\mathbf{R}}\right|}$$

$$=\left(\underline{\underline{\mathbf{I}}}-\frac{1}{k_0^2}\nabla\nabla\right)\frac{\mathrm{e}^{\mathrm{j}k_0R}}{4\pi R}$$

Dyadic Electric Green’s Function of Free-Space / Dyadische Greensche Funktion des Freiraumes

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}},\omega)=\left(\underline{\underline{\mathbf{I}}}-\frac{1}{k_0^2}\nabla\nabla\right)\frac{\mathrm{e}^{\mathrm{j}k_0R}}{4\pi R}$$

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}},\omega)=\left[\underline{\underline{\mathbf{I}}}-\hat{\underline{\mathbf{R}}}\hat{\underline{\mathbf{R}}}+\frac{\mathrm{j}}{k_0R}\left(\underline{\underline{\mathbf{I}}}-3\hat{\underline{\mathbf{R}}}\hat{\underline{\mathbf{R}}}\right)-\frac{1}{k_0^2R^2}\left(\underline{\underline{\mathbf{I}}}-3\hat{\underline{\mathbf{R}}}\hat{\underline{\mathbf{R}}}\right)\right]\frac{\mathrm{e}^{\mathrm{j}k_0R}}{4\pi R}\qquad \text{für / for}\qquad \underline{\mathbf{R}}\neq\underline{\mathbf{0}}$$

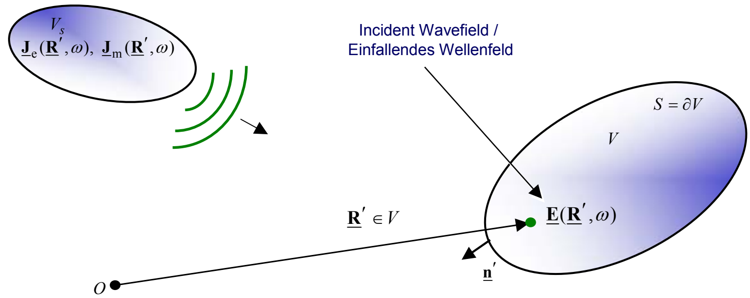
$$=\underline{\underline{\mathbf{G}}}^{(0)}(\underline{\mathbf{R}},\omega)$$

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}},\omega)=\mathrm{PV}\underline{\underline{\mathbf{G}}}^{(0)}(\underline{\mathbf{R}},\omega)-\frac{1}{3k_0^2}\underline{\underline{\mathbf{I}}}\,\delta(\underline{\mathbf{R}})\qquad \underline{\mathbf{R}}\ \text{arbitrary / beliebig}$$

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\underline{\mathbf{G}}}^{21}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

$$\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\underline{\mathbf{G}}}\ (\underline{\mathbf{R}}'-\underline{\mathbf{R}},\omega)$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version



We start with the Governing Equations for the Electric Field Strength and the Electric Dyadic Green's Function /
Wir beginnen mit den Grundgleichung für die elektrische Feldstärke und die elektrische dyadische Greensche Funktion $\underline{\mathbf{R}}'\in V$

$$\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)-k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)=\underline{\mathbf{0}} \qquad (1)$$

$$\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-k_0^2\,\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\underline{\mathbf{I}}}\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}}) \qquad (2)$$

We multiply Eq. (1) scalar with $\underline{\underline{\mathbf{G}}}$ from right and Eq. (2) scalar with $\underline{\mathbf{E}}$ from the left and subtract the second from the first Equation /
Wir multiplizieren Gl. (1) skalar mit $\underline{\underline{\mathbf{G}}}$ von rechts und Gl. (2) skalar mit $\underline{\mathbf{E}}$ von links und subtrahieren die zweite von der ersten Gleichung

$$\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\mathbf{0}} \qquad (3)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]-k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{I}}}\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}}) \qquad (4)$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version

$$\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\mathbf{0}} \qquad (3)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]-k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{I}}}\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}}) \qquad (4)$$

$$\begin{aligned}&\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\cancel{k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)}\\&-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]+\cancel{k_0^2\,\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)}=-\underbrace{\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\underline{\underline{\mathbf{I}}}}_{=\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)}\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})\end{aligned} \qquad (5)$$

We find / Wir finden

$$\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]=-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}}) \qquad (6)$$

$$=-\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}}) \qquad (7)$$

with the Identity / mit der Identität $\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})=\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})$

Now we integrate the last equation over the volume V /
Nun integrieren wir die letzte Gleichung über das Volumen V

$$\iiint_{\underline{\mathbf{R}}'\in V}\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\cdot\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\cdot\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^3\underline{\mathbf{R}}'=-\iiint_{\underline{\mathbf{R}}'\in V}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})\mathrm{d}^3\underline{\mathbf{R}}' \qquad (8)$$

$$\iiint_{\underline{\mathbf{R}}'\in V}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\delta(\underline{\mathbf{R}}'-\underline{\mathbf{R}})\mathrm{d}^3\underline{\mathbf{R}}'=\begin{cases}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) & \underline{\mathbf{R}}\in V\\ \underline{\mathbf{0}} & \underline{\mathbf{R}}\in\mathbb{R}^3\setminus\overline{V}\end{cases} \qquad (9)$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version

$$\iiint_{\underline{\mathbf{R}}'\in V}\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\bullet\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\bullet\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^3\underline{\mathbf{R}}'=\begin{cases}-\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) & \underline{\mathbf{R}}\in V\\ \underline{\mathbf{0}} & \underline{\mathbf{R}}\in\mathbb{R}^3\setminus\overline{V}\end{cases}\qquad(10)$$

