Numerical Methods of Electromagnetic Field Theory II (NFT II) Numerische Methoden der Elektromagnetischen Feldtheorie II (NFT II) /

6th Lecture / 6. Vorlesung

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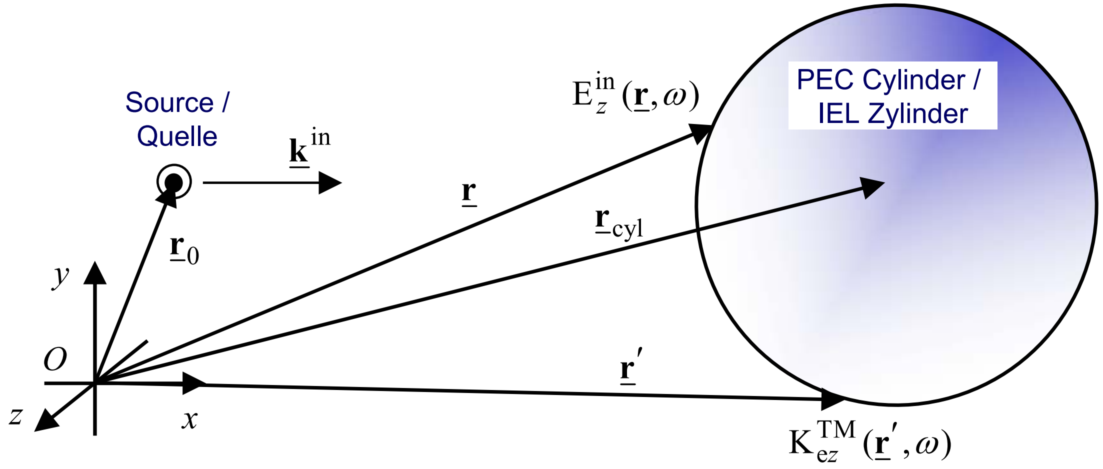
EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen



2-D Case /
2-D-Fall

$$\underline{\mathbf{R}} = \underbrace{r\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{r}}} + \underbrace{z\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{0}}} = \underline{\mathbf{r}}$$

2-D PEC TM EFIE / 2D-IEL-TM-EFIE

$$-\mathrm{j}\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} \mathrm{K}_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega)\, G\left(\underline{\mathbf{r}} - \underline{\mathbf{r}}',\omega\right) \mathrm{d}\underline{\mathbf{r}}' = \mathrm{E}_z^{\mathrm{in}}(\underline{\mathbf{r}},\omega), \quad \underline{\mathbf{r}} \in C_{\mathrm{sc}}$$

This is a *Fredholm integral equation of the 1. kind* in form of a *closed line integral* for the *unknown* electric surface current density for a *known* incident field. /
Dies ist eine *Fredholmsche Integralgleichung 1. Art* in Form eines *geschlossenen Linienintegrals* für die *unbekannte* elektrische Flächenladungsdichte für ein *bekanntes* einfallendes Feld.

$$G(\underline{\mathbf{r}} - \underline{\mathbf{r}}',\omega) = \frac{\mathrm{j}}{4}\mathrm{H}_0^{(1)}\left(k_0 \left|\underline{\mathbf{r}} - \underline{\mathbf{r}}'\right|\right)$$

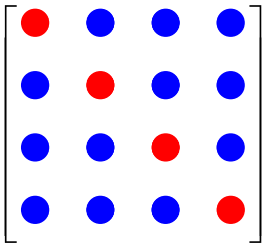
EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

2-D PEC TM EFIE / 2D-IEL-TM-EFIE

$$-\mathrm{j}\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} \mathrm{K}_{\mathrm{e}\,z}^{\mathrm{TM}}(\underline{\mathbf{r}}', \omega)\, G\big(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega\big)\, \mathrm{d}\underline{\mathbf{r}}' = \mathrm{E}_z^{\mathrm{in}}(\underline{\mathbf{r}}, \omega), \quad \underline{\mathbf{r}} \in C_{\mathrm{sc}}$$

We have to Consider Two Different Cases for the Elements of the Impedance Matrix /
Man unterscheidet zwei Verschiedene Fälle für die Elemente der Impedanzmatrix

$$\begin{bmatrix} Z_{11}(\omega) & Z_{12}(\omega) & \dots & Z_{1N}(\omega) \\ Z_{21}(\omega) & Z_{22}(\omega) & \dots & Z_{2N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1}(\omega) & Z_{N2}(\omega) & \dots & Z_{NN}(\omega) \end{bmatrix} \Leftrightarrow$$



Main Diagonal Elements / Hauptdiagonalelemente

For / Für: $\underline{\mathbf{r}} = \underline{\mathbf{r}}'$ (Self Cell / Eigenzelle)

Off Diagonal Elements / Nebendiagonalelemente

For / Für $\underline{\mathbf{r}} \neq \underline{\mathbf{r}}'$

Main Diagonal Elements / Hauptdiagonalelemente

1. Flat Cell Approximation / Ebene-Zelle-Approximation
2. Power Series Expansion of the Hankel Function for Small Arguments / Potenzreihen-Approximation der Hankel-Funktion für kleine Argumente

Off Diagonal Elements / Nebendiagonalelemente

1. Flat Cell Approximation / Ebene-Zelle-Approximation
2. Application of the Midpoint Rule / Anwendung der Mittelpunktsregel

EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

Elements of the Impedance Matrix /
Elemente der Impedanzmatrix

$$Z_{mn}(\omega) = \frac{\omega\mu_0}{4}\Delta^{(n)} \begin{cases} 1 + \mathrm{j}\dfrac{2}{\pi}\left[\ln\left(\dfrac{k}{4}\Delta^{(n)}\right) + \gamma - 1\right] & m = n \\ \mathrm{H}_0^{(1)}(k\, r_{mn}) & m \neq n \end{cases}$$

Matrix Equation / Matrixgleichung

$$\underbrace{\left[\mathbf{Z}\right]}_{=\mathrm{V/A}}(\omega)\underbrace{\left\{\mathbf{K}_{\mathrm{e}z}^{\mathrm{TM}}\right\}}_{=\mathrm{A/m}}(\omega) = \underbrace{\left\{\mathbf{E}_z^{\mathrm{in}}\right\}}_{=\mathrm{V/m}}(\omega)$$

Problem: Large Impedance Matrix /
Problem: Große Impedanzmatrix **!**

Iterative Solution via Conjugate Gradient (CG) Method /
Iterative Lösung durch Konjugierte Gradienten (KG) Methode

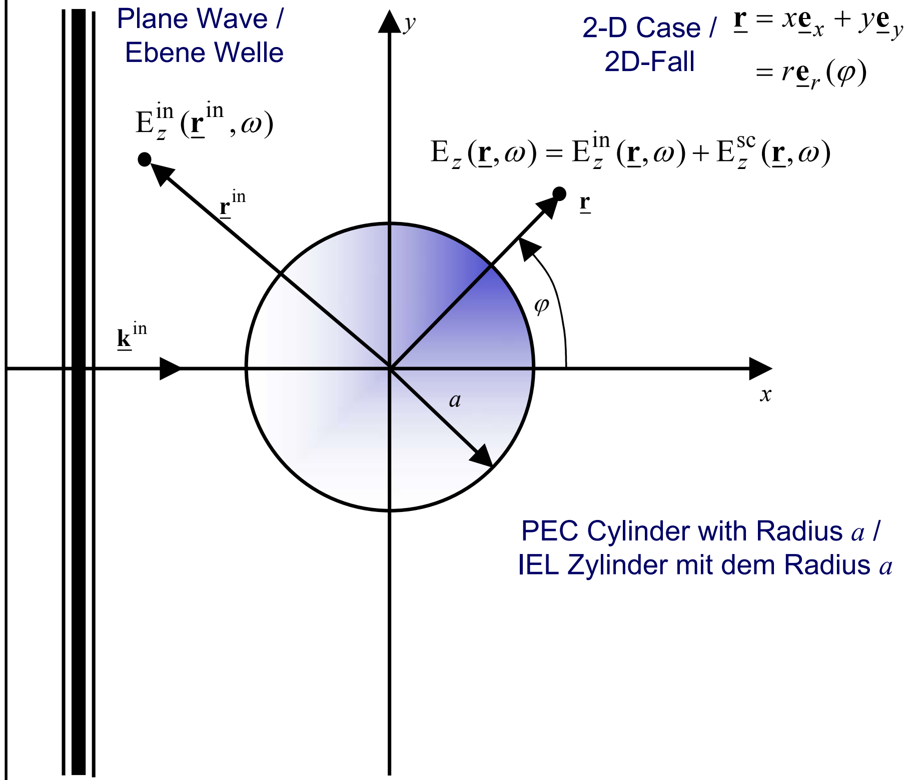
Solution of the Matrix Equation / Lösung der Matrixgleichung

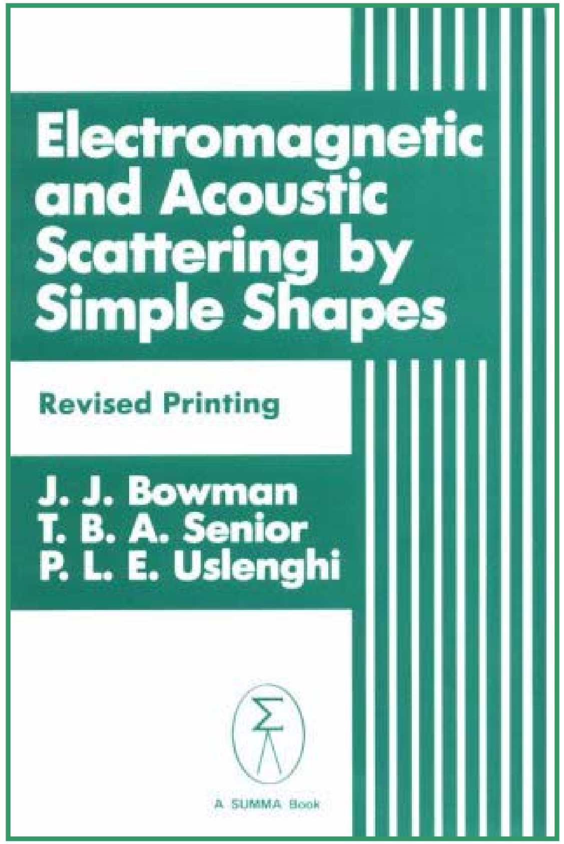
$$\underbrace{\left\{\mathbf{K}_{\mathrm{e}z}^{\mathrm{TM}}\right\}}_{=\mathrm{A/m}}(\omega) = \underbrace{\left[\mathbf{Z}\right]^{-1}}_{=\mathrm{A/V}}(\omega)\underbrace{\left\{\mathbf{E}_z^{\mathrm{in}}\right\}}_{=\mathrm{V/m}}(\omega)$$

Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall

Separation of Variables
Analytic Solution in Terms of Eigenfunctions /
Separation der Variablen
Analytische Lösung in Form von Eigenfunktionen

J. J. Bowman, T. B. A. Senior, P. L. E. Uslenghi (Editors):
Electromagnetic and Acoustic Scattering by Simple Shapes.
Taylor & Francis Inc, New York, 1988.





Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case – Analytic Solution: Separation of Variables / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall – Analytische Lösung: Separation der Variablen

Electric Field Strength of the Incident Wave /
Elektrische Feldstärke der einfallenden Welle

$$E_z^{\mathrm{in}}(r,\varphi,\varphi_{\mathrm{in}},\omega) = \underbrace{E_0(\omega)}_{=1\ \mathrm{V/m}}\, \mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}^{\mathrm{in}}\bullet\underline{\mathbf{r}}}$$

Boundary Condition at the PEC Cylinder /
Randbedingung am IEL-Zylinder

$$E_z(r=a,\varphi,\varphi_{\mathrm{in}},\omega) = E_z^{\mathrm{in}}(r=a,\varphi,\varphi_{\mathrm{in}},\omega) + E_z^{\mathrm{sca}}(r=a,\varphi,\varphi_{\mathrm{in}},\omega) = 0$$

Solution /
Lösung

Electric Field Strength of the Scattered Wave /
Elektrische Feldstärke der gestreuten Welle

$$E_z^{\mathrm{sc}}(r,\varphi,\varphi_{\mathrm{in}},\omega) = -\sum_{n=0}^{\infty} \varepsilon_n\,(-\mathrm{j})^n \frac{\mathrm{J}_n(ka)}{\mathrm{H}_n^{(1)}(ka)}\,\mathrm{H}_n^{(1)}(kr)\cos\left[n\left(\varphi-\varphi_{\mathrm{in}}\right)\right]$$

Neumann Function /
Neumann-Funktion

$$\varepsilon_n = \begin{cases} 1 & n=0 \\ 2 & n=1,2,3,\ldots \end{cases}$$

Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case – Analytic Solution: Separation of Variables / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall – Analytische Lösung: Separation der Variablen

Boundary Condition at the PEC Cylinder /
Randbedingung am IEL-Zylinder

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(r = a, \varphi, \varphi_{\text{in}}, \omega) = \underline{\mathbf{0}}$$

$$E_z(r = a, \varphi, \varphi_{\text{in}}, \omega) = 0$$

Induced Electric Surface Current Density at /
Induzierte elektrische Flächenstromdichte bei $r = a$

$$\underline{\mathbf{n}} \times \underline{\mathbf{H}}(r = a, \varphi, \varphi_{\text{in}}, \omega) = \underline{\mathbf{K}}_{\text{e}}(r = a, \varphi, \varphi_{\text{in}}, \omega), \qquad \underline{\mathbf{n}} = \underline{\mathbf{e}}_R$$

$$\begin{aligned} K_{\text{e}z}^{\text{TM}}(\varphi, \varphi_{\text{in}}, \omega) &= H_\varphi(r = a, \varphi, \varphi_{\text{in}}, \omega) \\ &= H_\varphi^{\text{in}}(r = a, \varphi, \varphi_{\text{in}}, \omega) + H_\varphi^{\text{sc}}(r = a, \varphi, \varphi_{\text{in}}, \omega) \\ &= 2 \frac{Y_0}{\pi} \frac{1}{ka} \sum_{n=0}^{\infty} \varepsilon_n \frac{(-\text{j})^n}{\text{H}_n^{(1)}(ka)} \cos\left[n \left(\varphi - \varphi_{\text{in}} \right) \right] \end{aligned}$$

$$K_{\text{e}z}^{\text{TM}}(\varphi, \varphi_{\text{in}}, \omega) = 2 \frac{Y_0}{\pi} \frac{1}{ka} \sum_{n=0}^{\infty} \varepsilon_n \frac{(-\text{j})^n}{\text{H}_n^{(1)}(ka)} \cos\left[n \left(\varphi - \varphi_{\text{in}} \right) \right]$$

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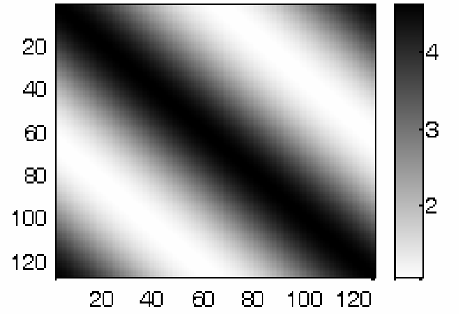
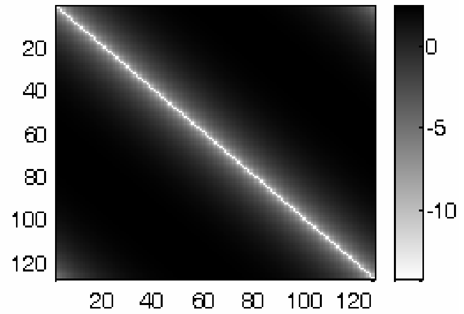
$\mathrm{Re}\{[\mathbf{Z}](\omega)\}$