We apply now the second vector-dyadic Green's theorem (see Tai [1997, pp. 125-126]) /
Wir wenden nun den zweiten vektor-dyadischen Greenschen Satz an (siehe Tai [1997, pp. 125-126])

$$\iiint_{\underline{\mathbf{R}}'\in V}\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\bullet\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\bullet\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^3\underline{\mathbf{R}}'$$
$$=\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\underline{\mathbf{n}}'\bullet\left\{\left[\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)+\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\times\left[\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\right\}\mathrm{d}^2\underline{\mathbf{R}}'\qquad(11)$$

We find for Eq. (10) / Wir finden für Gl. (10)

$$\iiint_{\underline{\mathbf{R}}'\in V}\left[\nabla'\times\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\bullet\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\bullet\left[\nabla'\times\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^3\underline{\mathbf{R}}'=\begin{cases}-\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) & \underline{\mathbf{R}}\in V\\ \underline{\mathbf{0}} & \underline{\mathbf{R}}\in\mathbb{R}^3\setminus\overline{V}\end{cases}\qquad(12)$$
$$=\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\underline{\mathbf{n}}'\bullet\left\{\left[\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)+\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\times\left[\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\right\}\mathrm{d}^2\underline{\mathbf{R}}'$$

We find from the 1st Maxwell Equation / Wir finden von der 1. Maxwellschen Gleichung

$$\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)=\mathrm{j}\omega\underline{\mathbf{B}}(\underline{\mathbf{R}}',\omega)\qquad(13)$$
$$=\mathrm{j}\omega\mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega)\qquad(14)$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version

It follows / Es folgt

$$\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\underline{\mathbf{n}}'\bullet\left\{\underbrace{\left[\nabla'\times\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\right]}_{=\mathrm{j}\omega\mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega)}\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)+\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\times\underbrace{\left[\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]}_{=-\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)}\right\}\mathrm{d}^2\underline{\mathbf{R}}'\qquad(15)$$

$$=\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\underline{\mathbf{n}}'\bullet\left\{\mathrm{j}\omega\mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega)\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)-\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)\times\left[\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\right\}\mathrm{d}^2\underline{\mathbf{R}}'\qquad(16)$$

Dyadic Magnetic Green's Function /
Dyadische magnetische Greensche Funktion

$$\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=\nabla G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\times\underline{\underline{\mathbf{I}}}=\nabla\times\left[G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\underline{\underline{\mathbf{I}}}\right]$$
$$=-\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\times\underline{\underline{\mathbf{I}}}=-\nabla'\times\left[G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\underline{\underline{\mathbf{I}}}\right]$$
$$=\nabla\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=-\nabla'\times\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

$$\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=-\underline{\underline{\mathbf{G}}}_{\mathrm{m}}^{21}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$
$$\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)=-\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}'-\underline{\mathbf{R}},\omega)$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version

It follows / Es folgt

$$\oiint_{\underline{\mathbf{R}}'\in S=\partial V} \underline{\mathbf{n}}' \cdot \left\{ \Big[\underbrace{\nabla'\times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega)}_{=\mathrm{j}\omega\mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega)} \Big] \times \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \times \Big[\underbrace{\nabla'\times \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)}_{=-\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)} \Big] \right\} \mathrm{d}^2\underline{\mathbf{R}}' \qquad (15)$$

$$= \oiint_{\underline{\mathbf{R}}'\in S=\partial V} \underline{\mathbf{n}}' \cdot \left\{ \mathrm{j}\omega\mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \times \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \times \Big[\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big] \right\} \mathrm{d}^2\underline{\mathbf{R}}' \qquad (16)$$

Dyadic Magnetic Green's Function / Dyadische magnetische Greensche Funktion

And the Vector Identity / Und der Vektoridentität

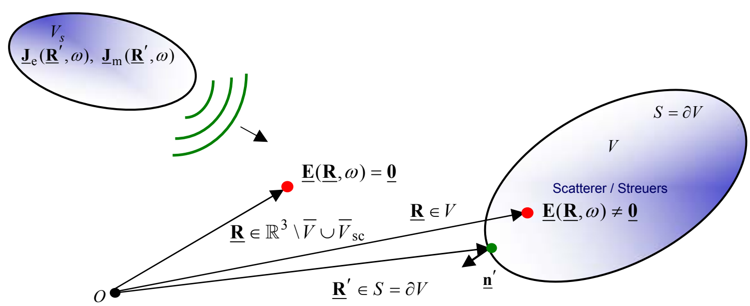
$$\underline{\mathbf{A}} \cdot (\underline{\mathbf{B}} \times \underline{\mathbf{D}}) = (\underline{\mathbf{A}} \times \underline{\mathbf{B}}) \cdot \underline{\mathbf{D}} \qquad (17)$$

$$= \oiint_{\underline{\mathbf{R}}'\in S=\partial V} \left\{ \mathrm{j}\omega\mu_0 \underbrace{\Big[\underline{\mathbf{n}}' \cdot \Big(\underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \times \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big) \Big]} - \underbrace{\left\{ \underline{\mathbf{n}}' \cdot \Big(\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \times \Big[\underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big] \Big) \right\}} \right\} \mathrm{d}^2\underline{\mathbf{R}}' \qquad (18)$$

$$\underline{\mathbf{n}}' \cdot \Big(\underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \times \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big) = \Big[\underline{\mathbf{n}}' \times \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \Big] \cdot \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