Elements of the Impedance Matrix /
Elemente der Impedanzmatrix

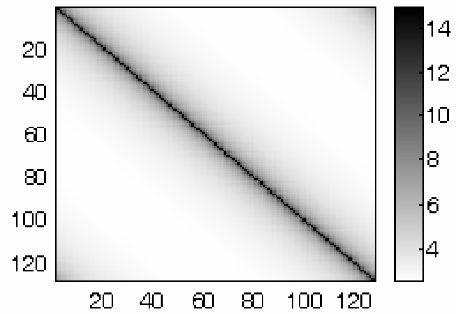
$$Z_{mn}(\omega) = \frac{\omega\mu_0}{4}\Delta^{(n)} \begin{cases} 1 + \mathrm{j}\dfrac{2}{\pi}\left[\ln\left(\dfrac{k}{4}\Delta^{(n)}\right) + \gamma - 1\right] & m = n \\ \mathrm{H}_0^{(1)}(k\, r_{mn}) & m \neq n \end{cases}$$

$\mathrm{Im}\{[\mathbf{Z}](\omega)\}$



$$ka = 1, \qquad N = 128$$

$|[\mathbf{Z}](\omega)|$

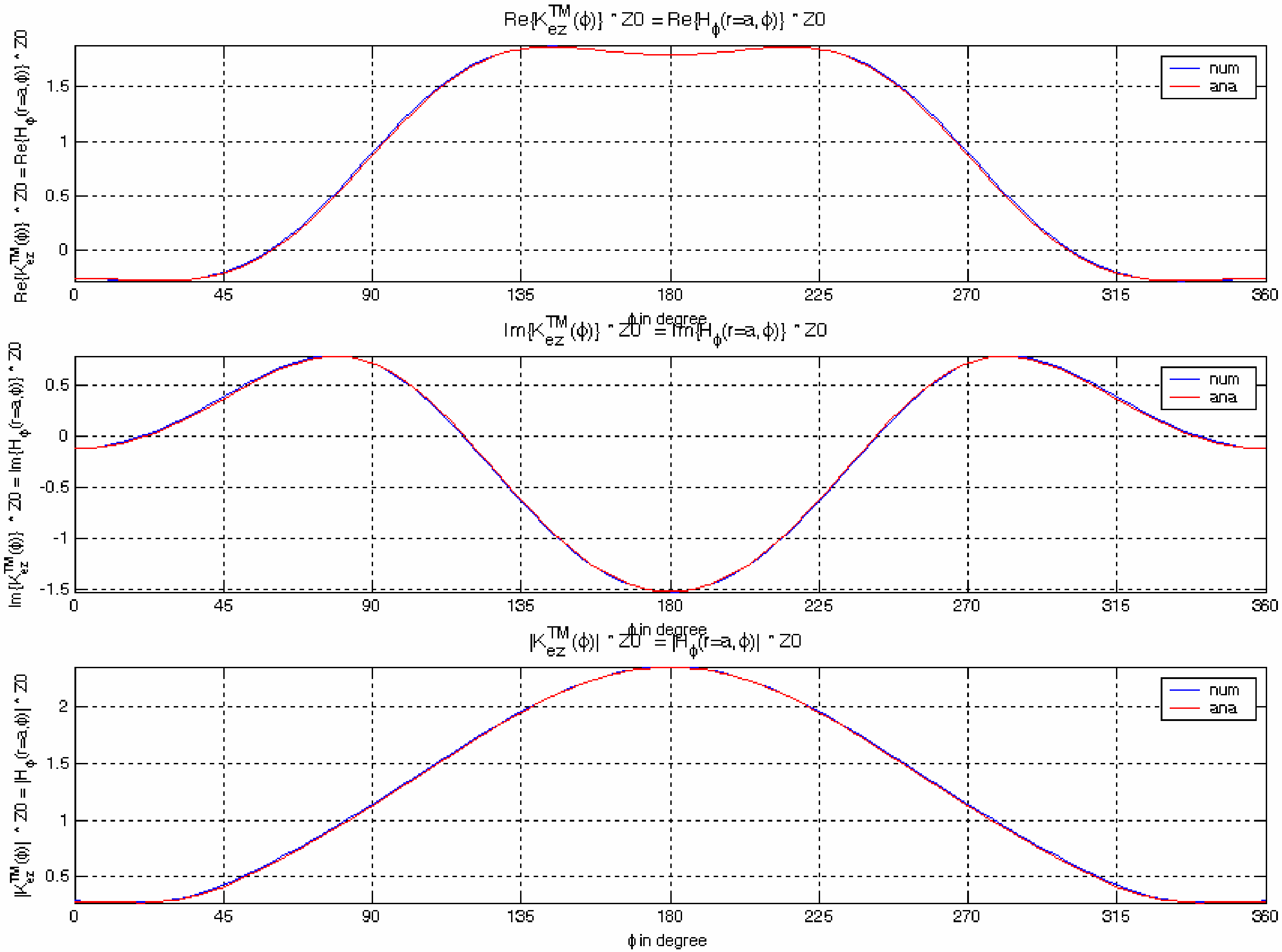


EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Resultate

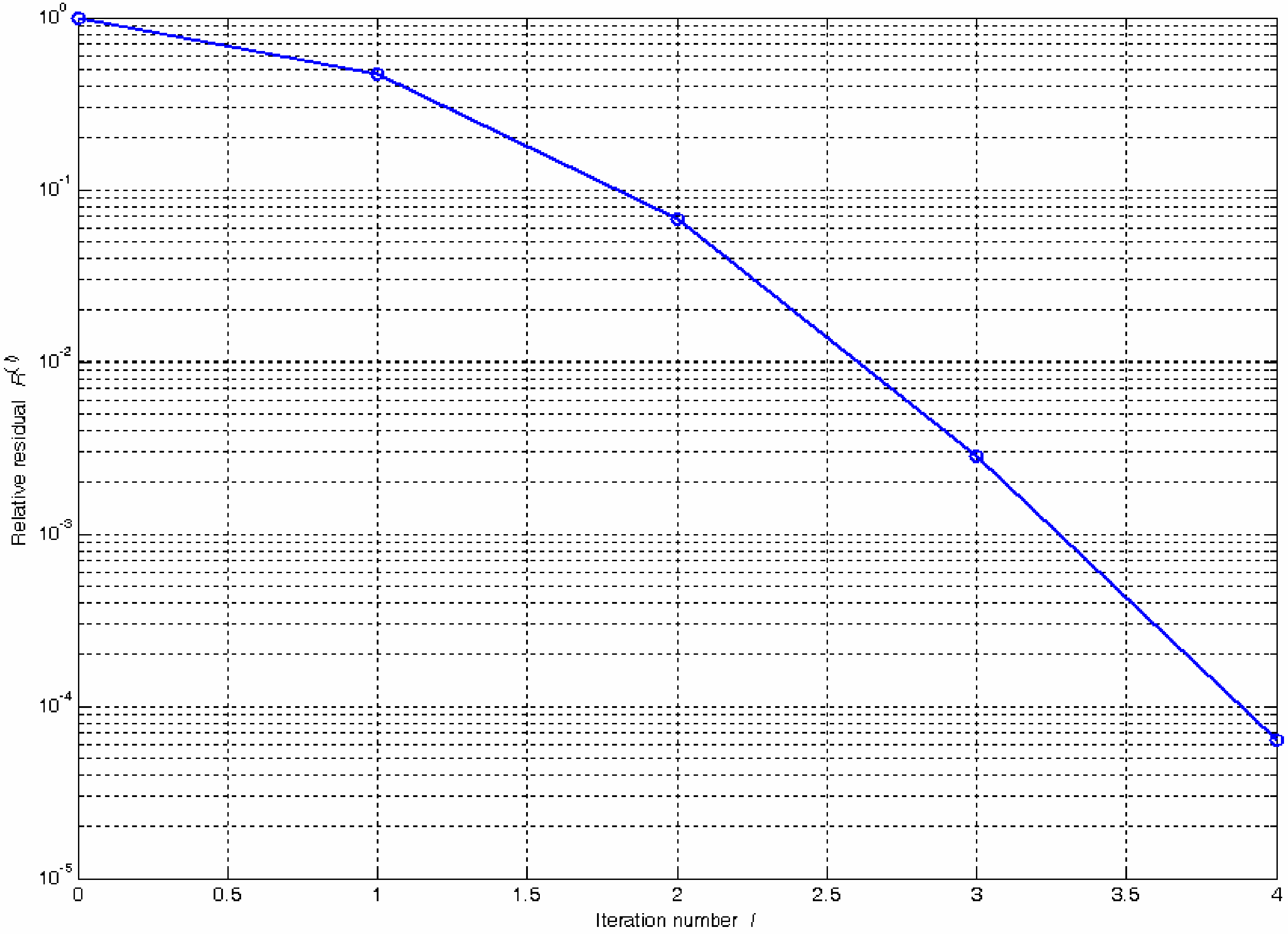
$$Z_0 \operatorname{Re}\left\{K_{\text{ez}}^{\text{TM}}(\varphi, \varphi_{\text{in}} = 180°, \omega)\right\}$$

$$Z_0 \operatorname{Im}\left\{K_{\text{ez}}^{\text{TM}}(\varphi, \varphi_{\text{in}} = 180°, \omega)\right\}$$

$$Z_0 \left|K_{\text{ez}}^{\text{TM}}(\varphi, \varphi_{\text{in}} = 180°, \omega)\right|$$



EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Resultate



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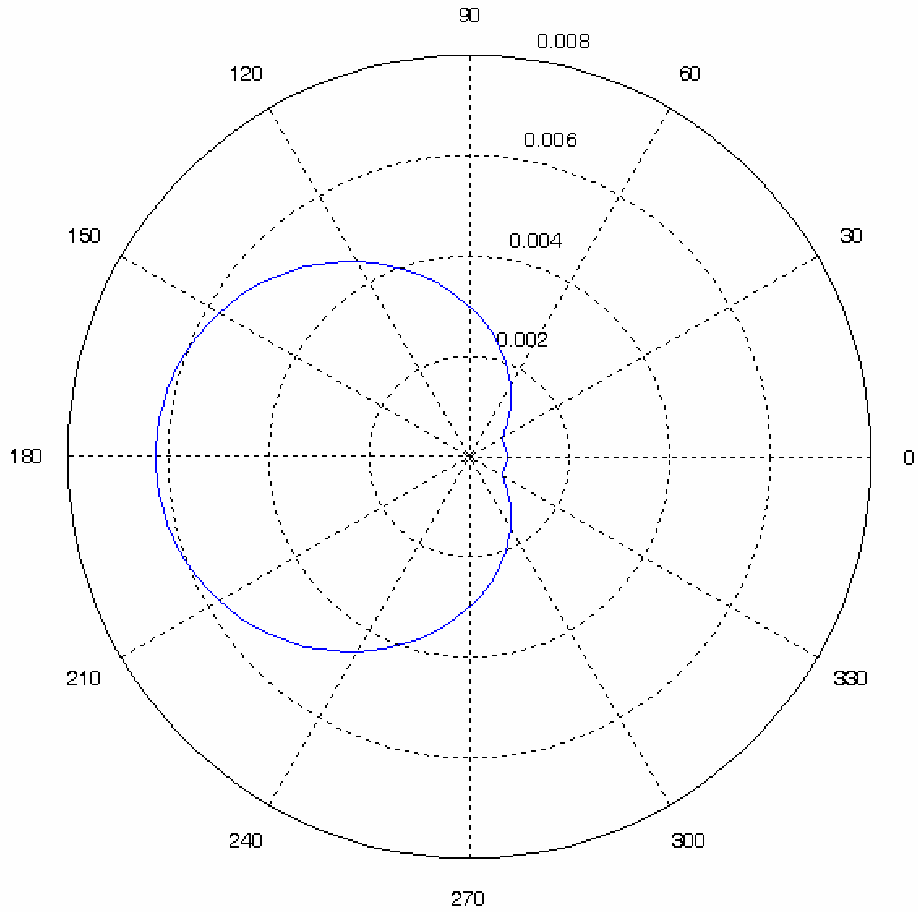
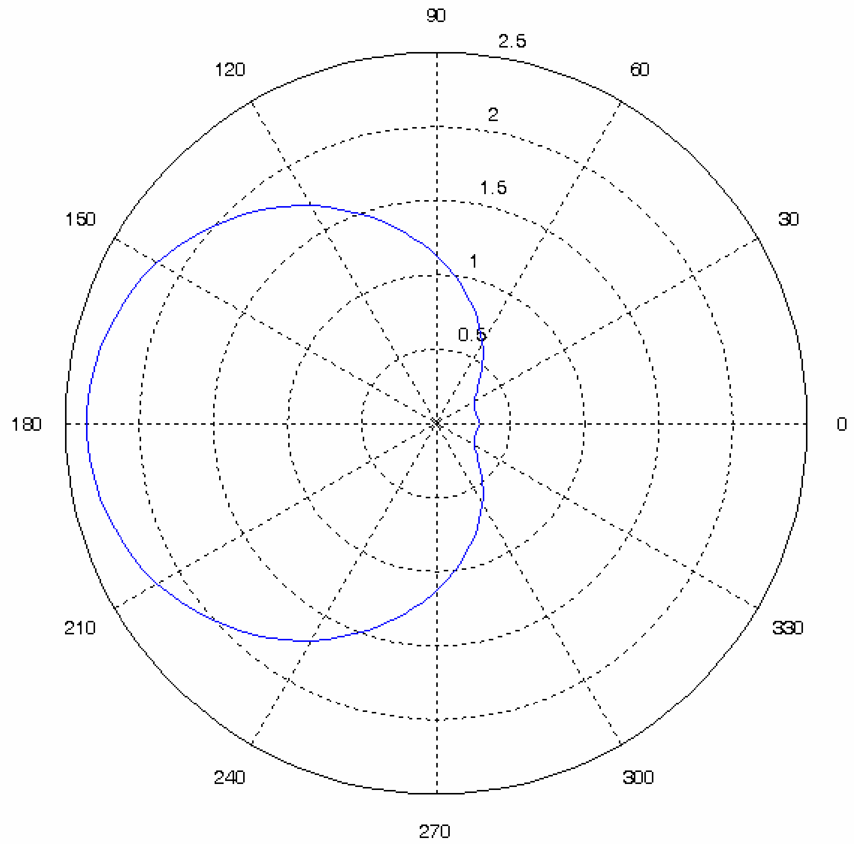
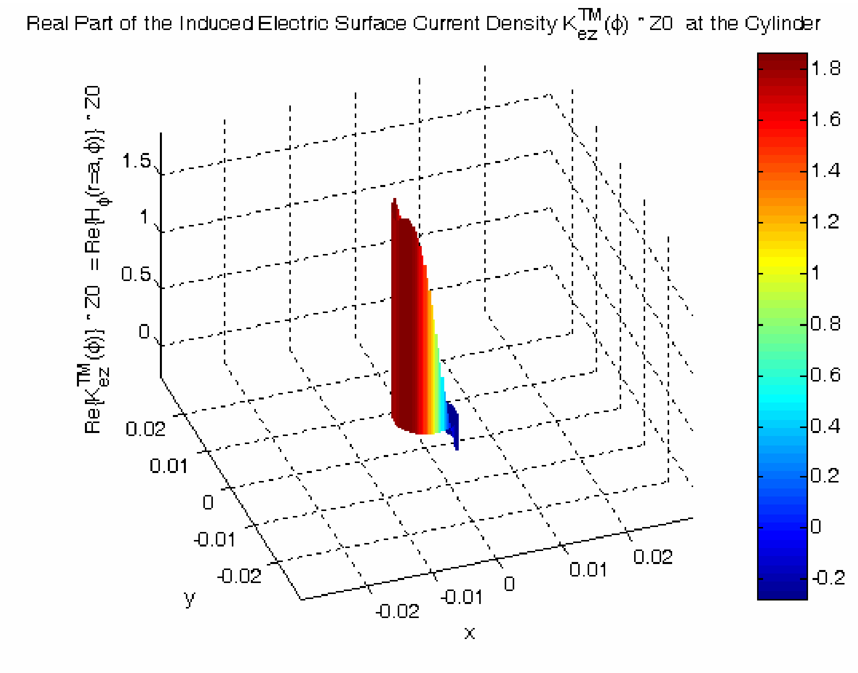
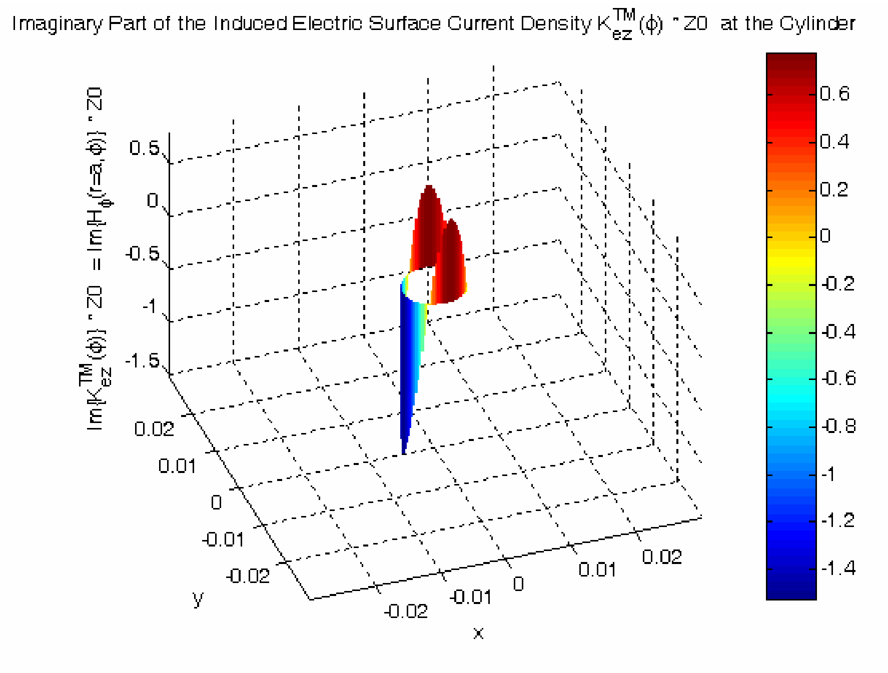


EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Results

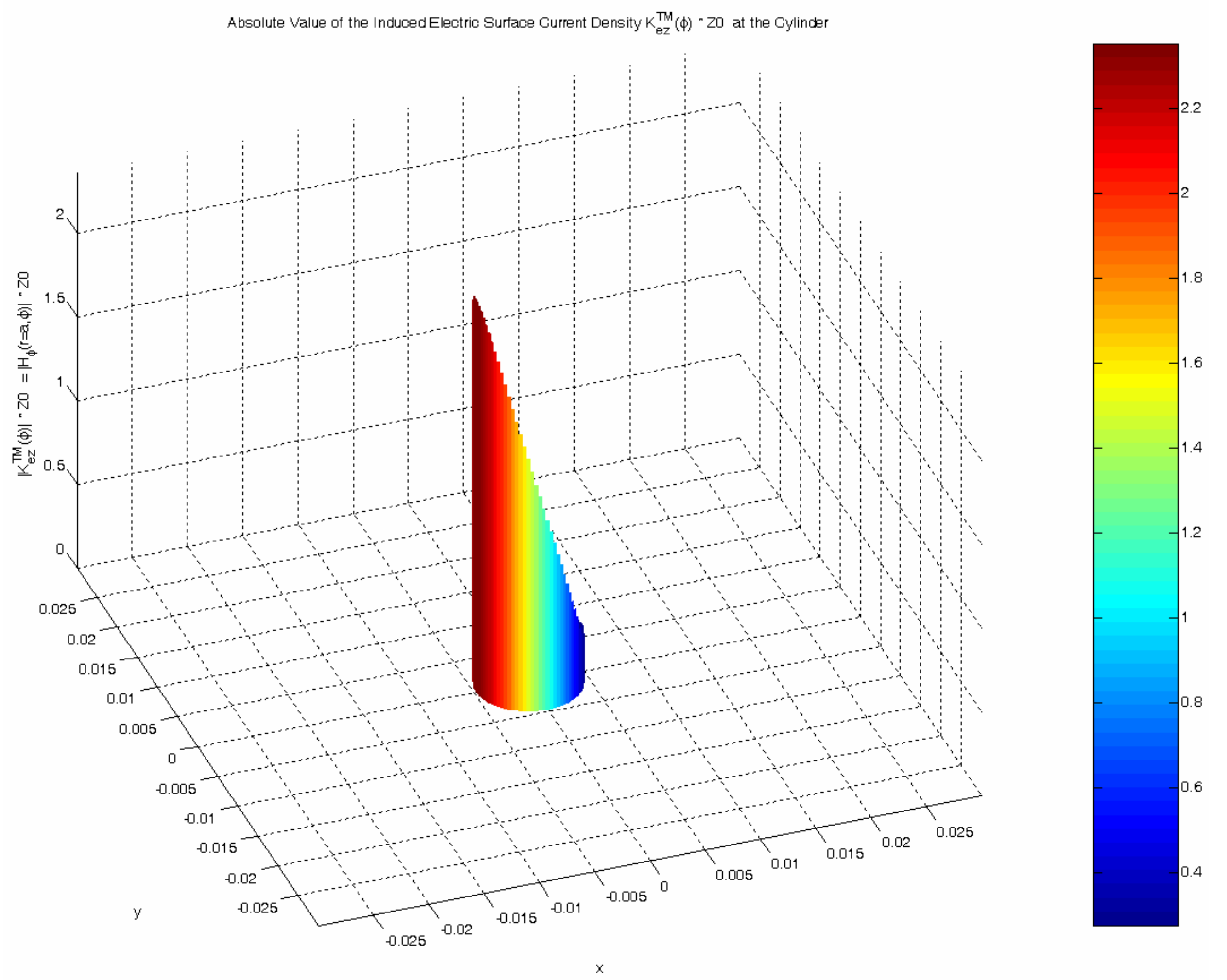


EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Resultate





EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Resultate



Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall

Number of cells N	Magnitude of induced electric surface current density, $\lvert K_z^{TM}(\varphi)\rvert$ for $\varphi = 0$	$\varphi = \pi/2$	$\varphi = \pi$
8	0.00082611	0.00291920	0.00573690
16	0.00077377	0.00299660	0.00613630
32	0.00076747	0.00300135	0.00622450
64	0.00076414	0.00299755	0.00623880
128	0.00076188	0.00299445	0.00623820
Exact	0.00076000	0.00299300	0.00623700
8	0.00084500	0.00298300	0.00639100
16	0.00078400	0.00302000	0.00630200
32	0.00077300	0.00300900	0.00627100
64	0.00076600	0.00300100	0.00625400
128	0.00076300	0.00299700	0.00624500

Table 1: Comparison between ours (top) and published (bottom) results, having circumference of one wavelength, $C = \lambda_0 = 0.3$ m

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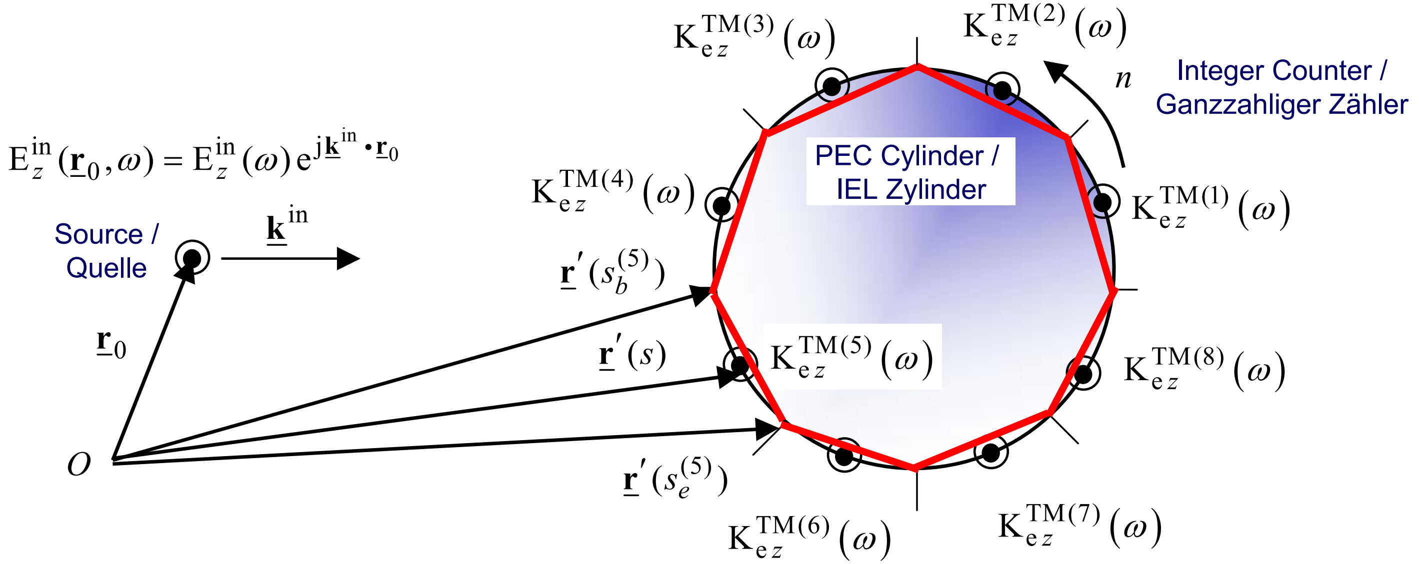
Calculation of the Scattered Field / Berechnung des Streufeldes

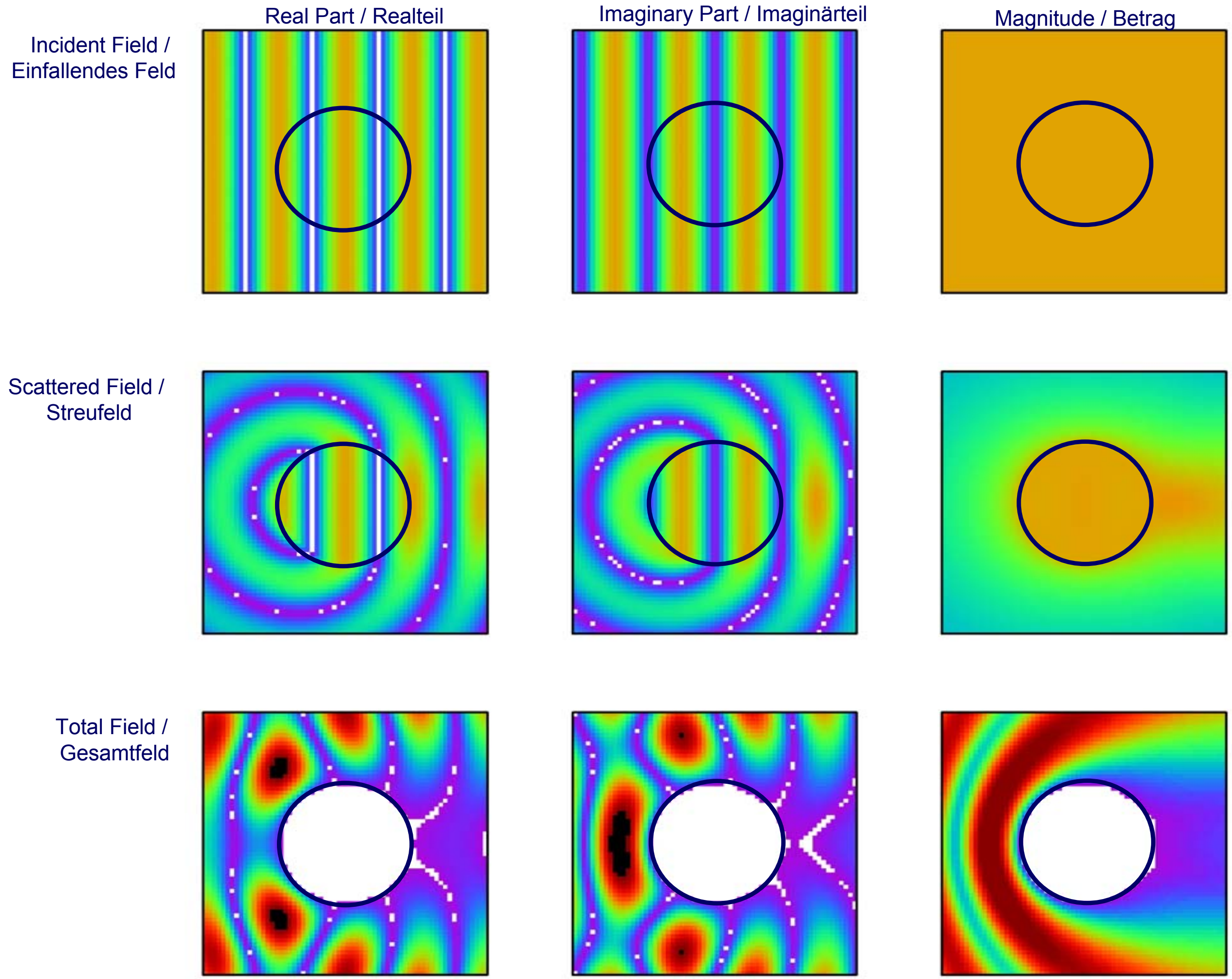
$$E_z^{sc}(\underline{\mathbf{r}},\omega) = j\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{sc} = \partial S_{sc}} K_{ez}^{TM}(\underline{\mathbf{r}}',\omega)\, G\left(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega\right) d\underline{\mathbf{r}}', \qquad \underline{\mathbf{r}} \in \mathbb{R}^3 \setminus \overline{C}_{sc}$$

Calculation of the Total Field / Berechnung des Gesamtfeldes

$$E_z(\underline{\mathbf{r}},\omega) = E_z^{in}(\underline{\mathbf{r}},\omega) + E_z^{sc}(\underline{\mathbf{r}},\omega), \qquad \underline{\mathbf{r}} \in \mathbb{R}^3 \setminus \overline{C}_{sc}$$



Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall



Iterative Methods for the Solution of Discrete Integral Equations / Iterative Methode zur Lösung von diskreten Integralgleichungen

CG Method – Conjugate Gradient (CG) Method

M. R. Hestenes & E. Stiefel, 1952

BiCG Method – Biconjugate Gradient (BiCG) Method

C. Lanczos, 1952
D. A. H. Jacobs, 1981
C. F. Smith et al., 1990
R. Barret et al., 1994

CGS Method – Conjugate Gradient Squared (CGS) Method (MATLAB Function)