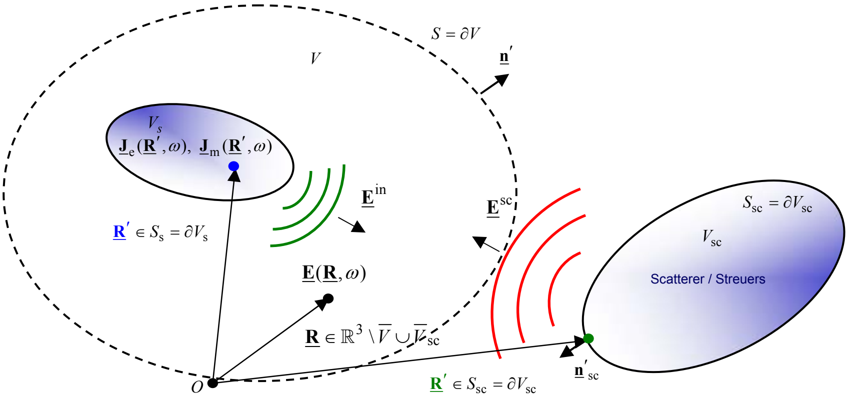
$$\underline{\mathbf{n}}' \cdot \Big(\underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \times \underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big) = \Big[\underline{\mathbf{n}}' \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \Big] \cdot \underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version

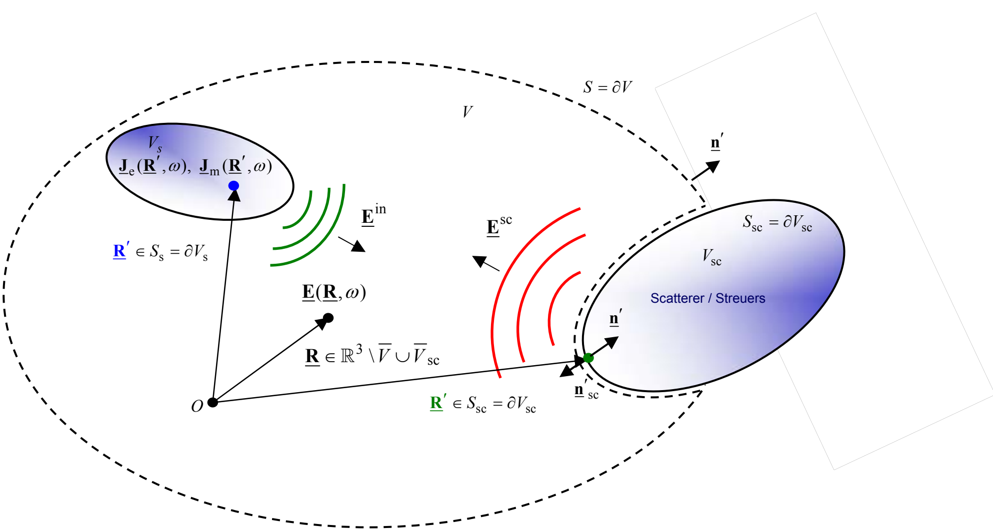


$$\left.\begin{matrix} \underline{\mathbf{R}}\in V & \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \\ \underline{\mathbf{R}}\in\mathbb{R}^3\setminus\overline{V}\cup\overline{V}_{\mathrm{sc}} & \underline{\mathbf{0}} \end{matrix}\right\} = -\oiint_{\underline{\mathbf{R}}'\in S=\partial V} \left\{ \mathrm{j}\omega\mu_0 \Big[\Big[\underline{\mathbf{n}}' \times \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \Big] \cdot \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big] - \left\{ \Big[\underline{\mathbf{n}}' \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \Big] \cdot \underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right\} \right\} \mathrm{d}^2\underline{\mathbf{R}}'$$

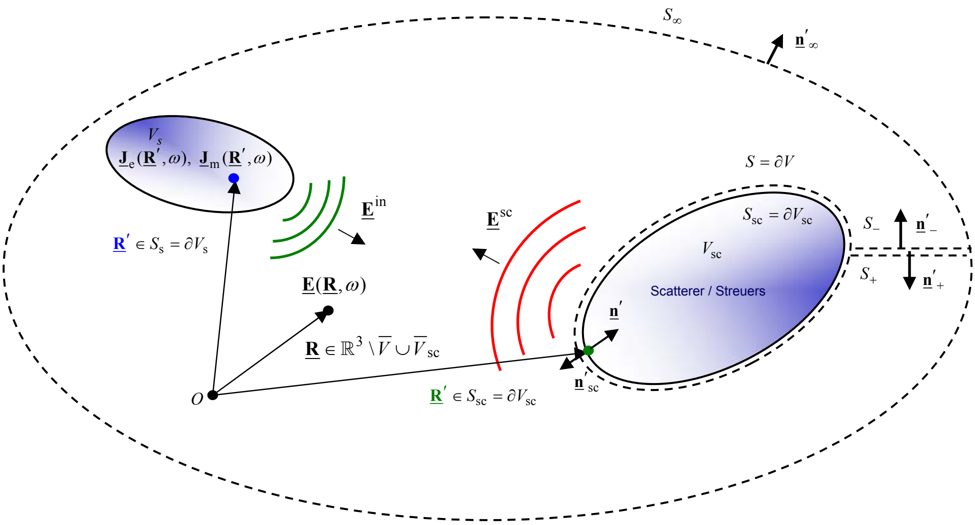
Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version



Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version



Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version



$$S = S_\infty \cup S_+ \cup S_- \cup S_{sc}$$

Electromagnetic Huygens' Principle – Franz Version / Elektromagnetisches Huygenssches Prinzip – Franz Version

$$S = S_\infty \cup S_+ \cup S_- \cup S_{sc}$$

$$\oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S = \partial V} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}'$$

$$= \oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_\infty = \partial V_\infty} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}' + \oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_+ = \partial V_+} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}' + \oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_- = \partial V_-} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}' + \oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_{sc} = \partial V_{sc}} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}'$$

$$\oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_+ = \partial V_+} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}' = -\oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_- = \partial V_-} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}'$$

The contribution of / Der Beitrag von S_+ and / und S_- vanishes / verschwindet

$$\oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_\infty = \partial V_\infty} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}' = 0$$

vanishes if there is no more source outside of / verschwindet, falls keine Quellen mehr außerhalb von S_∞

and if the Silver-Müller Radiation Conditions hold / und wenn die Silver-Müller-Ausstrahlungsbedingungen erfüllt sind

$$Z_0 \hat{\underline{\mathbf{R}}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \circ\left(\frac{1}{R}\right)$$

$$\hat{\underline{\mathbf{R}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + Z_0 \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \circ\left(\frac{1}{R}\right)$$

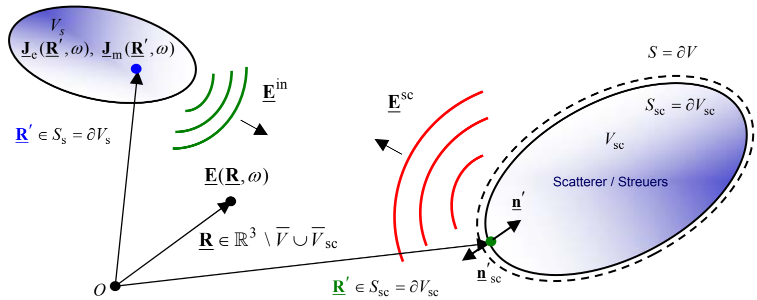
This includes the Conditions / Dies schließt die Bedingung ein

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = O\left(\frac{1}{R}\right)$$

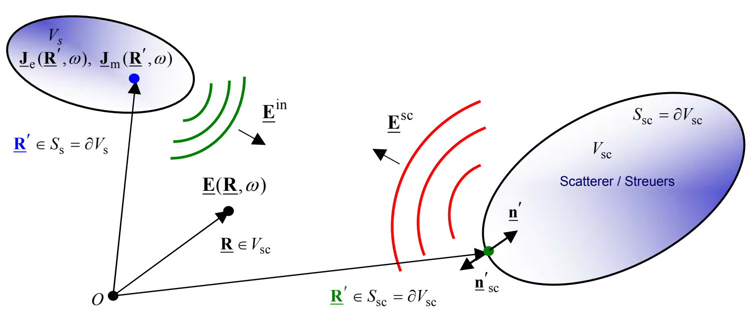
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = O\left(\frac{1}{R}\right)$$

$$\oint\!\!\!\oint_{\underline{\mathbf{R}}' \in S_{sc} = \partial V_{sc}} \left\{ \left(\underline{\mathbf{n}}' \right) \right\} \mathrm{d}^2 \underline{\mathbf{R}}'$$

remains / verbleibt



$$S = S_{sc}$$

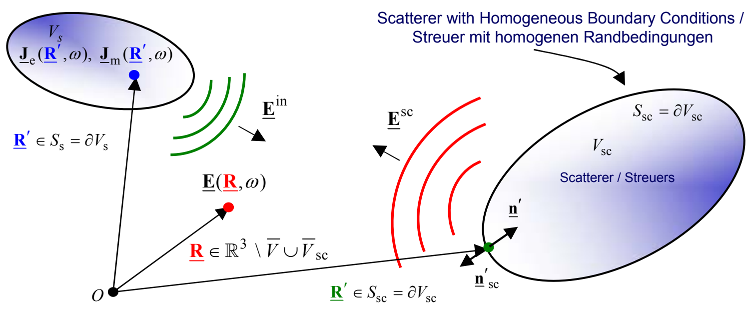


$$\begin{matrix} \underline{\mathbf{R}} \in \mathbb{R}^3 \setminus \overline{V} & \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \\ \underline{\mathbf{R}} \in V & \underline{\mathbf{0}} \end{matrix} \Bigg\} = -\oiint_{\underline{\mathbf{R}}' \in S = \partial V} \Big\{ \mathrm{j}\omega\mu_0 \left[\underline{\mathbf{n}}' \times \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \right] \cdot \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - \left[\underline{\mathbf{n}}' \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \right] \cdot \underline{\underline{\mathbf{G}}}_m(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big\} \, \mathrm{d}^2\underline{\mathbf{R}}'$$