P. Sonneveld, 1989

GMRES Method – Generalized Minimal – Residual (GMRES) Method

Y. Saad & M. H. Schultz, 1986
R. Barret et al., 1994
Y. Saad, 1996

QMR Method – Quasi–Minimal–Residual (QMR) Method

R. Freund & N. Nachtigal, 1990
N. Nachtigal, 1991
R. Barret et al., 1994
Y. Saad, 1996

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Non-singular Matrix Equation / Nicht singuläre Matrixgleichung

$$[\mathbf{A}]\{\mathbf{x}\} = \{\mathbf{b}\}$$

Non-singular Matrix / Nicht singuläre Matrix

$$[\mathbf{A}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix}_{N\times N}$$

Solution Vector / Lösungsvektor

$$\{\mathbf{x}\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix}_N$$

Right-hand Side / Rechte Seite

$$\{\mathbf{b}\} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{Bmatrix}_N$$

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Inner Vector Product (Scalar Vector Product) /
Inneres Vektorprodukt (Skalares Vektorprodukt)

$$\langle\{\mathbf{x}\},\{\mathbf{y}\}\rangle = \left(\{\mathbf{x}\}^{\mathrm{T}}\right)^{*}\{\mathbf{y}\}$$
$$= \{\mathbf{x}\}^{\dagger}\{\mathbf{y}\}$$

ℓ_2-Norm / ℓ_2-Norm

$$\|\{\mathbf{x}\}\|_2 = \sqrt{\langle\{\mathbf{x}\},\{\mathbf{x}\}\rangle}$$
$$=: \|\{\mathbf{x}\}\|$$

Used Vector Norms in Linear Algebra – Special Cases of the Hölder Norm / Verwendete Vektornormen in der Linearen Algebra – Spezialfälle der Hölder-Norm

$$\|\{\mathbf{x}\}\|_p = \left(\sum_{n=1}^{N}|x_n|^p\right)^{1/p}$$

$p = 1$ — ℓ_1-Norm / ℓ_1-Norm

$$\|\{\mathbf{x}\}\|_1 = |x_1| + |x_2| + \cdots + |x_N| = \sum_{n=1}^{N}|x_n|$$

$p = 2$ — ℓ_2-Norm / ℓ_2-Norm

$$\|\{\mathbf{x}\}\|_2 = \sqrt{x_1^* x_1 + x_2^* x_2 + \cdots + x_N^* x_N} = \sqrt{\sum_{n=1}^{N} x_n^* x_n}$$

$p = \infty$ — ℓ_∞-Norm / ℓ_∞-Norm

$$\|\{\mathbf{x}\}\|_\infty = \max_{n=1,\ldots,N}|x_n|$$

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Inner Vector Product (Scalar Vector Product) /
Inneres Vektorprodukt (Skalares Vektorprodukt)

$$\left\langle \{\mathbf{x}\},\{\mathbf{y}\}\right\rangle = \left(\{\mathbf{x}\}^{\mathrm{T}}\right)^{*}\{\mathbf{y}\}$$
$$= \{\mathbf{x}\}^{\dagger}\{\mathbf{y}\}$$

ℓ_2-Norm / ℓ_2-Norm

$$\left\|\{\mathbf{x}\}\right\|_2 = \sqrt{\left\langle \{\mathbf{x}\},\{\mathbf{x}\}\right\rangle}$$
$$=: \left\|\{\mathbf{x}\}\right\|$$

$$\{\mathbf{x}\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix}_N \qquad \{\mathbf{x}\}^{\mathrm{T}} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix}^{\mathrm{T}} = \{x_1, x_2, \ldots, x_N\} \qquad \left(\{\mathbf{x}\}^{\mathrm{T}}\right)^{*} = \left(\begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix}^{\mathrm{T}}\right)^{*} = \left\{\overset{*}{x}_1, \overset{*}{x}_2, \ldots, \overset{*}{x}_N\right\}$$

$$\left\|\{\mathbf{x}\}\right\| = \left\|\{\mathbf{x}\}\right\|_2 = \sqrt{\left\langle \{\mathbf{x}\},\{\mathbf{x}\}\right\rangle} = \sqrt{\left\{\overset{*}{x}_1, \overset{*}{x}_2, \ldots, \overset{*}{x}_N\right\}\begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix}} = \sqrt{\overset{*}{x}_1 x_1 + \overset{*}{x}_2 x_2 + \cdots + \overset{*}{x}_N x_N} = \sqrt{\sum_{n=1}^{N} \overset{*}{x}_n x_n}$$

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Non-singular Matrix Equation / Nicht singuläre Matrixgleichung

$$[\mathbf{A}]\{\mathbf{x}\} = \{\mathbf{b}\}$$

Iterative Method / Iterative Methode

$$\{\mathbf{x}\}^{(l)} = \{\mathbf{x}\}^{(l-1)} + \alpha^{(l)}\{\mathbf{p}\}^{(l)} \qquad l = 1, 2, \ldots, L$$

$$\begin{pmatrix}\text{New Approximation /}\\ \text{Neue Approximation}\end{pmatrix}^{(l)} \quad \begin{pmatrix}\text{Old Approximation /}\\ \text{Alte Approximation}\end{pmatrix}^{(l-1)} \quad \begin{pmatrix}\text{Correction Term /}\\ \text{Korrekturterm}\end{pmatrix}^{(l)}$$

$\alpha^{(l)}$ **Scalar Coefficient at Iteration Step l / Skalarer Koeffizient zum Iterationsschritt l**

$\{\mathbf{p}\}^{(l)}$ **lth Direction in the N-Dimensional Space / l-te Richtung im N-dimensionalen Raum**

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Iterative Method / Iterative Methode

$$\{\mathbf{x}\}^{(l)} \quad = \quad \{\mathbf{x}\}^{(l-1)} \quad + \quad \alpha^{(l)}\,\{\mathbf{p}\}^{(l)}$$

$$\begin{pmatrix}\text{New Approximation /}\\ \text{Neue Approximation}\end{pmatrix}^{(l)} \quad \begin{pmatrix}\text{Old Approximation /}\\ \text{Alte Approximation}\end{pmatrix}^{(l-1)} \quad \begin{pmatrix}\text{Correction Term /}\\ \text{Korrekturterm}\end{pmatrix}^{(l)}$$

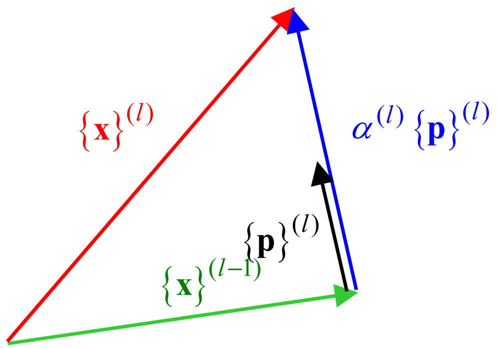
$\alpha^{(l)}$ Scalar Coefficient at Iteration Step l / Skalarer Koeffizient zum Iterationsschritt l

$\{\mathbf{p}\}^{(l)}$ lth Direction in the N-Dimensional Space / l-te Richtung im N-dimensionalen Raum

$$\{\mathbf{x}\}^{(l)} \quad = \quad \{\mathbf{x}\}^{(l-1)} \quad + \quad \alpha^{(l)}\,\{\mathbf{p}\}^{(l)}$$

$$\begin{pmatrix}\text{New Approximation /}\\ \text{Neue Approximation}\end{pmatrix}^{(l)} \quad \begin{pmatrix}\text{Old Approximation /}\\ \text{Alte Approximation}\end{pmatrix}^{(l-1)} \quad \begin{pmatrix}\text{Correction Term /}\\ \text{Korrekturterm}\end{pmatrix}^{(l)}$$

Geometric Interpretation / Geometrische Iterpretation



Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Error Functional / Fehlerfunktional

$$E^{(l)}\left(\{\mathbf{x}\}^{(l)}\right) = \left\|[\mathbf{A}]\{\mathbf{x}\}^{(l)} - \{\mathbf{b}\}\right\|$$

Scalar Coefficient, which Minimizes the Error Functional /
Skalarer Koeffizient, welcher das Fehlerfunktional minimiert

$$\alpha^{(l)} = -\frac{\left\langle [\mathbf{A}]\{\mathbf{p}\}^{(l)}, \{\mathbf{r}\}^{(l-1)} \right\rangle}{\left\|[\mathbf{A}]\{\mathbf{p}\}^{(l)}\right\|^2}$$

with the Residual Vector /
mit dem Fehlervektor

$$\{\mathbf{r}\}^{(l)} = [\mathbf{A}]\{\mathbf{x}\}^{(l)} - \{\mathbf{b}\}$$

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Non-singular Matrix Equation / Nicht singuläre Matrixgleichung

$$[\mathbf{A}]\{\mathbf{x}\} = \{\mathbf{b}\}$$

Iterative Method / Iterative Methode

$$\{\mathbf{x}\}^{(l)} = \{\mathbf{x}\}^{(l-1)} + \alpha^{(l)}\{\mathbf{p}\}^{(l)} \qquad l = 1, 2, \ldots, L$$

$$\begin{pmatrix}\text{New Approximation /} \\ \text{Neue Approximation}\end{pmatrix}^{(l)} \quad \begin{pmatrix}\text{Old Approximation /} \\ \text{Alte Approximation}\end{pmatrix}^{(l-1)} \quad \begin{pmatrix}\text{Correction Term /} \\ \text{Korrekturterm}\end{pmatrix}^{(l)}$$

⇨ Solution in Form of a Finite Sum / Lösung in Form einer endlichen Summe

$$\{\mathbf{x}\} = \{\mathbf{x}\}^{(0)} + \alpha^{(1)}\{\mathbf{p}\}^{(1)} + \alpha^{(2)}\{\mathbf{p}\}^{(2)} + \cdots + \alpha^{(L)}\{\mathbf{p}\}^{(L)}$$

$\{\mathbf{p}\}^{(i)}$ and $\{\mathbf{p}\}^{(j)}$ are Mutually Conjugate if / $\{\mathbf{p}\}^{(i)}$ und $\{\mathbf{p}\}^{(j)}$ sind gegenseitig konjugiert

⇨

$$\left\langle [\mathbf{A}]\{\mathbf{p}\}^{(i)}, [\mathbf{A}]\{\mathbf{p}\}^{(j)} \right\rangle = 0 \qquad \text{for / für} \qquad i \neq j$$

Conjugate Gradient Method (CG Method) – Convergence / Konjugierte Gradientenmethode (KG Methode) – Konvergenz

$$[\mathbf{A}]_{N\times N}\{\mathbf{x}\}_N=\{\mathbf{b}\}_N \qquad l=0,1,\dots,L \qquad L<N$$

Important Property of the CG Method:

For an arbitrary non-singular $N\times N$ Matrix $[\mathbf{A}]$, the CG algorithm produces in at most N iteration steps (assuming infinite-precision arithmetic). This is a direct consequence of the fact that $L=N$ $\{\mathbf{p}\}$ vectors span the solution space. Finite-step termination is a significant advantage of the CG method over other iterative algorithms.

Wichtige Eigenschaft der CG Methode:

Für eine beliebige nicht-singuläre $N\times N$-Matrix $[\mathbf{A}]$, der CG-Algorithmus produziert in höchstens N-Iterationsschritten (bei unendlich genauer Arithmetik). Dies ist eine direkte Konsequenz des Faktum, dass $L=N$ $\{\mathbf{p}\}$-Vektoren den Lösungsraum aufspannen. Dass die Lösung in einer endlichen Anzahl von Iterationsschritten generiert wird, ist ein entscheidender Vorteil der KG-Methode gegenüber anderen iterativen Algorithmen.

Convergence: Yes or No? / Konvergenz: Ja oder Nein?

$$\left\|\{\mathbf{x}\}-\{\mathbf{x}\}^{(l)}\right\|\le\left\|\{\mathbf{x}\}-\{\mathbf{x}\}^{(m)}\right\| \qquad l>m$$

$$\{\mathbf{e}\}^{(l)}\le\left\|\{\mathbf{x}\}-\{\mathbf{x}\}^{(l)}\right\|$$

$$R^{(l)}=\frac{\left\|\{\mathbf{r}\}^{(l)}\right\|}{\left\|\{\mathbf{b}\}^{(l)}\right\|}=\frac{\left\|[\mathbf{A}]\{\mathbf{x}\}^{(l)}-\{\mathbf{b}\}\right\|}{\left\|\{\mathbf{b}\}^{(l)}\right\|} \qquad R^{(l)}<10^{-4}$$

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Initialization / Initialisierung $(l = 0)$

Guess / Schätze $\{\mathbf{x}\}^{(0)}$

$$\{\mathbf{r}\}^{(0)} = [\mathbf{A}]\{\mathbf{x}\}^{(0)} - \{\mathbf{b}\}$$

$$\{\mathbf{p}\}^{(1)} = -[\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(0)}$$

Iterate / Iteriere $(l = 1, 2, \ldots)$

$$\alpha^{(l)} = \frac{\left\langle [\mathbf{A}]\{\mathbf{p}\}^{(l)}, \{\mathbf{r}\}^{(l-1)} \right\rangle}{\left\| [\mathbf{A}]\{\mathbf{p}\}^{(l)} \right\|^2} = \frac{\left\| [\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l-1)} \right\|}{\left\| [\mathbf{A}]\{\mathbf{p}\}^{(l)} \right\|^2}$$

$$\{\mathbf{x}\}^{(l)} = \{\mathbf{x}\}^{(l-1)} + \alpha^{(l)}\{\mathbf{p}\}^{(l)}$$

$$\{\mathbf{r}\}^{(l)} = [\mathbf{A}]\{\mathbf{x}\}^{(l)} - \{\mathbf{b}\} = \{\mathbf{r}\}^{(l-1)} + \alpha^{(l)}[\mathbf{A}]\{\mathbf{p}\}^{(l)}$$

$$R^{(l)} = \frac{\left\| \{\mathbf{r}\}^{(l)} \right\|}{\left\| \{\mathbf{b}\}^{(l)} \right\|} = \frac{\left\| [\mathbf{A}]\{\mathbf{x}\}^{(l)} - \{\mathbf{b}\} \right\|}{\left\| \{\mathbf{b}\}^{(l)} \right\|}$$

Stop here, if the error falls below some predefined value!

$$\beta^{(l)} = \frac{\left\langle [\mathbf{A}]^{\mathrm{a}}\left(\{\mathbf{r}\}^{(l)} - \{\mathbf{r}\}^{(l-1)}\right), [\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l)} \right\rangle}{\left\| [\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l-1)} \right\|^2}$$

Polak-Ribière

$$\{\mathbf{p}\}^{(l+1)} = -[\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l)} + \beta^{(l)}\{\mathbf{p}\}^{(l)}$$

Conjugate Gradient Method (CG Method) / Konjugierte Gradientenmethode (KG Methode)

Initialization / Initialisierung $(l = 0)$ Guess / Schätze $\{\mathbf{x}\}^{(0)}$

$$\{\mathbf{r}\}^{(0)} = [\mathbf{A}]\{\mathbf{x}\}^{(0)} - \{\mathbf{b}\}$$

$$\{\mathbf{p}\}^{(1)} = -[\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(0)}$$