$$\underline{\mathbf{n}}' = -\underline{\mathbf{n}}'_{sc}$$

$$\begin{matrix} \underline{\mathbf{R}}V \in & \underline{\mathbf{E}}^{sc}(\underline{\mathbf{R}},\omega) \\ \underline{\mathbf{R}} \in \mathbb{R}^3 \setminus \overline{V} & \underline{\mathbf{0}} \end{matrix} \Bigg\} = \oiint_{\underline{\mathbf{R}}' \in S_{sc} = \partial V_{sc}} \Big\{ \mathrm{j}\omega\mu_0 \left[\underline{\mathbf{n}}'_{sc} \times \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \right] \cdot \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - \left[\underline{\mathbf{n}}'_{sc} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \right] \cdot \underline{\underline{\mathbf{G}}}_m(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \Big\} \, \mathrm{d}^2\underline{\mathbf{R}}'$$

Electromagnetic Exterior Boundary Value Problem with Homogeneous Boundary Conditions: Scatterer with Perfectly Electric or Magnetic Conductivity / Elektromagnetisches Außenraum Randwertproblem mit homogenen Randbedingung: Streuer mit ideal elektrischer und magnetischer Leitfähigkeit



$$\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega) = \iiint_{\underline{\mathbf{R}}'=V_s} \left[\mathrm{j}\omega\mu_0 \underline{\mathbf{J}}_{\text{e}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \underline{\mathbf{J}}_{\text{m}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}_{\text{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right] \mathrm{d}^3\underline{\mathbf{R}}'$$

$$\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega) = \oiint_{\underline{\mathbf{R}}'\in S_{\text{sc}}=\partial V_{\text{sc}}} \left\{ \mathrm{j}\omega\mu_0 \left[\underline{\mathbf{n}}'_{\text{sc}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \right] \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - \left[\underline{\mathbf{n}}'_{\text{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \right] \bullet \underline{\underline{\mathbf{G}}}_{\text{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right\} \mathrm{d}^2\underline{\mathbf{R}}'$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)$$

Electromagnetic Exterior Boundary Value Problem with Homogeneous Boundary Conditions: Scatterer with Perfectly Electric or Magnetic Conductivity / Elektromagnetisches Außenraum Randwertproblem mit homogenen Randbedingung: Streuer mit ideal elektrischer und magnetischer Leitfähigkeit

Incident Wavefield / Einfallendes Wellenfeld

$$\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega) = \iiint_{\underline{\mathbf{R}}'=V_s} \left[\mathrm{j}\omega\mu_0 \underline{\mathbf{J}}_{\text{e}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \underline{\mathbf{J}}_{\text{m}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}_{\text{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right] \mathrm{d}^3\underline{\mathbf{R}}'$$

$$\underline{\mathbf{H}}^{\text{in}}(\underline{\mathbf{R}},\omega) = \iiint_{\underline{\mathbf{R}}'=V_s} \left[-\underline{\mathbf{J}}_{\text{e}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \mathrm{j}\omega\varepsilon_0 \underline{\mathbf{J}}_{\text{m}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}_{\text{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right] \mathrm{d}^3\underline{\mathbf{R}}'$$

Scattered Wavefield / Gestreutes Wellenfeld

$$\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega) = \oiint_{\underline{\mathbf{R}}'\in S_{\text{sc}}=\partial V_{\text{sc}}} \left\{ \mathrm{j}\omega\mu_0 \underbrace{\left[\underline{\mathbf{n}}'_{\text{sc}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}}',\omega) \right]}_{=\underline{\mathbf{K}}_{\text{e}}(\underline{\mathbf{R}}',\omega)} \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - \underbrace{\left[\underline{\mathbf{n}}'_{\text{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}}',\omega) \right]}_{=-\underline{\mathbf{K}}_{\text{m}}(\underline{\mathbf{R}}',\omega)} \bullet \underline{\underline{\mathbf{G}}}_{\text{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right\} \mathrm{d}^2\underline{\mathbf{R}}'$$

Electric Surface Current Density / Elektrische Flächenstromdichte

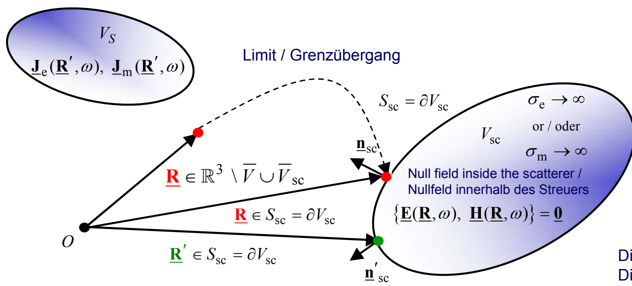
Magnetic Surface Current Density / Magnetische Flächenstromdichte

$$\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega) = \oiint_{\underline{\mathbf{R}}'\in S_{\text{sc}}=\partial V_{\text{sc}}} \left\{ \mathrm{j}\omega\mu_0 \underline{\mathbf{K}}_{\text{e}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \underline{\mathbf{K}}_{\text{m}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}_{\text{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right\} \mathrm{d}^2\underline{\mathbf{R}}'$$

Total Wavefield / Gesamtwellenfeld

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)$$

Electromagnetic Exterior Boundary Value Problem with Homogeneous Boundary Conditions: Scatterer with Perfectly Electric or Magnetic Conductivity / Elektromagnetisches Außenraum Randwertproblem mit homogenen Randbedingung: Streuer mit ideal elektrischer und magnetischer Leitfähigkeit



Boundary condition for $\mathbf{R} \in S_{sc}$
Randbedingung für $\mathbf{R} \in S_{sc}$

PEC / IEL
$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_m(\mathbf{R},\omega) = \underline{\mathbf{0}}$$

or / oder

PMC / IML
$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{H}}(\mathbf{R},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_e(\mathbf{R},\omega) = \underline{\mathbf{0}}$$

Direct scattering problem for PEC or PMC scatterer /
Direktes Streuproblem für IEL oder IML Streuer

PEC Case / IEL-Fall:
$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_m(\mathbf{R},\omega) = \underline{\mathbf{0}}$$

$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\mathbf{R},\omega) + \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{sc}(\mathbf{R},\omega)$$
$$= \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\mathbf{R},\omega) + \underline{\mathbf{n}}_{sc} \times \left\{ \oint\!\!\oint_{\mathbf{R}' \in S_{sc} = \partial V_{sc}} \left\{ j\omega\mu_0 \underline{\mathbf{K}}_e(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\mathbf{R}-\mathbf{R}',\omega) + \underline{\mathbf{K}}_m(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}_m(\mathbf{R}-\mathbf{R}',\omega) \right\} d^2\mathbf{R}' \right\}$$

Electromagnetic Exterior Boundary Value Problem with Homogeneous Boundary Conditions: Scatterer with Perfectly Electric Conductivity / Elektromagnetisches Außenraum Randwertproblem mit homogenen Randbedingung: Streuer mit ideal elektrischer Leitfähigkeit

PEC Case / IEL-Fall:
$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_m(\mathbf{R},\omega) = \underline{\mathbf{0}}$$

$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\mathbf{R},\omega) + \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{sc}(\mathbf{R},\omega)$$
$$= \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\mathbf{R},\omega) + \underline{\mathbf{n}}_{sc} \times \left\{ \oint\!\!\oint_{\mathbf{R}' \in S_{sc} = \partial V_{sc}} \left\{ j\omega\mu_0 \underline{\mathbf{K}}_e(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\mathbf{R}-\mathbf{R}',\omega) + \underline{\mathbf{K}}_m(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}_m(\mathbf{R}-\mathbf{R}',\omega) \right\} d^2\mathbf{R}' \right\}$$

$\mathbf{R} \in \mathbb{R}^3 \setminus \overline{V} \cup \overline{V}_{sc} \to \mathbf{R} \in S_{sc} = \partial V_{sc}$ Limit / Grenzübergang

$$\frac{1}{2}\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\mathbf{R},\omega)$$
$$+ \mathrm{PV}_\varepsilon \underline{\mathbf{n}}_{sc} \times \left\{ \oint\!\!\oint_{\mathbf{R}' \in S_{sc} = \partial V_{sc}} \left\{ j\omega\mu_0 \underline{\mathbf{K}}_e(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\mathbf{R}-\mathbf{R}',\omega) + \underline{\mathbf{K}}_m(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}_m(\mathbf{R}-\mathbf{R}',\omega) \right\} d^2\mathbf{R}' \right\}$$

$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}(\mathbf{R},\omega) = \underline{\mathbf{0}}$ Boundary condition for $\mathbf{R} \in S_{sc}$
Randbedingung für $\mathbf{R} \in S_{sc}$

PEC Case: EFIE /
IEL-Fall: EFIE
$$\underline{\mathbf{0}} = \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\mathbf{R},\omega) + j\omega\mu_0 \mathrm{PV}_\varepsilon \underline{\mathbf{n}}_{sc} \times \oint\!\!\oint_{\mathbf{R}' \in S_{sc} = \partial V_{sc}} \underline{\mathbf{K}}_e(\mathbf{R}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\mathbf{R}-\mathbf{R}',\omega)\, d^2\mathbf{R}'$$

EFIE: Electric Field Integral Equation

Electromagnetic Exterior Boundary Value Problem with Homogeneous Boundary Conditions: Scatterer with Perfectly Electric Conductivity / Elektromagnetisches Außenraum Randwertproblem mit homogenen Randbedingung: Streuer mit ideal elektrischer Leitfähigkeit