Iterate / Iteriere $(l = 1, 2, \ldots)$

$$\alpha^{(l)} = \frac{\left\langle [\mathbf{A}]\{\mathbf{p}\}^{(l)}, \{\mathbf{r}\}^{(l-1)} \right\rangle}{\left\| [\mathbf{A}]\{\mathbf{p}\}^{(l)} \right\|^2} = \frac{\left\| [\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l-1)} \right\|}{\left\| [\mathbf{A}]\{\mathbf{p}\}^{(l)} \right\|^2}$$

$$\{\mathbf{x}\}^{(l)} = \{\mathbf{x}\}^{(l-1)} + \alpha^{(l)}\{\mathbf{p}\}^{(l)}$$

$$\{\mathbf{r}\}^{(l)} = [\mathbf{A}]\{\mathbf{x}\}^{(l)} - \{\mathbf{b}\} = \{\mathbf{r}\}^{(l-1)} + \alpha^{(l)}[\mathbf{A}]\{\mathbf{p}\}^{(l)}$$

$$R^{(l)} = \frac{\left\| \{\mathbf{r}\}^{(l)} \right\|}{\left\| \{\mathbf{b}\}^{(l)} \right\|} = \frac{\left\| [\mathbf{A}]\{\mathbf{x}\}^{(l)} - \{\mathbf{b}\} \right\|}{\left\| \{\mathbf{b}\}^{(l)} \right\|}$$

Stop here, if the error falls below some predefined value!

$$\beta^{(l)} = \frac{\left\| [\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l)} \right\|^2}{\left\| [\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l-1)} \right\|^2}$$

Fletcher-Reeves

$$\{\mathbf{p}\}^{(l+1)} = -[\mathbf{A}]^{\mathrm{a}}\{\mathbf{r}\}^{(l)} + \beta^{(l)}\{\mathbf{p}\}^{(l)}$$

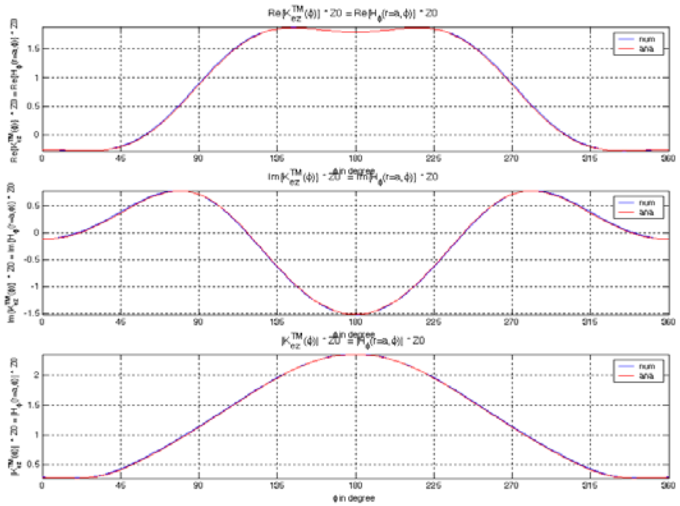
EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Resultate

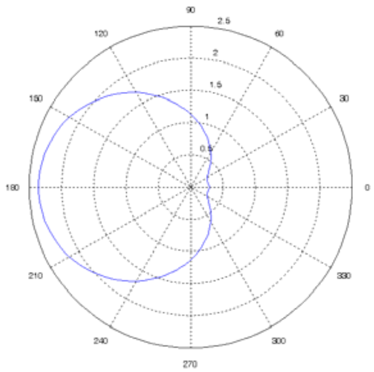
$$Z_0 K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 180°, \omega)$$

$Z_0 \operatorname{Re}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}\right\}$

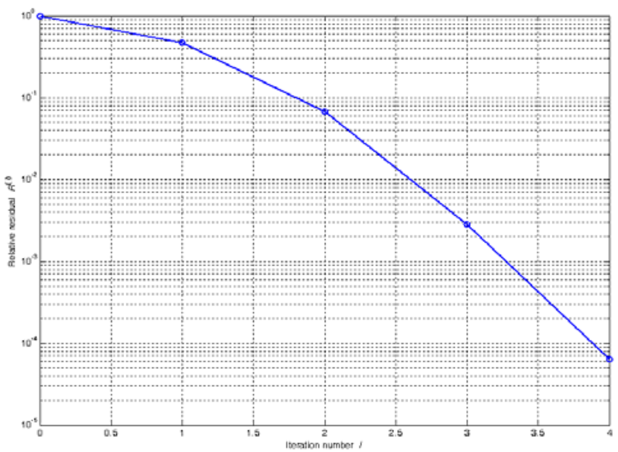
$Z_0 \operatorname{Im}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}\right\}$

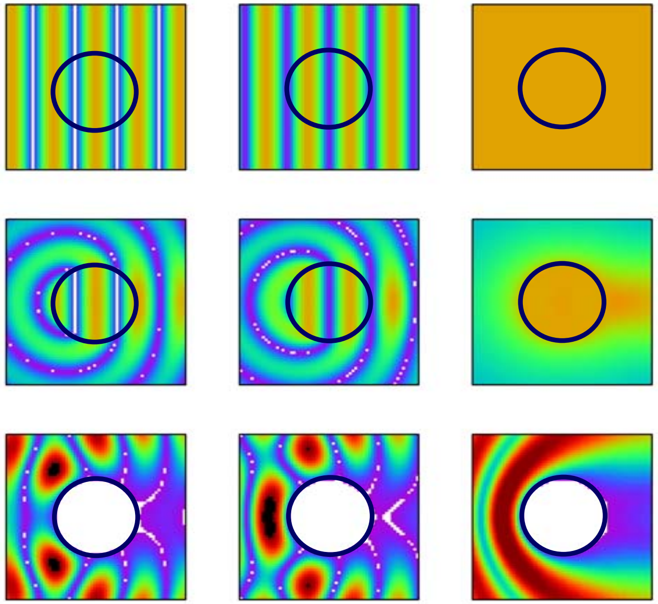
$Z_0 \left|K_{\mathrm{ez}}^{\mathrm{TM}}\right|$



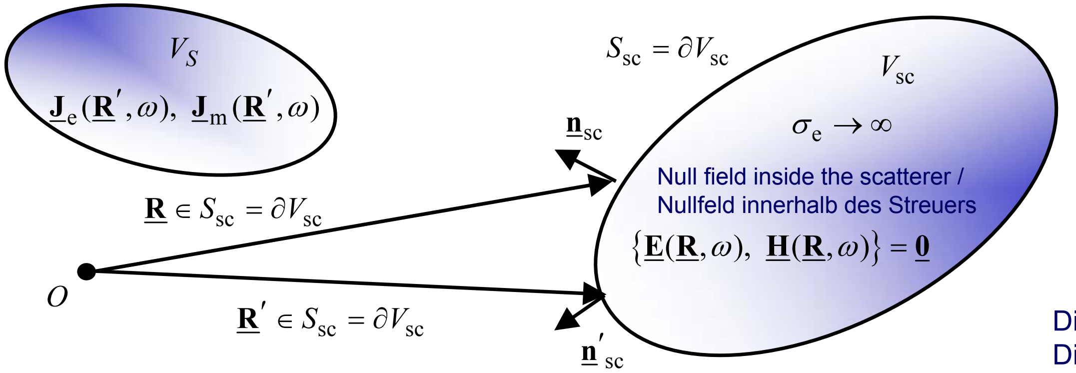


$R^{(l)}$





PEC Scatterer: Franz, Stratton-Chu, and Franz-Larmor Version of EFIE and MFIE / IEL Streuer: Franz, Stratton-Chu und Franz-Larmor Version von EFIE und MFIE



Boundary condition for $\underline{\mathbf{R}} \in S_\mathrm{sc}$
Randbedingung für $\underline{\mathbf{R}} \in S_\mathrm{sc}$

$$\underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\left(\to \underline{\mathbf{K}}_\mathrm{m}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}\right)$$

$$\underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},\omega)$$

Direct scattering problem for PEC scatterer /
Direktes Streuproblem für IEL Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{R}} \in S_\mathrm{sc}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{R}} \in S_\mathrm{sc}$):

Franz version / Franz-Version:

$$\mathrm{j}\omega\mu_0 \mathrm{PV}_\varepsilon\, \underline{\mathbf{n}}_\mathrm{sc} \times \oiint_{\underline{\mathbf{R}}' \in S_\mathrm{sc} = \partial V_\mathrm{sc}} \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega) \cdot \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{E}}^\mathrm{in}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{2}\underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_\mathrm{sc} \times \oiint_{\underline{\mathbf{R}}' \in S_\mathrm{sc} = \partial V_\mathrm{sc}} \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega) \cdot \underline{\underline{\mathbf{G}}}_\mathrm{m}(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{H}}^\mathrm{in}(\underline{\mathbf{R}},\omega)$$

Stratton-Chu version / Stratton-Chu-Version:

$$\underline{\mathbf{n}}_\mathrm{sc} \times \oiint_{\underline{\mathbf{R}}' \in S_\mathrm{sc} = \partial V_\mathrm{sc}} \left[\mathrm{j}\omega\mu_0 \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega) + \tfrac{1}{\mathrm{j}\omega\varepsilon_0} \nabla' \cdot \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega) \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega) \right] \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{E}}^\mathrm{in}(\underline{\mathbf{R}},\omega)$$

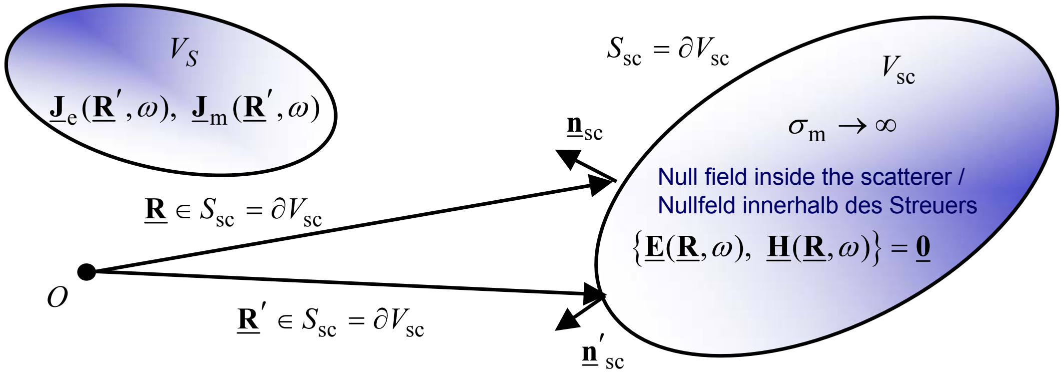
$$\frac{1}{2}\underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_\mathrm{sc} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_\mathrm{sc} = \partial V_\mathrm{sc}} \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega) \times \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{H}}^\mathrm{in}(\underline{\mathbf{R}},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$\frac{1}{\mathrm{j}\omega\varepsilon_0}\underline{\mathbf{n}}_\mathrm{sc} \times \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_\mathrm{sc} = \partial V_\mathrm{sc}} \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{E}}^\mathrm{in}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{2}\underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_\mathrm{sc} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_\mathrm{sc} = \partial V_\mathrm{sc}} \underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}}',\omega)\, G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_\mathrm{sc} \times \underline{\mathbf{H}}^\mathrm{in}(\underline{\mathbf{R}},\omega)$$

PMC Scatterer: Franz, Stratton-Chu, and Franz-Larmor Version of EFIE and MFIE / IML Streuer: Franz, Stratton-Chu und Franz-Larmor Version von EFIE und MFIE



Boundary condition for $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$
Randbedingung für $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\left(\to \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}\right)$$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)$$

Direct scattering problem for PEC scatterer /
Direktes Streuproblem für IEL Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$):

Franz version / Franz-Version:

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}} = \partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}_{\mathrm{m}}(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

$$\mathrm{j}\omega\varepsilon_0 \mathrm{PV}_{\varepsilon}\, \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}} = \partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

Stratton-Chu version / Stratton-Chu-Version:

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}} = \partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) \times \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

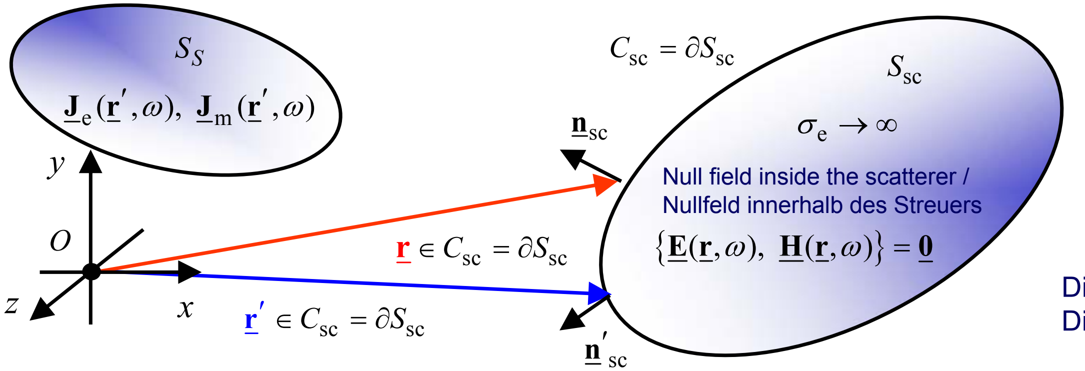
$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}} = \partial V_{\mathrm{sc}}} \left[\mathrm{j}\omega\varepsilon_0 \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega) + \frac{1}{\mathrm{j}\omega\mu_0} \nabla' \bullet \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) \nabla' G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\right] \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}} = \partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{\mathrm{j}\omega\mu_0}\underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}} = \partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}}',\omega) G(\underline{\mathbf{R}} - \underline{\mathbf{R}}',\omega)\, \mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

PEC Scatterer: EFIE and MFIE for the 2-D TM Case and 2-D TE Case / IEL Streuer: EFIE und MFIE für den 2D-TM-Fall und 2D-TE-Fall



Boundary condition for $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$
Randbedingung für $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{0}}$$

$$\left(\to \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{0}}\right)$$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{r}},\omega)$$

Direct scattering problem for a PEC scatterer /
Direktes Streuproblem für einen IEL Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$):

TM Case / TM-Fall:

$$\mathrm{j}\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega)\, G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)\, \mathrm{d}\underline{\mathbf{r}}' = -E_z^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{EFIE}$$

$$\frac{1}{2} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}},\omega) + \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = -\underline{\mathbf{e}}_s \bullet \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{MFIE}$$

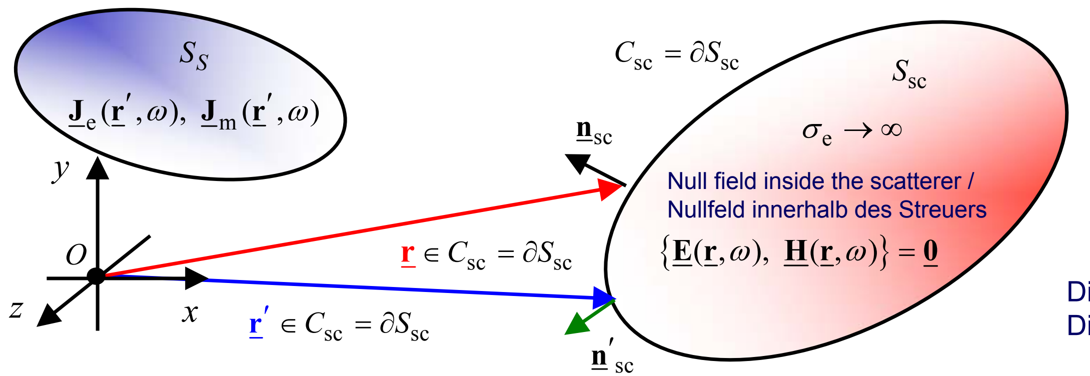
TE Case / TE-Fall

$$\frac{1}{\mathrm{j}\omega\varepsilon_0} \frac{\partial}{\partial n_{\mathrm{sc}}} \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TE}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = \underline{\mathbf{e}}_s \bullet \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{EFIE}$$

$$\frac{1}{2} K_{\mathrm{e}z}^{\mathrm{TE}}(\underline{\mathbf{r}}',\omega) - \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TE}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = H_z^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{MFIE}$$

with / mit $\underline{\mathbf{e}}_s = \underline{\mathbf{n}}'_{\mathrm{sc}} \times \underline{\mathbf{e}}_z = -\underline{\mathbf{e}}_t$