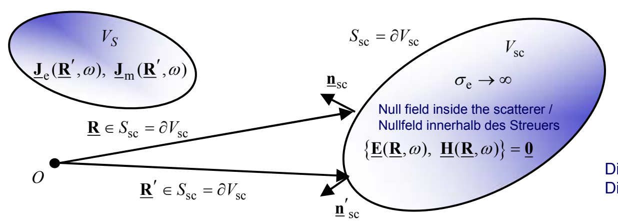
PEC Case: EFIE /
IEL-Fall: EFIE

$$\underline{\mathbf{0}} = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \mathrm{j}\omega\mu_0 \mathrm{PV}_\varepsilon\, \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}'$$

EFIE: Electric Field Integral Equation

$$\mathrm{j}\omega\mu_0 \mathrm{PV}_\varepsilon\, \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

PEC Scatterer: – Franz, Stratton-Chu, and Franz-Larmor Version of EFIE and MFIE / IEL Streuer: Franz, Stratton-Chu und Franz-Larmor Version von EFIE und MFIE



Boundary condition for $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$
Randbedingung für $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

Direct scattering problem for PEC scatterer /
Direktes Streuproblem für IEL Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$):

Franz version / Franz-Version:

$$\mathrm{j}\omega\mu_0 \mathrm{PV}_\varepsilon\, \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\bullet \underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

Stratton-Chu version / Stratton-Chu-Version:

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \left[\mathrm{j}\omega\mu_0 \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\,G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \frac{1}{\mathrm{j}\omega\varepsilon_0}\nabla'\bullet \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

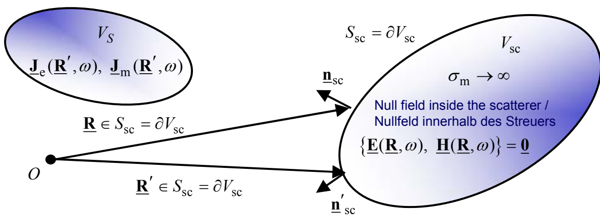
$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\times \nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$\frac{1}{\mathrm{j}\omega\varepsilon_0}\underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\,G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \oiint_{\underline{\mathbf{R}}'\in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\,G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

PMC Scatterer: – Franz, Stratton-Chu, and Franz-Larmor Version of EFIE and MFIE / IML Streuer: Franz, Stratton-Chu und Franz-Larmor Version von EFIE und MFIE



Boundary condition for $\underline{\mathbf{R}} \in S_{sc}$
Randbedingung für $\underline{\mathbf{R}} \in S_{sc}$

$$\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_e(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

Direct scattering problem for PEC scatterer /
Direktes Streuproblem für IEL Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{R}} \in S_{sc}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{R}} \in S_{sc}$):

Franz version / Franz-Version:

$$\frac{1}{2}\underline{\mathbf{K}}_m(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_{sc} \times \oiint_{\underline{\mathbf{R}}' \in S_{sc}=\partial V_{sc}} \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) \cdot \underline{\underline{\mathbf{G}}}_m(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\, d^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)$$

$$j\omega\varepsilon_0 \mathrm{PV}_\varepsilon \underline{\mathbf{n}}_{sc} \times \oiint_{\underline{\mathbf{R}}' \in S_{sc}=\partial V_{sc}} \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) \cdot \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\, d^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{H}}^{in}(\underline{\mathbf{R}},\omega)$$

Stratton-Chu version / Stratton-Chu-Version:

$$\frac{1}{2}\underline{\mathbf{K}}_m(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_{sc} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{sc}=\partial V_{sc}} \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) \times \nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\, d^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{n}}_{sc} \times \oiint_{\underline{\mathbf{R}}' \in S_{sc}=\partial V_{sc}} \left[j\omega\varepsilon_0 \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \frac{1}{j\omega\mu_0} \nabla' \cdot \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) \nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) \right] d^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{H}}^{in}(\underline{\mathbf{R}},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$\frac{1}{2}\underline{\mathbf{K}}_m(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_{sc} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{sc}=\partial V_{sc}} \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\, d^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{sc} \times \underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{j\omega\mu_0} \underline{\mathbf{n}}_{sc} \times \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{sc}=\partial V_{sc}} \underline{\mathbf{K}}_m(\underline{\mathbf{R}}',\omega) G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\, d^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{sc} \times \underline{\mathbf{H}}^{in}(\underline{\mathbf{R}},\omega)$$

2-D Versions: TM and TE Case / 2D-Versionen: TM- und TE-Fall

2-D Case /
2D-Fall

Position Vector / Ortsvektor

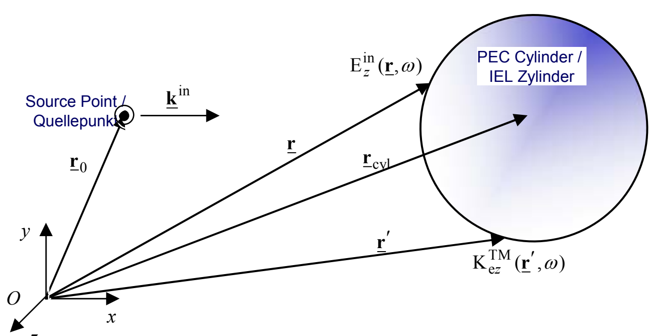
$$\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z\Big|_{z=0} = \underbrace{x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y}_{=r\underline{\mathbf{e}}_r(\varphi)} + \underbrace{z\underline{\mathbf{e}}_z}_{=\underline{\mathbf{0}}} = \underbrace{r\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{r}}} = \underline{\mathbf{r}}$$