PMC Scatterer: EFIE and MFIE for the 2-D TM Case and 2-D TE Case / IML Streuer: EFIE und MFIE für den 2D-TM-Fall und 2D-TE-Fall



Boundary condition for $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$
Randbedingung für $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{0}}$$
$$\left(\to \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{r}},\omega) = \underline{\mathbf{0}}\right)$$
$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{r}},\omega) = -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{r}},\omega)$$

Direct scattering problem for a PMC scatterer /
Direktes Streuproblem für einen IML Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{r}} \in C_{\mathrm{sc}}$):

TM Case / TM-Fall:

$$\mathrm{j}\omega\mu_0 \frac{\partial}{\partial n_{\mathrm{sc}}} \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{m}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = \underline{\mathbf{e}}_s \bullet \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{EFIE}$$

$$\frac{1}{2} K_{\mathrm{m}z}^{\mathrm{TM}}(\underline{\mathbf{r}},\omega) - \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{m}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = -E_z^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{MFIE}$$

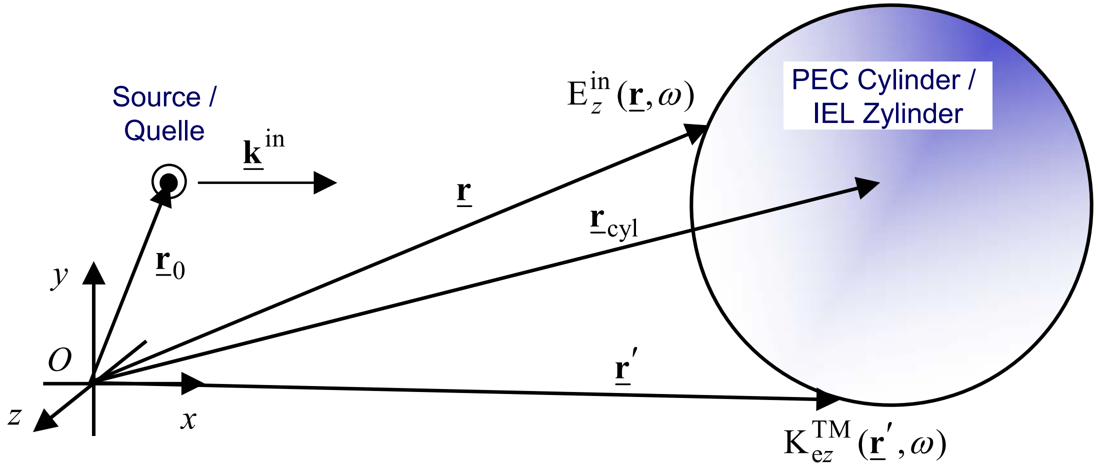
TE Case / TE-Fall

$$\mathrm{j}\omega\varepsilon_0 \frac{\partial}{\partial n_{\mathrm{sc}}} \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{m}z}^{\mathrm{TE}}(\underline{\mathbf{r}}',\omega) G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega) \mathrm{d}\underline{\mathbf{r}}' = H_z^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{EFIE}$$

$$\frac{1}{2} K_{\mathrm{m}z}^{\mathrm{TE}}(\underline{\mathbf{r}}',\omega) + \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{m}z}^{\mathrm{TE}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = \underline{\mathbf{e}}_s \bullet \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{r}},\omega) \qquad \text{MFIE}$$

with / mit $\underline{\mathbf{e}}_s = \underline{\mathbf{n}}'_{\mathrm{sc}} \times \underline{\mathbf{e}}_z = -\underline{\mathbf{e}}_t$

EM Scattering by a Perfectly Electrically Conducting Cylinder: MFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: MFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen



2-D Case /
2D-Fall

$$\underline{\mathbf{R}} = \underbrace{r\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{r}}} + \underbrace{z\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{0}}} = \underline{\mathbf{r}}$$

2-D PEC TM MFIE / 2D-IEL-TM-MFIE

$$\frac{1}{2}K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}},\omega) + \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega) \frac{\partial G(\underline{\mathbf{r}} - \underline{\mathbf{r}}',\omega)}{\partial n_{\mathrm{sc}}} \mathrm{d}\underline{\mathbf{r}}' = -\underline{\mathbf{e}}_s \bullet \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{r}},\omega)$$

This is a *Fredholm integral equation of the 2. kind* in form of a *closed line integral*
for the *unknown* electric surface current density for a *known* incident field. /
Dies ist eine *Fredholmsche Integralgleichung 2. Art* in Form eines *geschlossenen Linienintegrals*
für die *unbekannte* elektrische Flächenladungsdichte für ein *bekanntes* einfallendes Feld.

$$G(\underline{\mathbf{r}} - \underline{\mathbf{r}}',\omega) = \frac{\mathrm{j}}{4}\mathrm{H}_0^{(1)}\left(k_0\left|\underline{\mathbf{r}} - \underline{\mathbf{r}}'\right|\right)$$

PEC Scatterer: Combined Field Integral Equation – CFIE = EFIE and MFIE / IEL Streuer: Kombinierte Feldintegralgleichung – CFIE = EFIE und MFIE

Internal Resonance Problem /
Probleme mit internen Resonanzen

⇩

Ill-Conditioned Matrix Equation – The Matrix Operator has a Null Space at these Resonance Frequencies /
Schlechtgestellte Matrixgleichung – Der Matrixoperator besitzt einen Nullraum bei den Resonanzfrequenzen

⇩

Non-Uniqueness Due to Internal Resonance Problem /
Nichteindeutigkeit wegen den internen Resonanzen

Remedy of the Non-Uniqueness /
Lösung der Nichteindeutigkeit ⇩

CFIE is a Linear Combination of the EFIE and MFIE /
CFIE ist eine linear Kombination von EFIE und MFIE

$$\mathrm{CFIE} = \alpha\ \mathrm{EFIE} + (\alpha - 1)\ Z\ \mathrm{MFIE}$$

with / mit $0 \leq \alpha \leq 1 \qquad \rightarrow \alpha = 0.2$

PEC Scatterer: Combined Field Integral Equation – CFIE = EFIE and MFIE / IEL Streuer: Kombinierte Feldintegralgleichung – CFIE = EFIE und MFIE

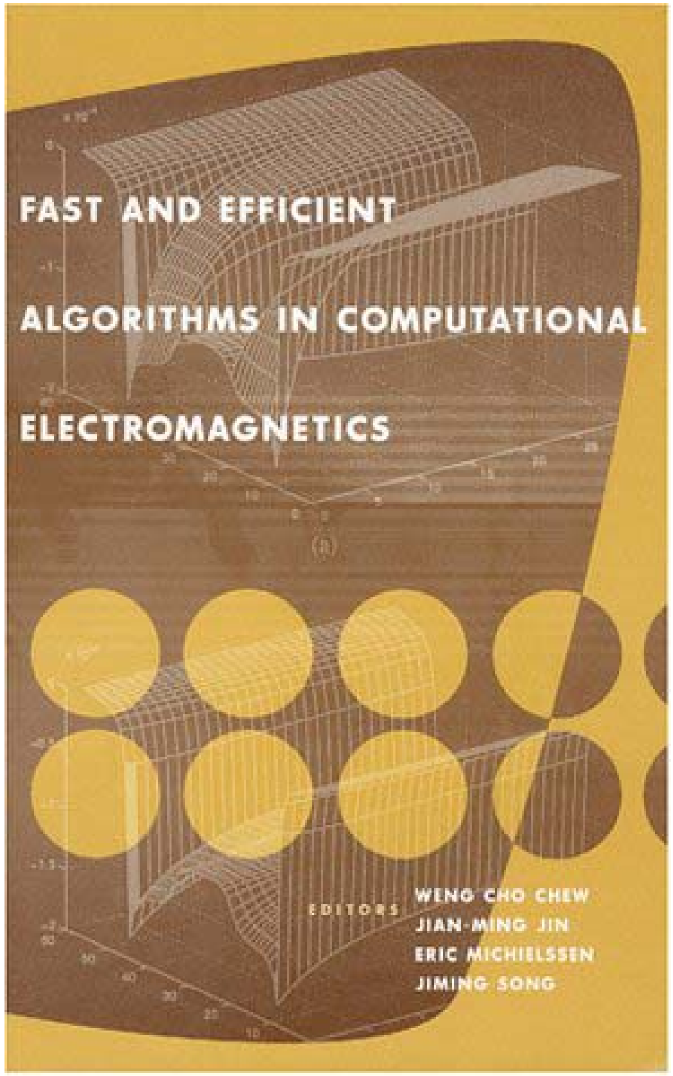
CFIE is a Linear Combination of the EFIE and MFIE /
CFIE ist eine linear Kombination von EFIE und MFIE

$$\mathrm{CFIE} = \alpha \;\mathrm{EFIE} + (\alpha - 1)\, Z \;\mathrm{MFIE}$$

with /
mit $\quad 0 \le \alpha \le 1 \qquad \rightarrow \alpha = 0.2$

Antilla, G., N. G. Alexopoulos:
Scattering from Complex Three-Dimensional Geometries using
a Curvilinear Hybrid Finite-Element-Integral Equation Approach,
Optical Soc. America, Vol. 11, pp. 1445-1457, April 1994.

PEC Scatterer: Combined Field Integral Equation – CFIE = EFIE and MFIE / IEL Streuer: Kombinierte Feldintegralgleichung – CFIE = EFIE und MFIE



PEC Scatterer: Combined Field Integral Equation – CFIE = EFIE and MFIE / IEL Streuer: Kombinierte Feldintegralgleichung – CFIE = EFIE und MFIE

CFIE is a Linear Combination of the EFIE and MFIE /
CFIE ist eine linear Kombination von EFIE und MFIE

$$\mathrm{CFIE} = \alpha\ \mathrm{EFIE} + (\alpha - 1)\ Z\ \mathrm{MFIE} \qquad \alpha = 0.2$$

$$\underbrace{\mathrm{j}\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}', \omega)\, G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)\, \mathrm{d}\underline{\mathbf{r}}' = -E_z^{\mathrm{in}}(\underline{\mathbf{r}}, \omega)}_{=\mathrm{EFIE}}$$

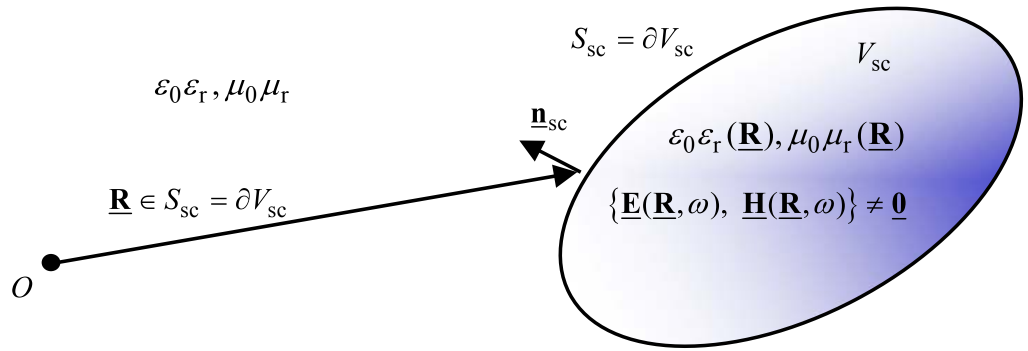
$$\underbrace{\frac{1}{2} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}, \omega) + \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}', \omega) \frac{\partial G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)}{\partial n_{\mathrm{sc}}}\, \mathrm{d}\underline{\mathbf{r}}' = -\underline{\mathbf{e}}_s \bullet \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{r}}, \omega)}_{=\mathrm{MFIE}}$$

$$\alpha\ \mathrm{j}\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}', \omega)\, G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)\, \mathrm{d}\underline{\mathbf{r}}'$$

$$+ (\alpha - 1) Z_0 \frac{1}{2} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}, \omega) + \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} K_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}', \omega) \frac{\partial G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)}{\partial n_{\mathrm{sc}}}\, \mathrm{d}\underline{\mathbf{r}}'$$

$$= -\alpha E_z^{\mathrm{in}}(\underline{\mathbf{r}}, \omega) - (\alpha - 1) Z_0 \underline{\mathbf{e}}_s \bullet \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{r}}, \omega)$$

Penetrable Scatterer: Transition Conditions / Penetrable Streuer: Übergangsbedingungen



Transition Condition for $\underline{\mathbf{R}} \in S_{sc}$/
Übergangsbedingungen für $\underline{\mathbf{R}} \in S_{sc}$

$$\underline{\mathbf{n}}_{sc} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} -\underline{\mathbf{K}}_m(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \times \left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \underline{\mathbf{K}}_e(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \cdot \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \eta_e(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \cdot \left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \eta_m(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \times \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t) - \frac{\varepsilon_r^{(2)}}{\varepsilon_r^{(1)}}\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} -\varepsilon_0 \varepsilon_r^{(2)} \underline{\mathbf{K}}_m(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \times \left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t) - \frac{\mu_r^{(2)}}{\mu_r^{(1)}}\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \mu_0 \mu_r^{(2)} \underline{\mathbf{K}}_e(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \cdot \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \frac{\varepsilon_r^{(1)}}{\varepsilon_r^{(2)}}\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \dfrac{1}{\varepsilon_0 \varepsilon_r^{(2)}} \eta_e(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}}_{sc} \cdot \left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t) - \frac{\mu_r^{(1)}}{\mu_r^{(2)}}\underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \dfrac{1}{\mu_0 \mu_r^{(2)}} \eta_m(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) + \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = -\mathrm{j}\omega\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},\omega)$$

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) = \varepsilon_0\varepsilon_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$
$$\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = \mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)$$