Field Quantities / Feldgrößen

$$\{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega), \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega), \underline{\mathbf{K}}_e(\underline{\mathbf{R}},\omega), \underline{\mathbf{K}}_m(\underline{\mathbf{R}},\omega)\}\Big|_{z=0} \to \{\underline{\mathbf{E}}(\underline{\mathbf{r}},\omega), \underline{\mathbf{H}}(\underline{\mathbf{r}},\omega), \underline{\mathbf{K}}_e(\underline{\mathbf{r}},\omega), \underline{\mathbf{K}}_m(\underline{\mathbf{r}},\omega)\}$$

$$\frac{\partial}{\partial z} \equiv 0$$

All Field Quantities and the Geometry are Independent of z /
Alle Feldgrößen und die Geometrie sind von z unabhängig



2-D Versions of EFIE: TM Case / 2D-Versionen von EFIE: TM-Fall

Boundary Condition / Randbedingung

$$\underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{0}} \quad \rightarrow \underline{\mathbf{K}}_{\rm m}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{0}} \qquad \Longrightarrow \qquad \underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{H}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{r}},\omega) \quad \rightarrow \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{r}},\omega) \neq \underline{\mathbf{0}}$$

$$\underbrace{\underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}(\underline{\mathbf{r}},\omega)}_{=\underline{\mathbf{0}}} = \underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}^{\rm in}(\underline{\mathbf{r}},\omega) + \underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}^{\rm sc}(\underline{\mathbf{r}},\omega)$$

$$\underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}^{\rm sc}(\underline{\mathbf{r}},\omega) = -\underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}^{\rm in}(\underline{\mathbf{r}},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$-\frac{1}{\mathrm{j}\omega\varepsilon_0}\underline{\mathbf{n}}_{\rm sc} \times \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\rm sc} = \partial V_{\rm sc}} \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}^{\rm in}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{n}}_{\rm sc} \times \underbrace{\left[-\frac{1}{\mathrm{j}\omega\varepsilon_0}\nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\rm sc} = \partial V_{\rm sc}} \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}'\right]}_{=\underline{\mathbf{E}}^{\rm sc}(\underline{\mathbf{R}},\omega)} = -\underline{\mathbf{n}}_{\rm sc} \times \underline{\mathbf{E}}^{\rm in}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{E}}^{\rm sc}(\underline{\mathbf{R}},\omega) = -\frac{1}{\mathrm{j}\omega\varepsilon_0}\nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\rm sc} = \partial V_{\rm sc}} \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}'$$

2-D Versions of EFIE: TM Case / 2D-Versionen von EFIE: TM-Fall

In the 2-D TM is only the E_z Component Unequal of Zero. That's Because we project the Electric Field Strength Onto the Unit Vector in z Direction / Im 2D-TM-Fall ist nur die E_z Komponente ungleich von Null. Deshalb projizieren wir den elektrische Feldstärkevektor auf den Einheitsvektor in z-Richtung

$$\underline{\mathbf{e}}_z \bullet \underline{\mathbf{E}}^{\rm sc}(\underline{\mathbf{R}},\omega) = \mathrm{E}_z^{\rm sc}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{e}}_z \bullet \underline{\mathbf{E}}^{\rm sc}(\underline{\mathbf{R}},\omega) = -\frac{1}{\mathrm{j}\omega\varepsilon_0}\underline{\mathbf{e}}_z \bullet \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\rm sc} = \partial V_{\rm sc}} \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}'$$

$$\underline{\mathbf{e}}_z \bullet \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\rm sc} = \partial V_{\rm sc}} \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{e}}_z \bullet \oiint_{\underline{\mathbf{R}}' \in S_{\rm sc} = \partial V_{\rm sc}} \nabla \times \nabla \times \left[\underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^2\underline{\mathbf{R}}'$$

$$\nabla \times \nabla \times \left[\underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right] = \nabla\nabla \bullet \left[\underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right] - \nabla \bullet \nabla\left[\underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]$$

$$= \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega) \bullet \nabla\nabla G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - \underline{\mathbf{K}}_{\rm e}(\underline{\mathbf{R}}',\omega)\nabla \bullet \nabla G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)$$

$$\nabla\nabla G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = k_0^2 \ldots$$

2-D-PEC-TM-EFIE / 2D-IEL-TM-EFIE

$$\Longrightarrow \qquad \mathrm{j}kZ\int_{\underline{\mathbf{r}}' \in C_{\rm sc}} G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)\,\mathrm{K}_{\rm ez}^{\rm TM}(\underline{\mathbf{r}}',\omega)\,\mathrm{d}\underline{\mathbf{r}}' = -\mathrm{E}_z^{\rm in}(\underline{\mathbf{r}},\omega), \qquad \underline{\mathbf{r}} \in C_{\rm sc}$$

End of 2nd Lecture /
Ende der 2. Vorlesung