$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\nabla \times \underbrace{\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega)}_{=\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)} - \nabla \times \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)$$

$$\nabla \times \nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = -\mathrm{j}\omega\nabla \times \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega)}_{=\varepsilon_0\varepsilon_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)} + \nabla \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$
\begin{aligned}
&\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\\
&=\mathrm{j}\omega\nabla\times\underbrace{\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega)}_{=\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)}-\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)\\
&=\mathrm{j}\omega\mu_0\nabla\times\left[\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)\right]-\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)\\
&=\mathrm{j}\omega\mu_0\left\{\mu_\mathrm{r}(\underline{\mathbf{R}})\underbrace{\left[\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)\right]}_{-\mathrm{j}\omega\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega)}_{=\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)}+\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega)}+\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\underbrace{\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)}_{=\frac{1}{\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)}\right\}-\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)\\
&=\mathrm{j}\omega\mu_0\left\{\left[-\mathrm{j}\omega\mu_\mathrm{r}(\underline{\mathbf{R}})\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega)\right]+\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\left[\frac{1}{\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)\right]\right\}\\
&=\omega^2\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\\
&\quad+\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega)+\frac{1}{\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)
\end{aligned}
$$

$$
\frac{1}{\mu_\mathrm{r}(\underline{\mathbf{R}})}\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]=\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})
$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$\nabla \ln \mu_{\mathrm{r}}(\underline{\mathbf{R}}) = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right) \ln \mu_{\mathrm{r}}(x,y,z)$$

$$= \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} \ln \mu_{\mathrm{r}}(x,y,z) + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} \ln \mu_{\mathrm{r}}(x,y,z) + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \ln \mu_{\mathrm{r}}(x,y,z) \right)$$

$$\frac{\partial}{\partial x} \ln \underbrace{\mu_{\mathrm{r}}(x,y,z)}_{=\alpha} = \frac{\partial}{\partial x} \frac{\partial \alpha}{\partial \alpha} \ln \underbrace{\mu_{\mathrm{r}}(x,y,z)}_{=\alpha} = \frac{\partial}{\partial \alpha} \frac{\partial \alpha}{\partial x} \ln \underbrace{\mu_{\mathrm{r}}(x,y,z)}_{=\alpha} = \underbrace{\frac{\partial}{\partial \alpha} \ln \underbrace{\alpha}_{=\mu_{\mathrm{r}}(x,y,z)}}_{=\frac{1}{\alpha}} \frac{\partial \alpha}{\partial x} = \frac{1}{\alpha} \frac{\partial \alpha}{\partial x} = \frac{1}{\mu_{\mathrm{r}}(x,y,z)} \frac{\partial}{\partial x} \mu_{\mathrm{r}}(x,y,z)$$

$$\frac{\partial}{\partial x} \ln \mu_{\mathrm{r}}(x,y,z) = \frac{1}{\mu_{\mathrm{r}}(x,y,z)} \frac{\partial}{\partial x} \mu_{\mathrm{r}}(x,y,z)$$

$$\frac{\partial}{\partial y} \ln \mu_{\mathrm{r}}(x,y,z) = \frac{1}{\mu_{\mathrm{r}}(x,y,z)} \frac{\partial}{\partial y} \mu_{\mathrm{r}}(x,y,z)$$

$$\frac{\partial}{\partial z} \ln \mu_{\mathrm{r}}(x,y,z) = \frac{1}{\mu_{\mathrm{r}}(x,y,z)} \frac{\partial}{\partial z} \mu_{\mathrm{r}}(x,y,z)$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$\frac{\partial}{\partial x}\ln\mu_{\mathrm{r}}(x,y,z)=\frac{1}{\mu_{\mathrm{r}}(x,y,z)}\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)$$
$$\frac{\partial}{\partial y}\ln\mu_{\mathrm{r}}(x,y,z)=\frac{1}{\mu_{\mathrm{r}}(x,y,z)}\frac{\partial}{\partial y}\mu_{\mathrm{r}}(x,y,z)$$
$$\frac{\partial}{\partial z}\ln\mu_{\mathrm{r}}(x,y,z)=\frac{1}{\mu_{\mathrm{r}}(x,y,z)}\frac{\partial}{\partial z}\mu_{\mathrm{r}}(x,y,z)$$

$$\begin{aligned}
\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}}) &= \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x}\ln\mu_{\mathrm{r}}(x,y,z)+\underline{\mathbf{e}}_y\frac{\partial}{\partial y}\ln\mu_{\mathrm{r}}(x,y,z)+\underline{\mathbf{e}}_z\frac{\partial}{\partial z}\ln\mu_{\mathrm{r}}(x,y,z)\right)\\
&= \left(\underline{\mathbf{e}}_x\frac{1}{\mu_{\mathrm{r}}(x,y,z)}\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)+\underline{\mathbf{e}}_y\frac{1}{\mu_{\mathrm{r}}(x,y,z)}\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)+\underline{\mathbf{e}}_z\frac{1}{\mu_{\mathrm{r}}(x,y,z)}\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)\right)\\
&= \frac{1}{\mu_{\mathrm{r}}(x,y,z)}\left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)+\underline{\mathbf{e}}_y\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)+\underline{\mathbf{e}}_z\frac{\partial}{\partial x}\mu_{\mathrm{r}}(x,y,z)\right)\\
&= \frac{1}{\mu_{\mathrm{r}}(x,y,z)}\left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x}+\underline{\mathbf{e}}_y\frac{\partial}{\partial x}+\underline{\mathbf{e}}_z\frac{\partial}{\partial x}\right)\mu_{\mathrm{r}}(x,y,z)\\
&= \frac{1}{\mu_{\mathrm{r}}(\underline{\mathbf{R}})}\nabla\mu_{\mathrm{r}}(\underline{\mathbf{R}})
\end{aligned}$$

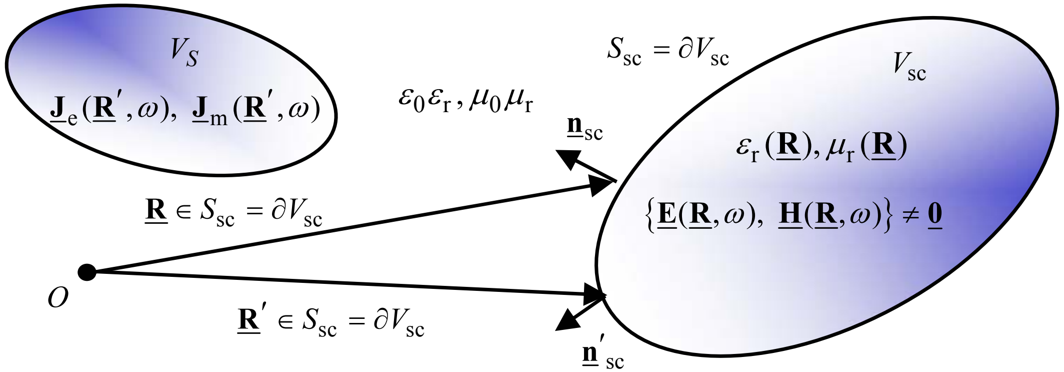
$$\frac{1}{\mu_{\mathrm{r}}(\underline{\mathbf{R}})}\left[\nabla\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]=\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

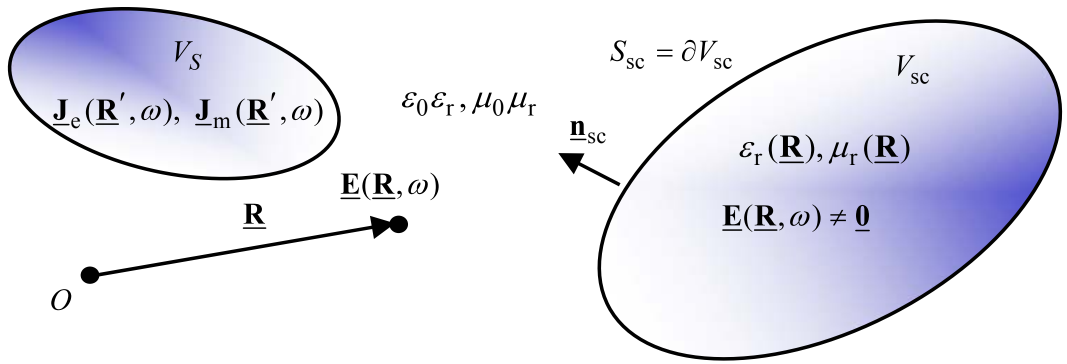
$$\frac{1}{\mu_\mathrm{r}(\underline{\mathbf{R}})}\left[\nabla \mu_\mathrm{r}(\underline{\mathbf{R}})\right] = \nabla \ln \mu_\mathrm{r}(\underline{\mathbf{R}})$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \omega^2\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + \frac{1}{\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$
$$+\,\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega) + \frac{1}{\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})}\left[\nabla\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega) - \nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) - \omega^2\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$
$$+\,\mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega) + \frac{1}{\mu_0}\left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega) - \nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)$$



Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem



$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) - \omega^2\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

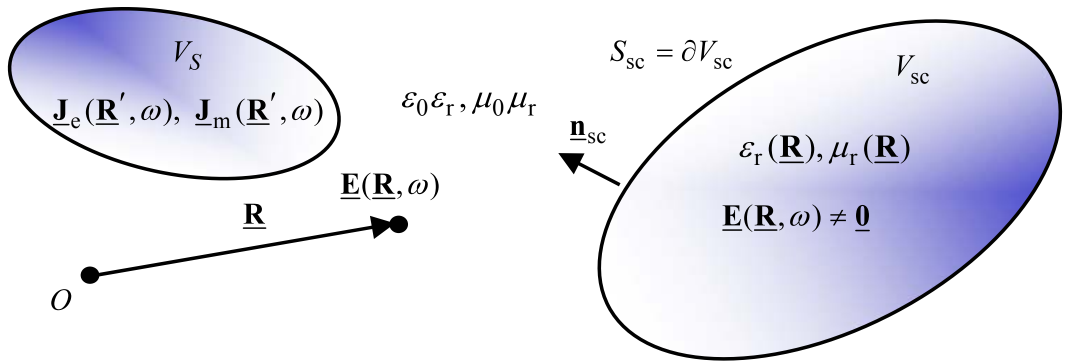
$$+\,\mathrm{j}\omega\mu_0\underbrace{\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega)}_{=\mu_\mathrm{r}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega)} + \frac{1}{\mu_0}\underbrace{\left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)}_{=0} - \nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)$$

$$\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega) = \begin{cases} \mu_\mathrm{r}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega) & \mu_\mathrm{r}(\underline{\mathbf{R}}) = \mu_\mathrm{r} \quad \underline{\mathbf{R}} = V_s \\ \underline{\mathbf{0}} & \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}} \quad \underline{\mathbf{R}} \neq V_s \end{cases}$$

$$\left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega) = \begin{cases} \underline{\mathbf{0}} & \left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right] = \underline{\mathbf{0}} \quad \underline{\mathbf{R}} \in V_s \\ \underline{\mathbf{0}} & \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}} \quad \underline{\mathbf{R}} \in V_\mathrm{sc} \end{cases}$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) - \omega^2\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + \mathrm{j}\omega\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},\omega) - \nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},\omega)$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem



$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-\omega^2\mu_0\mu_r(\underline{\mathbf{R}})\varepsilon_0\varepsilon_r(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_r(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\mathrm{j}\omega\mu_0\mu_r(\underline{\mathbf{R}})\underline{\mathbf{J}}_e(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_m(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{sc}(\underline{\mathbf{R}},\omega)$$

$$\begin{aligned}&\nabla\times\nabla\times\left[\underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{sc}(\underline{\mathbf{R}},\omega)\right]-\omega^2\mu_0\mu_r(\underline{\mathbf{R}})\varepsilon_0\varepsilon_r(\underline{\mathbf{R}})\left[\underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{sc}(\underline{\mathbf{R}},\omega)\right]\\&=\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_r(\underline{\mathbf{R}})\right]\times\nabla\times\left[\underline{\mathbf{E}}^{in}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{sc}(\underline{\mathbf{R}},\omega)\right]+\mathrm{j}\omega\mu_0\mu_r(\underline{\mathbf{R}})\underline{\mathbf{J}}_e(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_m(\underline{\mathbf{R}},\omega)\end{aligned}$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$
\begin{aligned}
&\nabla\times\nabla\times\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]-k^2\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]\\
&=\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]+\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)\\
&\quad+k^2(\underline{\mathbf{R}})\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]-k^2\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]\\
&=\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]+\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)\\
&\quad+\left[k^2(\underline{\mathbf{R}})-k^2\right]\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]
\end{aligned}
$$

$$
\begin{aligned}
&\nabla\times\nabla\times\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]-k^2\left[\underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}},\omega)\right]\\
&=\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\left[k^2(\underline{\mathbf{R}})-k^2\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)
\end{aligned}
$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$\nabla\times\nabla\times\left[\underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)\right]-k^2\left[\underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)+\underline{\mathbf{E}}^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)\right]$$

$$=\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\left[k^2(\underline{\mathbf{R}})-k^2\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

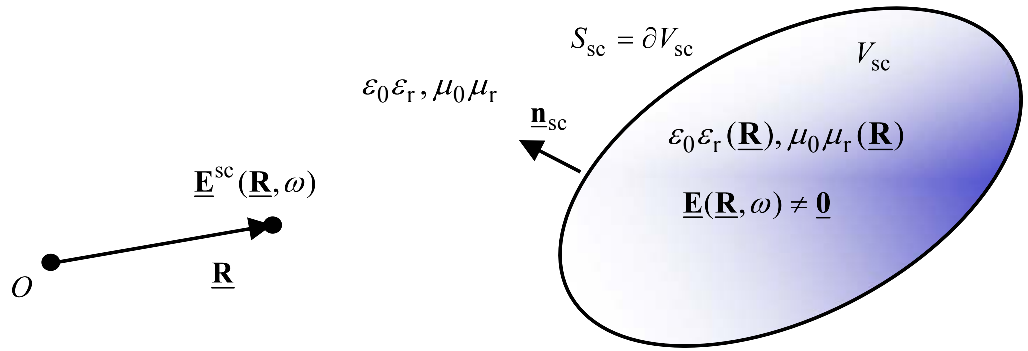
$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-k^2\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$
$$+\left[k^2(\underline{\mathbf{R}})-k^2\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+k^2(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-k^2(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}(\underline{\mathbf{R}})\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)+\frac{1}{\mathrm{j}\omega\mu_0}\left[\nabla\ln\mu_{\mathrm{r}}(\underline{\mathbf{R}})\right]\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)-\omega^2\mu_0\mu_{\mathrm{r}}\varepsilon_0\varepsilon_{\mathrm{r}}\underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega)$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem



$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) = \mathrm{j}\omega \underline{\mathbf{B}}(\underline{\mathbf{R}}, \omega)$$

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}}, \omega) = -\mathrm{j}\omega \underline{\mathbf{D}}(\underline{\mathbf{R}}, \omega) \qquad \underline{\mathbf{D}}(\underline{\mathbf{R}}, \omega) = \varepsilon_0 \varepsilon_r (\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega)$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}, \omega) = \rho_e (\underline{\mathbf{R}}, \omega) \qquad \underline{\mathbf{B}}(\underline{\mathbf{R}}, \omega) = \mu_0 \mu_r (\underline{\mathbf{R}}) \underline{\mathbf{H}}(\underline{\mathbf{R}}, \omega)$$

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}}, \omega) = \rho_m (\underline{\mathbf{R}}, \omega)$$

$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) - \omega^2 \varepsilon_0 \varepsilon_r (\underline{\mathbf{R}}) \mu_0 \mu_r (\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) = \mu_0 \nabla \ln \mu_r (\underline{\mathbf{R}}) \times \left[\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) \right]$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-\omega^2\varepsilon_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\mu_0\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mu_0\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\times\left[\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right]$$

Equivalent Current Densities / Äquivalente Stromdichten

$$\underline{\mathbf{J}}_\mathrm{e}^\mathrm{eq}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\varepsilon_0\varepsilon_\mathrm{r}\left[1-\frac{\varepsilon_\mathrm{r}(\underline{\mathbf{R}})}{\varepsilon_\mathrm{r}}\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{J}}_\mathrm{m}^\mathrm{eq}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\mu_0\mu_\mathrm{r}\left[1-\frac{\mu_\mathrm{r}(\underline{\mathbf{R}})}{\mu_\mathrm{r}}\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)+\omega^2\varepsilon_0\varepsilon_\mathrm{r}\mu_0\mu_\mathrm{r}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mathrm{j}\omega\mu_0\mu_\mathrm{r}\underline{\mathbf{J}}_\mathrm{e}^\mathrm{eq}(\underline{\mathbf{R}},\omega)-\nabla\times\underline{\mathbf{J}}_\mathrm{m}^\mathrm{eq}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mu_0\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\times\left[\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right]+\omega^2\varepsilon_0\mu_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-\omega^2\varepsilon_0\mu_0\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mu_0\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\times\left[\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right]+\omega^2\varepsilon_0\mu_0\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\mu_\mathrm{r}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-\omega^2\varepsilon_0\mu_0\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)-\omega^2\varepsilon_0\mu_0\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)=\mu_0\nabla\ln\mu_\mathrm{r}(\underline{\mathbf{R}})\times\left[\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right]+\omega^2\varepsilon_0\mu_0\left[\varepsilon_\mathrm{r}(\underline{\mathbf{R}})\mu_\mathrm{r}(\underline{\mathbf{R}})-\varepsilon_\mathrm{r}\mu_\mathrm{r}\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem

$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) - \omega^2 \varepsilon_0 \mu_0 \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \mu_0 \nabla \ln \mu_\mathrm{r}(\underline{\mathbf{R}}) \times \left[\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right] + \omega^2 \varepsilon_0 \mu_0 \left[\varepsilon_\mathrm{r}(\underline{\mathbf{R}}) \mu_\mathrm{r}(\underline{\mathbf{R}}) - \varepsilon_\mathrm{r}\mu_\mathrm{r}\right] \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

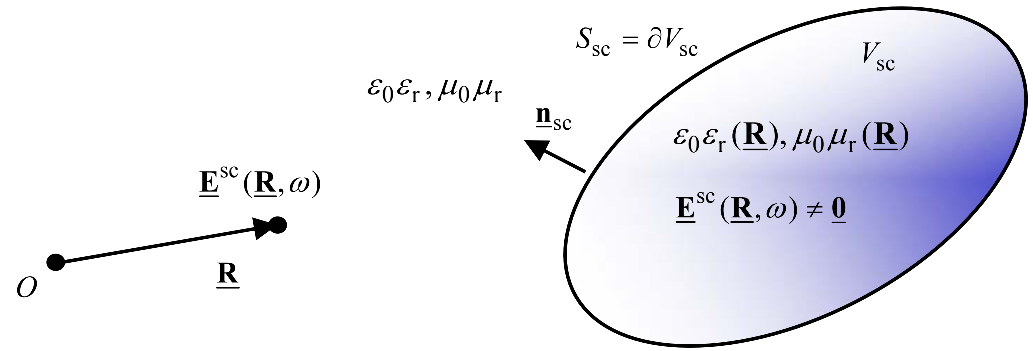
Equivalent Current Densities / Äquivalente Stromdichten

$$\underline{\mathbf{J}}_\mathrm{e}^\mathrm{eq}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\varepsilon_0\varepsilon_\mathrm{r}\left[1 - \frac{\varepsilon_\mathrm{r}(\underline{\mathbf{R}})}{\varepsilon_\mathrm{r}}\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\underline{\mathbf{J}}_\mathrm{m}^\mathrm{eq}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\mu_0\mu_\mathrm{r}\left[1 - \frac{\mu_\mathrm{r}(\underline{\mathbf{R}})}{\mu_\mathrm{r}}\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)$$

$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + \omega^2 \varepsilon_0 \varepsilon_\mathrm{r} \mu_0 \mu_\mathrm{r} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\mu_0\mu_\mathrm{r} \underline{\mathbf{J}}_\mathrm{e}^\mathrm{eq}(\underline{\mathbf{R}},\omega) - \nabla \times \underline{\mathbf{J}}_\mathrm{m}^\mathrm{eq}(\underline{\mathbf{R}},\omega)$$

Penetrable Scatterer: Representation Theorem / Penetrable Streuer: Repräsentationstheorem



$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) - \omega^2 \varepsilon_0 \varepsilon_r(\underline{\mathbf{R}}) \mu_0 \mu_r(\underline{\mathbf{R}}) \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) = \mu_0 \nabla \ln \mu_r(\underline{\mathbf{R}}) \times \left[\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega)\right]$$

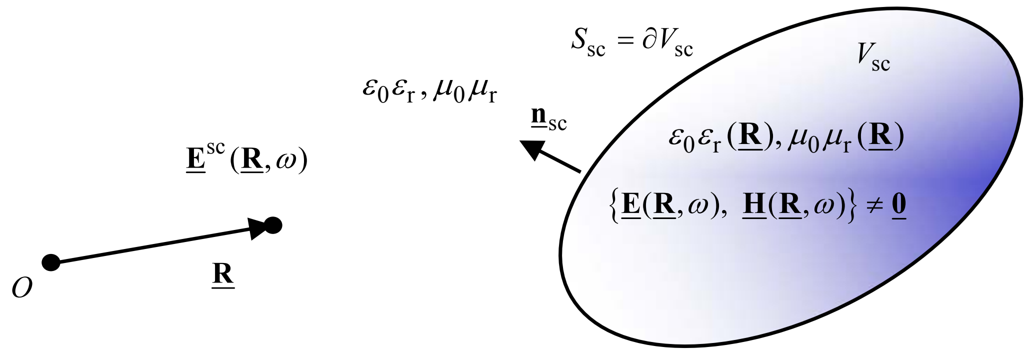
Equivalent Current Densities / Äquivalente Stromdichten

$$\underline{\mathbf{J}}_e^{eq}(\underline{\mathbf{R}}, \omega) = j\omega \varepsilon_0 \varepsilon_r \left[1 - \frac{\varepsilon_r(\underline{\mathbf{R}})}{\varepsilon_r}\right] \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega)$$

$$\underline{\mathbf{J}}_m^{eq}(\underline{\mathbf{R}}, \omega) = j\omega \mu_0 \mu_r \left[1 - \frac{\mu_r(\underline{\mathbf{R}})}{\mu_r}\right] \underline{\mathbf{H}}(\underline{\mathbf{R}}, \omega)$$

$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) + \omega^2 \varepsilon_0 \varepsilon_r \mu_0 \mu_r \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) = j\omega \mu_0 \mu_r \underline{\mathbf{J}}_e^{eq}(\underline{\mathbf{R}}, \omega) - \nabla \times \underline{\mathbf{J}}_m^{eq}(\underline{\mathbf{R}}, \omega)$$

Penetrable Scatterer: Data Equation and Lippmann-Schwinger Equation / Penetrable Streuer: Datengleichung und Lippmann-Schwinger Gleichung



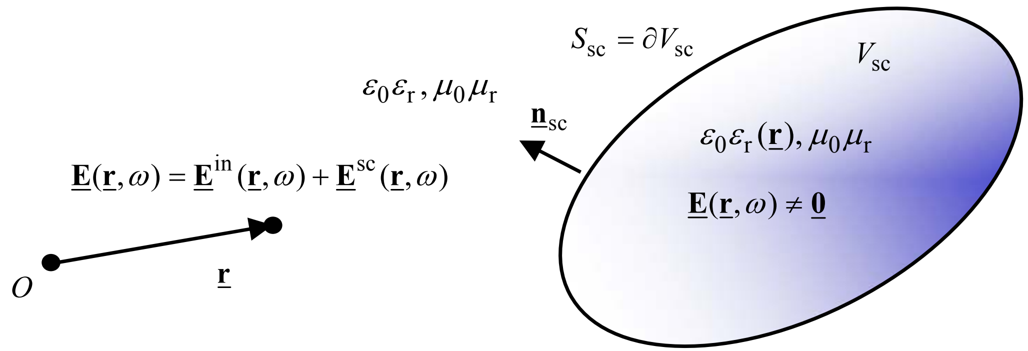
$$\nabla \times \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) + \omega^2 \varepsilon_0 \varepsilon_{\text{r}} \mu_0 \mu_{\text{r}} \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) = \mathrm{j} \omega \mu_0 \mu_{\text{r}} \underline{\mathbf{J}}_{\text{e}}^{\text{eq}}(\underline{\mathbf{R}}, \omega) - \nabla \times \underline{\mathbf{J}}_{\text{m}}^{\text{eq}}(\underline{\mathbf{R}}, \omega)$$

$$\chi(\underline{\mathbf{R}}) = \frac{k^2(\underline{\mathbf{R}})}{k^2} - 1$$

$$\underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}}, \omega) = k^2 \iiint_{\underline{\mathbf{R}}' \in V} \chi(\underline{\mathbf{R}}') \underline{\mathbf{E}}(\underline{\mathbf{R}}', \omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) \, \mathrm{d}^3 \underline{\mathbf{R}}'$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) = \underline{\mathbf{E}}^{\text{in}}(\underline{\mathbf{R}}, \omega) + \underbrace{k^2 \iiint_{\underline{\mathbf{R}}' \in V} \chi(\underline{\mathbf{R}}') \underline{\mathbf{E}}(\underline{\mathbf{R}}', \omega) \bullet \underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}} - \underline{\mathbf{R}}', \omega) \, \mathrm{d}^3 \underline{\mathbf{R}}'}_{= \underline{\mathbf{E}}^{\text{sc}}(\underline{\mathbf{R}}, \omega)}$$

Penetrable Scatterer: Data Equation and Lippmann-Schwinger Equation – 2-D TM Case/ Penetrable Streuer: Datengleichung und Lippmann-Schwinger Gleichung – 2-D TM Case

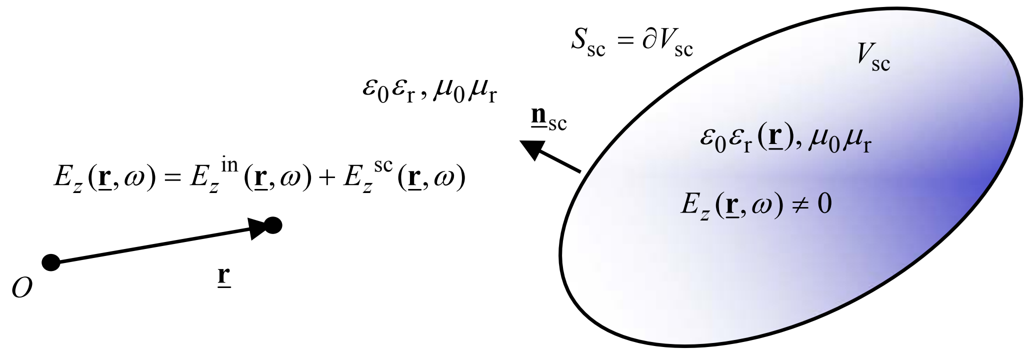


$$\Delta E_z(\underline{\mathbf{r}},\omega) + \omega^2 \varepsilon_0 \varepsilon_r(\underline{\mathbf{r}}) \mu_0 \mu_r E_z(\underline{\mathbf{r}},\omega) = 0$$

$$\Delta E_z(\underline{\mathbf{r}},\omega) + \underbrace{\omega^2 \varepsilon_0 \varepsilon_r \mu_0 \mu_r}_{=k^2} E_z(\underline{\mathbf{r}},\omega) = \underbrace{\omega^2 \varepsilon_0 \varepsilon_r \mu_0 \mu_r}_{=k^2} E_z(\underline{\mathbf{r}},\omega) - \underbrace{\omega^2 \varepsilon_0 \varepsilon_r(\underline{\mathbf{r}}) \mu_0 \mu_r}_{=k^2(\underline{\mathbf{r}})} E_z(\underline{\mathbf{r}},\omega)$$

$$\begin{aligned}\Delta E_z(\underline{\mathbf{r}},\omega) + k^2 E_z(\underline{\mathbf{r}},\omega) &= k^2 E_z(\underline{\mathbf{r}},\omega) - k^2(\underline{\mathbf{r}}) E_z(\underline{\mathbf{r}},\omega) \\ &= \left[k^2 - k^2(\underline{\mathbf{r}}) \right] E_z(\underline{\mathbf{r}},\omega) \\ &= k^2 \left[1 - \frac{k^2(\underline{\mathbf{r}})}{k^2} \right] E_z(\underline{\mathbf{r}},\omega)\end{aligned}$$

Penetrable Scatterer: Data Equation and Lippmann-Schwinger Equation / Penetrable Streuer: Datengleichung und Lippmann-Schwinger Gleichung



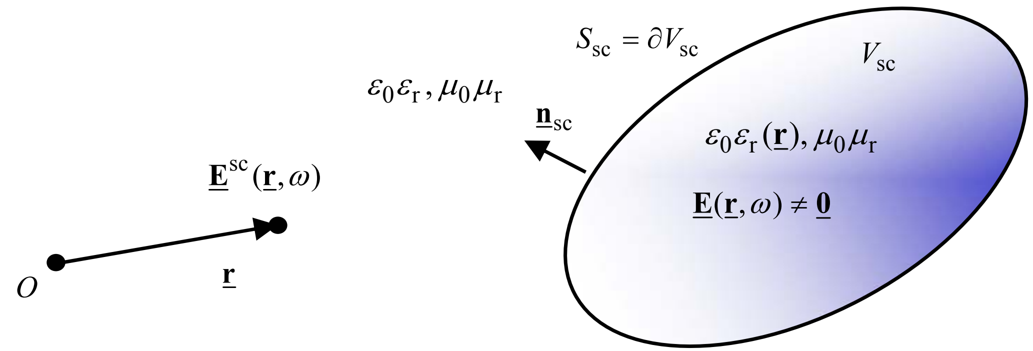
$$\Delta E_z(\underline{\mathbf{r}}, \omega) + k^2 E_z(\underline{\mathbf{r}}, \omega) = k^2 \underbrace{\left[1 - \frac{k^2(\underline{\mathbf{r}})}{k^2} \right]}_{= -\chi(\underline{\mathbf{r}})} E_z(\underline{\mathbf{r}}, \omega)$$

$$\Delta E_z(\underline{\mathbf{r}}, \omega) + k^2 E_z(\underline{\mathbf{r}}, \omega) = -k^2 \chi(\underline{\mathbf{r}}) E_z(\underline{\mathbf{r}}, \omega)$$

$$E_z^{\mathrm{sc}}(\underline{\mathbf{r}}, \omega) = k^2 \iint_{\underline{\mathbf{r}}' \in S} \chi(\underline{\mathbf{r}}')\, E_z(\underline{\mathbf{r}}', \omega)\, G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)\, \mathrm{d}^2 \underline{\mathbf{r}}'$$

$$E_z(\underline{\mathbf{r}}, \omega) = E_z^{\mathrm{in}}(\underline{\mathbf{r}}, \omega) + k^2 \iint_{\underline{\mathbf{r}}' \in S} \chi(\underline{\mathbf{r}}')\, E_z(\underline{\mathbf{r}}', \omega)\, G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)\, \mathrm{d}^2 \underline{\mathbf{r}}'$$

Penetrable Scatterer: Data Equation and Lippmann-Schwinger Equation / Penetrable Streuer: Datengleichung und Lippmann-Schwinger Gleichung



$$E_z^{sc}(\underline{\mathbf{r}}, \omega) = k^2 \iint_{\underline{\mathbf{r}}' \in S} G(\underline{\mathbf{r}} - \underline{\mathbf{r}}', \omega)\, \chi(\underline{\mathbf{r}}')\, E_z(\underline{\mathbf{r}}', \omega)\, \mathrm{d}^2 \underline{\mathbf{r}}'$$

$$\left\{ E_z^{sc} \right\} = [\mathbf{G}][\chi]\{E_z\}$$

$$[\mathbf{G}] \rightarrow G_{mn} = \begin{cases} \mathrm{j}\dfrac{\pi}{2} ka\, \mathrm{J}_1(ka)\, \mathrm{H}_0^{(1)}\left(k\left|\underline{\mathbf{r}}_m - \underline{\mathbf{r}}_n\right|\right) & m \neq n \\ -1 + \mathrm{j}\dfrac{\pi}{2} ka\, \mathrm{H}_1^{(1)}(ka) & m = n \end{cases}$$

End of 6th Lecture /
Ende der 6. Vorlesung